A

COMPLEAT SYSTEM

OF

OPTICKS

In Four BOOKS, viz.

A Popular, a Mathematical, a Mechanical, and a Philosophical Treatise.

To which are added,

REMARKS upon the Whole.

BY

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VOL. II.

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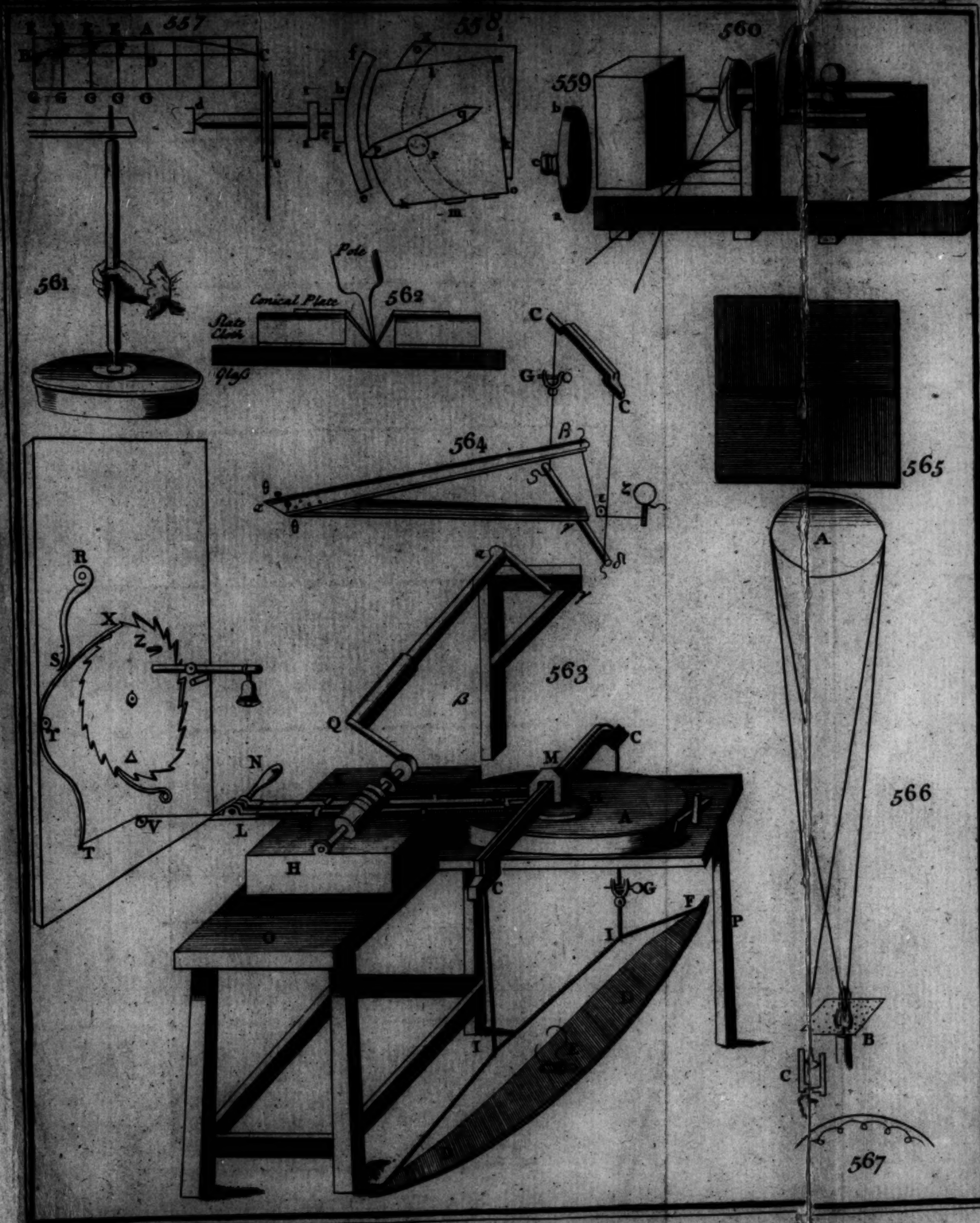
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A

COMPLEAT SYSTEM OF OPTICKS.

BOOK III.

A *Mechanical Treatise.*

CHAPTER I.

The method of grinding and polishing glasses for Telescopes, extracted from Mr. Huygens *and other Authors, by the Honourable* Samuel Molyneux *Esquire.*

THIS ingenious and honourable Author, out of his great regard for the improvement of Astronomy, by perfecting the methods of making telescopes both by refraction and reflection, did not only collect and consider what had been written and practised by others, but also made several new experiments of his own contriving, after he had procured a most compleat Apparatus of all sorts of instruments for that purpose. But in the midst of these thoughts, being appointed a Lord Commissioner of the Admiralty, he became so engaged in publick affairs, that he told me he had not leisure, to pursue these enquiries any farther. Then it was that he gave me the following papers, and was pleased to invite me to make use of his house and apparatus of instruments, in order to finish what he had left imperfect. But he dyed soon after, and so I lost that opportunity, and a most worthy friend. Having therefore seen nothing of the practice of grinding glasses, I durst not venture to add any thing of my own relating to it; but have supplied from Mr. *Huygens* what was left unfinished by our honourable Author. And as Mr. *Huygens*'s treatise is esteemed the best of any yet extant, I have taken care that nothing material therein contained should pass unobserved. If I have mistaken his meaning any where, it is the more excusable considering the difficulty of the subject, and that it was a posthumous trea- Advertisement.

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tise written originally in Dutch and translated into Latine by another hand. To distinguish Mr *Molyneux*'s papers from what I have translated from Mr. *Huygens*, I have put their names in the margine at the beginning of every piece.

§. 1. *Of making and polishing the Tools.*

Proportions in the object-glass.

742. It is easier to make an object-glass of equal convexities than of any other figure; because the same tools will serve for forming both its surfaces. And a glass of this figure will make as perfect an image in its focus as any other, because the aberrations of the rays caused by the sphericalness of the figures of the surfaces, whatever be the proportion of their semidiameters, are inconsiderable in long telescopes, in comparison to the aberrations caused by the different refrangibility of the rays [a]; and these latter aberrations are the same whether the semidiameters of the surfaces be equal or unequal, supposing the aperture and focal distance to be the same [b]. If it be proposed then to make a glass of equal convexities, that shall have a given focal distance, the radius of the spherical surface will be found by taking it in proportion to the given focal distance as 12 to 11, as will appear by art. 232, putting the sine of incidence to the sine of refraction out of air into glass as 17 to 11, as Sir *Isaac Newton* has accurately determined it [c]. The focal distance of the glass being given, its aperture is also given by the Table in art. 364; and because its figure cannot be formed exactly true to the very edges, the breadth of the glass may be taken about half an inch more than the diameter of its aperture, or even 3 quarters or a whole inch more, if its focal distance be between 50 and 200 feet.

a Art. 340.

b Art. 678.

c Newt. Opt. p. 86.

Proportions in the tools.

743. Mr. *Huygens* directs in general to make the breadth of the concave tool, or plate or dish or form, in which an object-glass must be ground, almost three times the breadth of the glass. Though in another place, he speaks of grinding a glass whose focal distance was 200 feet and breadth $8\frac{1}{4}$ inches in a plate only 15 inches broad [d]. But for eye-glasses and others of lesser spheres, the tool must be broader in proportion to the breadth of these glasses, to afford room enough for the motion of the hand in polishing. Mr. *Huygens* made his tools of copper or cast brass; which for fear they should change their figures by bending, can hardly be cast too thick; nevertheless he found by experience that a tool 14 inches broad and half an inch thick was strong enough for forming glasses to a sphere of 36 feet diameter; when the tool was strongly cemented upon a cylindrical stone an inch thick, with hard cement made of pitch and ashes.

d Dioptr. p. 280.

Patterns for the tools.

744. In order to make moulds for casting such tools as are pretty much concave, he directs that wooden patterns should be turned in a lathe, a little thicker and broader than the tools themselves. But for tools that belong

long to ſpheres above twenty or thirty feet diameter, he ſays it is ſufficient to make uſe of flat boards turned circular to the breadth and thickneſs required. When the plates are caſt, they muſt be turned in a lathe, exactly to the concavity required. And for this purpoſe it is requiſite to make a couple of braſs gages in the manner following.

745. Take a wooden pole a little longer than the radius of the ſpherical ſurface of the glaſs to be formed, and through the ends of it ſtrike two ſmall ſteel points, at a diſtance from each other equal to the radius of the ſphere intended; and by one of the points hang up the pole againſt a wall, ſo that this upper point may have a circular motion in a hole or ſocket made of braſs or iron fixt firmly to the wall. Then take two equal plates of braſs or copper, well hammered and ſmoothed, whoſe length is ſomewhat more than the breadth of the tool of caſt braſs; and whoſe thickneſs may be about a tenth or a twelfth of an inch, and whoſe breadth may be two or three inches. Then having faſtened theſe plates flat againſt the wall in an horizontal poſition, with the moveable point in the pole ſtrike a true arch upon each of them. Then file away the braſs on one ſide exactly to the arch ſtruck, ſo as to make one of the braſs edges convex and the other concave; and to make the arches correſpond more exactly, fix one of the plates flat upon a table and grind the other againſt it with emery. Theſe are the gages to be made uſe of in turning the braſs tools exactly to the ſphere required.

Gages for the tools. Mr. *Molyneux*

But if the radius of the ſphere be very great the gages muſt be made in this manner. Imagine the line AE drawn upon the braſs plate to be the tangent of the required arch AFB, whoſe radius for example is 36 feet and diameter 72. From A ſet off the parts AE, EE, &c. ſeverally equal to an inch, and let them be continued a little beyond half the breadth of the tool required. Then as 72 feet or 864 inches is to 1 inch, ſo let 1 inch be to a fourth number; this will be the number of decimal parts of an inch in the firſt line EF reckoning from A. Multiply this fourth number ſucceſſively by the numbers 4, 9, 16, 25, &c. the ſquares of 2, 3, 4, 5, &c; and the ſeveral products will be the number of parts contained in the 2d, 3d, 4th, 5th EF reſpectively. But becauſe theſe numbers of parts are too ſmall to be taken from a ſcale by a pair of compaſſes, ſubſtract them ſeverally from one inch, repreſented by the lines EG, and the remainders being taken from a ſcale of an inch divided into decimal parts and transferred by the compaſſes from G to F, will determine the points F, F, &c. of the arch required. And the ſame being done on the other ſide of the line AD, the braſs plate muſt be filed away exactly to the points of this arch, and poliſhed as before.

Mr. *Huygens*. Fig. 557.

746. To apply the braſs tool to a turning lathe in order to turn the concave ſurface of it exactly ſpherical, let the 558th figure repreſent a view of ſome part of the lathe, taken from a point directly over it; and here

Of turning the braſs tools in a lathe. Mr *Huygens*.

Fig. 558 to 560.

here let *ab* represent a ſtrong flat diſk of braſs, half an inch thick at leaſt, having a ſtrong iron ſkrew-pin fixt firmly in the center of it and ſtanding out exactly perpendicular to one ſide of it; by which it may be ſkrewed into the end *c* of the mandrel or axis of the lathe, repreſented by *cd*. This diſk is repreſented ſeparately in fig. 559. and muſt be well ſodered to the back ſide of the tool *ef*, which therefore in the middle of it muſt be made plane, and exactly parallel to the circumference of its oppoſite ſurface; to the intent that the circumference may be carried round the axis of the lathe in a plane perpendicular to it. The mandrel or axis *cd* turns upon a point *d* in the puppet head of the lathe, and in an iron collar repreſented by *st*, as uſual.

Let *ghik* repreſent a board nailed faſt upon the other puppet head; and let the concave gage *gh* be laid upon this board, with its concave arch parallel to the concavity of the tool *ef*, and be ſkrew'd down to the board with flat-headed ſkrews ſunk into the braſs. Let *lmno* repreſent ſuch another board lying upon the former, with the convex gage *lm* ſkrewed to the under ſide of it, ſo that by moving this upper board, the arch of the convex gage may be brought to touch the concave one and ſlide againſt it. The turning tool *pq* is laid upon this moveable board, and is held faſt to it by a broad headed ſkrew at *r*, to be turned or unturned by the hand upon occaſion. To know whether the concave gage be exactly parallel to the concavity of the tool *ef* ſkrewed faſt to the mandrel, direct the point *p* of the turning tool *pq* to touch any point of the tool *ef* near its circumference; then having fixt the turning tool *pq* by the ſkrew *r*, turn the braſs tool *ef* half round, and move the upper board till the point *p* of the turning tool be brought over againſt the ſame mark upon the tool *ef*; and if it juſt touches it as before when the gages coincide, all is right. If not, the poſition of the head of the lathe may be altered a little by ſtriking it with a mallet. But the beſt way is to make this examination of the ſituation of the concave gage, when only one end of it is fixt to the head of the lathe by a ſingle tack or ſkrew, about which it may eaſily be moved into its true poſition. And while the tool or plate *ef* is turning, the ſame examination of its paralleliſm to the gage muſt be frequently repeated; otherwiſe its ſurface will take a falſe figure. It is convenient that the upper board *lmno* ſhould project over both the gages; and to keep its ſurfaces parallel to that of the under board, two round headed nails, or a plate of braſs, as thick as the gages, muſt be fixt to its under ſurface, towards the oppoſite ſide *no*. Care muſt be taken to drill the holes in the gages, through which they are ſkrewed to the boards, not too near the poliſhed arches, for fear of altering their figure, by the yielding of the braſs. The tool and all the parts of the lathe muſt be fixt very firm; becauſe any trembling motion will cauſe the graving tool *pq* to indent the

the braſs. After the tool is well turned it muſt be ſeparated from the braſs back *ab* by melting the ſoder with live coals laid upon it.

It is eaſy to underſtand, that by tranſpoſing the gages, a convex tool may be turned in the ſame manner.

747. Mr. *Huygens* would have his plates or tools firſt formed in a turning lathe, and then ground together with emery; that is to ſay the concave and convex tool of the ſame ſphere together. But the tools of very large ſpheres, he would have ground at firſt quite plane by a ſtone-cutter; and then ground hollow with a round flat ſtone and emery to the deſired gage. And he preſcribes to uſe for this grinding firſt a ſtone half as broad as the tool, and after that another ſtone very nearly as broad as the tool; and in this way of forming the tools it is convenient to tye a little frame of thick paper, or rather of thin paſtboard about an inch high round about the tool to keep in the emery; and in grinding, the whole muſt be made extreamly firm and ſtable. And when the tool is to be poliſhed, it muſt ſtill remain upon the ſtone pedeſtal; otherwiſe it will be in danger of bending a little in the operation. Another way of forming large tools. Mr. *Molyneux*.

748. For poliſhing theſe tools thus ground, Mr. *Huygens* takes ſome ſoap and daubs the concave tool therewith, then he takes the laſt mentioned round ſtone, ſomewhat leſs than the tool, (or the convex tool it ſelf) and heats it, and then he pours upon it ſome hot melted cement (made of pitch and fine powdered and ſifted aſhes as much as he can mix with it) and then he turns over the ſtone and cement upon the concave tool, into which alſo he had poured a good quantity of the ſame cement, having firſt laid three little pieces of braſs of equal thickneſſes on the circumference thereof in order to preſs and keep this cruſt of cement of an exact equal thickneſs in all its parts; and thus he lets them cool together. Then taking the ſtone from the tool and turning it up, he ſifts upon the cement that ſticks to it, a cruſt of very fine emery, and with a flat iron ſpatula about one third of an inch thick, gently warmed, he preſſes lightly the emery, to ſtick to and incruſtate upon the cement; then he again gently warms the whole ſtone (or convex tool) cement and emery, and he again replaces it upon the concave tool and leaves it again to cool; ſo that he has by this means a cruſt of emery exactly of the figure of his tool; with this he poliſhes the tool dry, without the addition of any wet, preſſing it down hard on the ſurface of the tool. And to preſs it the harder he places upon it a long pole, a little bent to make it ſpring, whoſe upper end is fixt to the cieling of the room, or elſe is preſſed downwards by a ſtrong iron ſpring; and he ſays it will be neceſſary to have two perſons to rub the ſtone upon the tool. And here it is to be noted that great care muſt be taken in this and all caſes where this way of grinding by a pole is made uſe of, to fix the point of preſſure exactly and truly in the middle, To poliſh the tool by an incruſtation of pitch and emery.

as

as ſhall be more particularly explained hereafter when the glaſs comes to be ground in this tool.

Several ſorts of cement.

749. It is to be noted that this cement, and indeed all cements for ſticking glaſſes, is made in different manners by different perſons. *Pere Cherubin* [a] ſays it is uſually made of common black pitch and fine ſifted vine aſhes, but he himſelf made it of roſin and ocre, or of roſin and ſpaniſh white, pounding the roſin firſt and mixing it with a due quantity of the powder, and then ſifting the mixture upon hot melted pitch, and while hot, well mixing and incorporating the whole. The cement that Mr. *Scarlet* uſes is made of common pitch, and common coal-aſhes ſifted fine. In all caſes it is harder or ſofter as more or leſs of the aſhes or other powder is put into it; and in this preſent caſe for poliſhing theſe tools it muſt be made as hard as poſſible, by putting in as great a quantity of the aſhes as is poſſible: for otherwiſe if the cement be not hard enough, the particles of the emery will be looſened by the heat in grinding, and then will only run round upon the tool, and will not work out the little inequalities and roughneſſes thereof. If the emery ſhould be found to grow blunt, a very little more of it may be duſted dry upon the tool, by which its ſharpneſs and cutting quality will be a little recovered; but if the cement be ſufficiently hard at firſt, as is preſcribed, the emery will always remain ſharp enough.

[a] La Dioptrique oculaire p. 352.

To perfect the poliſh of the tool with blue hones.

750. To bring the concave tool ſtill to greater perfection, and ſtill to preſerve its true form, take equal pieces, about an inch ſquare, of blue hone, ſuch as engravers poliſh their copper withal, and place as many of them, laying the grain ſome one way and ſome another, as you can, upon the ſurface of the tool to be poliſhed; ſticking and ſteadying the ſame, as cloſe as you can to one another, with ſoap or common white ſtarch; then fill up all the interſtices between the ſaid pieces of hone with clean dry ſand, to about two thirds of the thickneſs of the ſaid hones; then having a border of paper or paſtboard put round the tool as before, ſhake the tool gently that the ſand may every where equally ſubſide, and blow it to an equal depth with a pair of bellows. Then take ſome hard cement extreamly hot and pour it all over the hones, and having cleaned the ſtone (or the convex tool) which before was incruſtated with pitch and emery, place this ſtone (or convex tool) warmed on the top of this cement and let all cool together. Then rubbing the tool with this poliſher made with hones, by applying your pole to the top of the ſtone as before, you ſhall know when the tool is brought to perfection, if wiping off the dirt and filth you find, by looking obliquely upon it againſt the light, that it ſhines equally bright and ſtrong in all parts. If you would uſe this poliſher again, it muſt be kept in a cool cellar, leaving the hones uppermoſt; otherwiſe in warm weather they will change their ſituation in the cement, even with their own weight.

751. And here it is to be obſerved that this method of grinding theſe tools by an incruſtation of emery on hard cement, formed to a convexity by the very tool it ſelf, and alſo the method of poliſhing the ſame by the blue hones fixt in hard cement, may probably as well be uſed and applyed to the grinding and poliſhing of a reflecting metal for a teleſcope. But in that caſe I believe it is beſt not to uſe a ſpringing pole, but one that ſhall turn upon a point above, being nearly equal in length to the radius of the ſphere intended. Applyed to the grinding and poliſhing an object-metal for a teleſcope.

§. 2. *Of chooſing glaſs.*

752. The beſt ſort of glaſs has generally a yellowiſh, reddiſh, or a greeniſh colour, when we look through it againſt the ſky-light, or lay it upon a ſheet of white paper. That which is perfectly white, though it tranſmits the moſt light, is generally fuller of veins, and is often ſubject to grow moiſt in the air, which in time deſtroys its poliſh. In theſe parts there is no better glaſs to be had, than pieces of broken looking-glaſſes. But after ſome time I found pretty good glaſs at a glaſs-houſe; and made uſe of the ſame matter with which they make drinking-glaſſes, which I found was always the beſt after the matter had reſted for two or three days, when the workmen kept holydays. I ordered pieces of glaſs to be made for my own uſe in the ſame manner as they make looking-glaſſes, that is by cutting off the top and bottom of the round globe they blow up, and by ſlitting the ſide of it, and then by flatting it upon the hearth of the furnace. Theſe pieces, which were about half an inch or three quarters thick, I ordered to be ground to an equal thickneſs by a ſtone-cutter, in the machine they poliſh marble with. Several ſorts of glaſs. Mr. *Huygens.*

753. To diſcover the veins in glaſs one ſhould look very obliquely againſt a ſmall light in a room otherwiſe dark. Thus one may examine the pieces of a poliſhed looking-glaſs. But becauſe they are ſeldom thick enough for object-glaſſes, it is neceſſary to take ſome pieces of the ſame ſort of glaſs before it is poliſhed, and to get it ground to an equal thickneſs and poliſhed a little by the common glaſs-grinders, in order to judge what pieces are fit for uſe. Sometimes little veins will appear like fine hairs or threads, which ſcarce do any harm. I have ſeveral very good glaſſes of this ſort. Sometimes their imperfections cannot be diſcovered by the former way of trial, and yet after the glaſs is well formed and poliſhed they will appear by reflection in this manner. In a dark room place the glaſs upright upon a table, turning that ſurface from you which is ſuſpected to be faulty, then holding a lighted candle in your hand, ſo that the middle of the broad light reflected from the firſt ſurface may fall upon your eye, recede from the glaſs till the rays reflected from the back-ſurface ſhall juſt begin to invert the candle; then the whole glaſs will Veins in glaſs how diſcovered.

appear

appear all over bright, and then you will discover its defects and the imperfections of the polish. When the glass is a portion of a large sphere, we use a small perspective 3 or 4 inches long to magnify the defects.

§. 3. *Of preparing the glasses before they can be ground and polished.*

To round the glass. Mr. *Huygens*.

754. The pieces of glass above mentioned, which we directed to be planed to an equal thickness and polished a little by a glass-grinder, should be much broader than the intended object-glass, that there may be room enough for choosing the best part of them. For planing and smoothing those large pieces of glass, I ordered the workmen to make use of plates of cast iron, such as are sold at the ironmongers shops, after they had been ground and planed in a stone-cutters engine. Upon the plate of glass, with a diamond-pointed compass, strike a circle representing the circumference of the object-glass; and also another concentrick circle with a radius about a tenth or twelfth part of an inch bigger. And also two other such circles, on the other side of the glass, directly opposite to the former: which may be done by means of a Circular glass to be described by and by. The larger parts of the glass may be separated from the outward circle by a red hot iron, or by a strong broad vice, opened exactly to the thickness of the glass. The remaining inequalities may be taken off by a grind-stone, beginning with the largest first, and taking care they do not splinter. Then having warmed the glass, cement a wooden handle to it, and in a common deep tool for eye-glasses, making use of white clear sand and water, grind the circumference of the glass exactly true to the innermost circle on each side of it.

To grind it to an equal thickness.

755. Then having made a great many small cavities with a punch upon one side of a round copper plate, and having fixt the other side of it upon the middle of the round glass, by cement made with two parts of rosin or hard pitch and one part of wax, place the steel point of the springing pole above described, being 14 or 15 feet long, into that cavity of the copper plate, which lyes nearest to the thickest part of the glass. Then work the glass by the pole with sand and water upon a flat plate of cast iron, of a round figure, the plate having been planed with sand and water by a stone-cutter. Then having examined the thickness of the glass in several places by a hand vice, which is better than a pair of callippers, by repeating the same operation it will soon be reduced to an equal thickness in all its parts.

Towards the end of this operation it is convenient to make use of sifted emery, because the sand will scratch too deep; and then it will also be necessary to place the steel point of the pole exactly over the center of the under surface of the glass; otherwise that surface will take a cylin-

cylindrical or ſort of a convex figure inſtead of a plane one, even though it was exactly plane before you began to grind it. The reaſon of which is well worth obſerving. And when convex glaſſes are to be poliſhed, it is alſo abſolutely neceſſary to place the point of preſſure exactly over the center of the under ſurface of the glaſs.

756. Therefore to bring one of the little cavities in the copper plate exactly over that center, we make uſe of a circular glaſs formed from a broken looking-glaſs, the quickſilver being rubbed off; having deſcribed upon it, with a diamond-pointed compaſs, 8 or 10 concentrick circles at the diſtance of about a quarter of an inch from each other, ſo that the larger circles may be ſomewhat bigger than the circumference of the glaſs to be poliſhed. Lay this circular glaſs upon the ſurface of the glaſs to be poliſhed, and move it to and fro till you perceive the circumference of the glaſs to be poliſhed is exactly parallel to the neareſt circle upon the circular glaſs; then having inverted both the glaſſes, lay the circular glaſs upon a table, and having laid a ſmall live coal upon the copper plate, to make it moveable upon the cement, place one point of a pair of compaſſes in one of the little cavities, and move the copper till a circumference deſcribed with the other point coincides exactly with any one circle upon the circular glaſs, and the buſineſs is done. With ſtarch it is convenient to paſte three ſlender ſhreds of fine linen, directed towards the center of the circular glaſs, that the other glaſs may not ſlide too eaſily upon it, and that they may not ſcratch one another. The circular glaſs deſcribed,

757. The cavities punched in the copper plate, and alſo the point of the pole, ſhould be triangular, to hinder the rotation of the glaſs; which is ſtill more neceſſary in perfecting the poliſh of the glaſs. And here it muſt be obſerved again whether the circumference of the glaſs remains exactly circular on both ſides of it, which muſt be tryed with a pair of compaſſes; and if it be not, it muſt be corrected again by grinding it exactly circular in a common tool for making eye-glaſſes; which will contribute very much to its taking an exact ſpherical ſurface when it comes to be ground in its proper tool. For if any part of the circumference be protuberant, it will hinder the adjoining parts of the ſurface from wearing ſo much as they ſhould do; and by conſequence will ſpoil the ſpherical figure of the ſurface. The object-glaſs muſt be exactly circular.

§. 4. *Of grinding the glaſſes.*

758. The glaſs being planed and rounded as above, take away the plate with ſeveral cavities, and with ſome of the ſame cement fix on a ſmaller round piece of braſs or rather ſteel, truly flat and turned about the bigneſs of a farthing but thicker, having firſt made in the center thereof, with a triangular ſteel punch, a hole about the bigneſs of a Mr. *Molyneux.*

 gooſe

goose quill, and about the depth of $\frac{1}{12}$ of an inch; and at the very bottom of this triangular hole, a little small round hole must be punched somewhat deeper with a very fine small steel punch. A small steel point of about an inch long must be truly shaped and fitted to this triangular hole, and at the very apex to the small round deeper impression. Nevertheless it must not be fitted so exactly to the same but that it may have some liberty to move a little to and fro; the apex always continuing to touch and press upon the surface of the round hole below. This steel triangular point must be fixed to the end of a pole, to the other end of which another round iron point must be fixed, of about five or six inches long, to play freely up and down in a round hole in a piece of brass let into a board fixed against the ceiling for that purpose, perpendicularly over the bench, and over the center of the tool, which must be strongly and truly fixed horizontally thereon.

Fig. 561.

759. Now here it is to be noted that Mr. *Huygens* prescribes to fix his brass plate to the glass by the means of cement, and takes no notice of any other method whatever; though a very small experience in these affairs will convince any body that it is hardly possible, in this or any other case, to bring the cement to a fluidity sufficient to fix two plane surfaces exactly parallel one to the other, without heating the glass and the brass also to a great degree, and so as to endanger the figure of the glass considerably. To avoid this in fixing glasses to brass or wood or the like, some have done it with plaister of Paris: Mr. *Scarlet* does it by cementing another intermediate glass to the brass (or wood) and then fixing the glass, to be ground, to the outward surface of the cemented glass with common glew. Without all this trouble I have done it only with common Icthyocolla or fish glew, which will run very fluid, and will fix the glass and the brass it self strongly together; and round the edges of the brass I stick on some common soft red wax, such as is used for the privy seal, to keep the wet from getting to the glew.

760. For grinding glasses truly plane, upon a plane tool, by this method, Mr. *Huygens* prescribes this pole to be about fifteen feet long; but in grinding upon a concave plate, the pole had best be made equal to the radius of the sphere of the tool; though I believe it would not be material if made considerably shorter, according as the height of the room will allow.

It is necessary to have lying by one an ordinary piece of course glass, ground in the same tool, called a bruiser; whereby when any new emery is necessary to be laid on the tool in grinding your glass, the said emery is to be constantly first run over and smoothed, for fear any little course grains should remain and scratch the glass to be ground.

761. Having these things prepared together with some pots of emery of various finenesses, take of your roughest sort a small half pugil, wetting the

the ſame and daubing it pretty equably on the tool; then lay on your glaſs and fix up your pole and continue to grind for a quarter of an hour, not preſſing upon the pole, but barely carrying the glaſs round thereby; then take the like quantity of ſome finer emery and work another quarter of an hour therewith; then take the like quantity of emery ſtill finer and work for the ſame time; laſt of all take a leſs quantity of ſome of the very fineſt you have, which will be ſufficient for a glaſs of five inches diameter, and work therewith for an hour and half, taking away by little and little ſome of the emery with a wet ſponge. Do not keep it too wet nor too dry, but about the conſiſtence of pap; for much depends on this. If it is too dry, your emery will clog and ſtick and incorporate, ſo as for the moſt part to cut little or not at all, unleſs here and there where its body chances to be broke, and there it will ſcratch and cut your glaſs irregularly; and if it is too wet, and too much diluted, it will from the irregular ſeparation of its parts cut in ſome places more than others juſt as in the other caſe. Fig. 561.

762. But Mr. *Huygens* tells us this method of uſing various ſorts of freſh emery is not good, finding by experience that the ſurfaces of large glaſſes are often ſcratched. And therefore he ſays it is beſt to take a large quantity of the firſt or ſecond ſort of emery, and ſo work with the ſame from the firſt to the laſt; taking away by little and little every half hour, or quarter of an hour, more and more of the emery with a wet ſponge; by which means he could bring the glaſs extreamly ſmooth and fine, ſo as to ſee pretty diſtinctly a candle or the ſaſh windows well defined through it; which is a mark when it is ground enough to be ready to receive a poliſh. But if the glaſs has not acquired this degree of tranſparency, it is certain, ſays Mr. *Huygens*, that too much emery remains; and therefore it muſt ſtill be diminiſhed and the operation continued. He found it beſt to make uſe of common well water in this grinding; and he took care to move the glaſs in circles, taking in an inch beyond the center of the tool and ſomewhat beyond the outſide of the tool; and he found in a glaſs of two hundred feet whoſe diameter was 8 inches $\frac{1}{4}$, which he ground in a tool of 15 inches diameter, that the figure of the tool in grinding would alter conſiderably, unleſs he carried the glaſs round an inch beyond the center of the tool one way, and 3 inches $\frac{1}{4}$ beyond the ſkirts of it another way; but if he carried it no more than a ſtraws breadth beyond the ſkirts of the tool, and accordingly farther beyond the center, the glaſs would always grind falſly, too much being taken off on the outſides ſo that he could never after bring the outſides of the glaſs to a true and fine poliſh.

763. When you firſt begin to grind and the emery begins to be ſmooth, the glaſs will ſtick a little to the tool and run ſtiff; then freſh emery is to be added. When it afterwards comes to be poliſhed it will, if

 large

large, require a conſiderable ſtrength to move it, but this inconvenience will happen leſs in grinding by the pole than in grinding by hand. For the warmth of the hand makes the ſubſtance of the glaſs ſwell, and not only increaſes the ſticking of the glaſs, but in ſome meaſure may alſo ſpoil the figure of it and alſo of the tool. When it is ground with the pole it never ſticks very ſtrongly, unleſs when you take the glaſs off from the tool and keep it from it for ſome time, and then apply it to the tool again; and this in large glaſſes; for by this means, ſays Mr. *Huygens*, the glaſs gets from the air a greater warmth than it had on the tool; and being again applyed to the tool, its lower ſurface is ſuddenly contracted by the coldneſs of the tool and ſo ſticks to it. Wherefore ſaith he you muſt, in that caſe, wait till the glaſs and the tool come to be of one temper. The like effect is obſerveable in grinding large glaſſes when there is a fire in the room. Perhaps the cauſe of theſe effects may be more truly deduced from the attractive qualities of warm glaſs. But whatever is the cauſe, we may from hence perceive the great nicety of grinding large glaſſes, and the neceſſity there is of grinding them ſlowly, and with the greateſt caution in the moſt minute circumſtances.

764. The method hitherto deſcribed of grinding with emery, is what is recommended by Mr. *Huygens*. Le Pere *Cherubin* preſcribes another material, and it is the grit of a hard grind-ſtone well beaten into a fine powder and ſifted pretty fine. And here in England the ſame thing was uſed to be performed by Mr. *Cox* with common clean fine white ſand, taking away by little and little the ſaid grit and ſand as it ground finer and finer. Nay Mr. *Cox* was uſed to continue his grinding till the matter of the ſand came to be ſo fine, and ſo little of it to remain in the tool, that he could, and frequently did uſe to poliſh off his glaſſes therein, without the uſe of any other material whatſoever; and I my ſelf have been preſent, while Mr. *Scarlet* ground and poliſhed, or dryed off, a glaſs of 16 feet in this manner. They call this way drying off on ſand, becauſe as the matter grows finer and finer, they wet it leſs and leſs, till for the laſt quarter of an hour (the whole work laſting near two hours) they only wet it by breathing upon it, and at the very laſt not at all.

765. It ſeems this method is now quite diſuſed: perhaps the violent labour requiſite at the laſt may be a reaſon of it. A better reaſon may be the great improbability of grinding or poliſhing true by this method, by the uncertain and unequal force of the hand, and if this be the true reaſon of its diſuſe, I cannot well ſee but that this method of grinding and poliſhing out and out in the ſame tool, and with the ſame material, *viz.* white ſand, might perhaps be again reſtored, and greatly improved by adding to the old way Mr. *Huygens*'s method of grinding and poliſhing with a pole and ſpring to preſs down the pole; or ſome analogous contrivance. And in relation to grinding by all or any of the methods above deſcribed, this

this one general remark muſt be made, that the artiſt muſt allow time and patience to bring his glaſs by grinding to the ſmootheſt and fineſt ſurface, that he poſſibly can, before he attempts to poliſh. For this and this only makes his glaſs poliſh truly ſmooth, well and eaſily: and the ſmoother you bring it in grinding, the leſs labour you will have in poliſhing; in which conſiſts not only the greateſt difficulty but the greateſt danger too of ſpoiling all you have done before.

§. 5. *Of giving glaſſes the laſt and fineſt poliſh.*

766. Having removed the little braſs plate from the glaſs, take ſays Mr. *Huygens* a very thick ſlate, or rather a block of blue or grey ſtone; let it be half an inch thick, and let it be ground true and round at the ſtone-cutters; its diameter being ſomewhat ſmaller than the diameter of your glaſs, leaving a hole, quite through, in the center, of about an inch diameter. Then make ſome cement two parts roſin or hard pitch and one part wax; and taking a piece of thick kerſey cloth, truly and equally wrought, cut this cloth round, and leave a like hole, one inch diameter, in the middle. Then warming the ſtone and alſo warming the glaſs, and ſpreading thinly and equably upon them ſome of this cement, lay on the cloth and thereupon lay on alſo the glaſs, having left in the middle a ſpace the breadth of a ſhilling uncemented and blackt with a candle. Then provide an hollow conical plate of iron or ſteel (ſhaped like an high crowned hat) having the baſis of the cone 1 inch diameter, and having round the baſis a flat border about 2 $\frac{1}{2}$ inches diameter, and having the depth or altitude of the cone exactly of the thickneſs of the ſlate, cloth and cement, to which the glaſs is fixt. The vertex of this cone muſt go down through the ſlate and cloth, ſo that being cemented on the ſlate, the ſaid vertex may approach to the glaſs within a hair's breadth, and lye perpendicularly over the center of the lower ſurface of the glaſs: and this muſt be adjuſted by the circular glaſs deſcribed above. Within the vertex of this hollow cone, the lower point of the pole is to be applyed in poliſhing; but it may be firſt proper to be obſerved, that perhaps fiſh glew and a braſs plate, in lieu, and of the dimenſions, of the aboveſaid ſlate, may perhaps be better. Mr. *Huygens* obſerves alſo that the angle of the cone ſhould be 80 or 90 degrees; and that the hollow vertex of it ſhould be ſolid enough to receive a ſmall impreſſion from a round ſteel punch, to put the point of the pole into, which might otherwiſe have too much liberty and ſlip from the vertex. The deſign of the black ſpot in the middle of the glaſs, is to diſcover by the light of a candle obliquely reflected from your glaſs, after it has been poliſhed ſome time, whether it be perfectly clear, and free from the appearance of any bluiſh colour like that of aſhes.

Fig. 562. Mr. *Molyneux*.

767.

767. Before the work of poliſhing is begun, it is proper to ſtretch an even well wrought piece of linen over the tool, duſting thereupon ſome very fine tripoly. Then taking the glaſs in your hand, run it round 40 or 50 times thereupon; and this will chiefly take off the roughneſs of the glaſs about the border of it, which otherwiſe might too much wear away the lower parts of the tool, in which the glaſs is chiefly to obtain its laſt poliſh. If I underſtand Mr. *Huygens* right, this cloth is then to be removed, and the glaſs to be begun to be poliſhed upon the very naked tool it ſelf. But firſt there is to be prepared ſome very fine tripoly, and alſo ſome blue vitriol, otherwiſe called cyprion, engliſh and hungarian vitriol finely powdered: mix four parts of tripoly with one of vitriol: 6 or 8 grains of this mixture (which is about the quantity of two large peas) is ſufficient for a glaſs 5 inches broad. This compound powder muſt be wetted with about 8 or 10 drops of clear vinegar in the middle of the tool; and it muſt be mixed and ſoftened throughly with a very fine ſmall mullet. Then with a courſe painting bruſh take great care to ſpread it thinly and equably upon the tool, or at leaſt upon a much larger ſpace in the middle of it, than the glaſs ſhall run over in the poliſhing. This coat muſt be laid on very thin (but not too thin neither) otherwiſe it will waſte away too much in the poliſhing, and the tool will be apt to be furrowed thereby, and to have its figure impaired; inſomuch that ſometimes a new daubing thereof muſt be laid on, which it is not eaſy to do ſo equably as at firſt. This daubing muſt be perfectly dryed by holding over it a hot clean frying pan, or a thin pan of iron, with lighted charcoal therein for that purpoſe; then leave all till the tool is perfectly cold. Then having ſome other very fine tripoly very well waſhed and ground with a mullet, and afterwards dryed and finely powdered, take ſome of the ſame and ſtrow it thinly and equably on the tool ſo prepared; then take your courſe glaſs which lay by you, and ſmooth all the ſaid tripoly, very equably and finely: then take your glaſs to be poliſhed and wipe it thoroughly clean from all cement, greaſe or other filth which may ſtick to it, with a clean cloth dipped in water, a little tinged with tripoly and vitriol; then taking your glaſs in your hand apply it on the tool, and move it gently twice or thrice, in a ſtraight line backwards and forwards; then take it off and obſerve whether the marks of the tripoly, ſticking to the glaſs, ſeem to be equably ſpread over the whole ſurface thereof; if not, it is a ſign that either the tool or the glaſs is too warm; then you muſt wait a little and try again till you find the glaſs takes the tripoly every where alike. Then you may begin boldly to poliſh, and there will be no great danger of ſpoiling the figure of the glaſs; which in the other caſe would infallibly happen. If the tool be warmer than the glaſs, it will touch the glaſs harder in the middle than towards its circumference; becauſe the upper ſurface of the tool being ſwelled by heat will become

too

too flat. On the contrary if the glaſs be warmer than the tool, it will bear harder towards its circumference than at the center; becauſe the inferior ſurface of the glaſs is contracted by the coldneſs of the plate, more than the ſuperior.

768. Mr. *Huygens* ſays that if the work of poliſhing were to be performed by ſtrength of hand only, it would be a work of very great labour, and even could not be performed in glaſſes of 5 or 6 feet focal diſtance: and he ſeems to think it abſolutely neceſſary that an extraordinary great force or preſſure ſhould be applyed upon the glaſs. For this purpoſe he has therefore contrived and deſcribed two methods for ſufficiently increaſing the preſſure; for the explanation of which, recourſe muſt be had to the book it ſelf and his figures [a]; it may ſuffice here to ſay that they chiefly conſiſt in applying the force of a ſtrong ſpring to preſs down the center of the glaſs upon the poliſher.

[a] See Art. 774.

769. This operation of poliſhing, as it is one of the moſt difficult and nice points of the whole, hath been very variouſly attempted and deſcribed by various Authors. Sir *Iſaac Newton*, Pere *Cherubin*, Mr. *Huygens* and the common glaſs grinders, have taken different methods in this matter. Sir *Iſaac* is the only perſon who ſeems not to inſiſt on the neceſſity of a very violent and ſtrong preſſure. In the engliſh 8° edition of his Opticks page 95. he hath theſe words "An object-glaſs of a fourteen foot teleſcope, made by an artificer at *London*, I once mended conſiderably, by grinding it on pitch with putty, and leaning very eaſily on it in the grinding, leſt the putty ſhould ſcratch it. Whether this may not do well enough for poliſhing theſe reflecting glaſſes, I have not yet tried. But he that ſhall try either this or any other way of poliſhing which he may think better, may do well to make his glaſſes ready for poliſhing by grinding them without that violence, wherewith our *London* workmen preſs their glaſſes in grinding. For by ſuch violent preſſure, glaſſes are apt to bend a little in the grinding, and ſuch bending will certainly ſpoil their figure.

770. As to his own method of poliſhing glaſs, I do not know that he any where expreſly deſcribes it, but his method of poliſhing reflecting metals he doth, and it was thus in his own words p. 92. "The poliſh I uſed was in this manner. I had two round copper plates each ſix inches in diameter, the one convex the other concave, ground very true to one another. On the convex I ground the object-metal or concave which was to be poliſhed, till it had taken the figure of the convex and was ready for a poliſh. Then I pitched over the convex very thinly, by dropping melted pitch upon it, and warming it to keep the pitch ſoft, whilſt I ground it with the concave copper wetted to make it ſpread evenly all over the convex. Thus by working it well, I made it as thin as a groat, and after the convex was cold I ground it again, to give it as true a figure

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as I could. Then I took putty which I had made very fine by washing it from all its grosser particles, and laying a little of this upon the pitch, I ground it upon the pitch with the concave copper till it had done making a noise; and then upon the pitch I ground the object-metal with a brisk motion for about two or three minutes of time, leaning hard upon it. Then I put fresh putty upon the pitch and ground it again till it had done making a noise, and afterwards ground the object-metal upon it as before. And this work I repeated till the metal was polished, grinding it the last time with all my strength for a good while together, and frequently breathing upon the pitch to keep it moist, without laying on any more fresh putty. The object-metal was two inches broad and about one third part of an inch thick, to keep it from bending. I had two of these metals, and when I had polished them both I tried which was best, and ground the other again to see if I could make it better than that which I kept. And thus by many trials I learned the way of polishing, till I made those two reflecting perspectives I spake of above. For this art of polishing will be better learned by repeated practice than by my description. Before I ground the object-metal on the pitch, I always ground the putty on it with the concave copper till it had done making a noise, because if the particles of the putty were not by this means made to stick fast in the pitch, they would by rolling up and down grate and fret the object-metal and fill it full of little holes. It seems not improbable but that glass may also be polished with proper care by the same method.

771. The method of polishing described by Pere *Cherubin* seems to be chiefly thus: he polishes with tripoly or putty or first with tripoly and afterwards with putty. But what he seems most to approve of is putty alone. He polishes in the same tool he grinds in, and he very verbosely describes various ways of doing it. He prescribes to stretch very tight a very fine thin leather or fine english fustian, or fine holland, or any fine linen or fine silk taffety or satin, all of an equable thickness as near as may be, upon the tool; then he daubs thinly on this surface thus stretched a streak of putty, sufficiently wetted to the consistence of thick syrrup, about as broad as the glass or a little more, passing through the center of the tool directly from him; then smoothing the putty by running his bruiser and pressing it backwards and forwards to him and from him, he at length lays on the glass cemented to its handle; and giving it always the same motion, strongly pressing to him and from him along the streak of putty, and by such pressure forcing the surface of the silk, already somewhat stretched, close to the surface of the tool, to which the figure of the glass was exactly adapted, he says that he could by that means obtain an excellent fine polish on any of the abovementioned substances. Before every stroke he turned the glass a little on its axis, and its handle was on this occasion considerably heavier than usual in grinding, which he commends

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as very useful in this busineſs; and if new putty was wanting he made no difficulty of laying it on, as often as neceſſary, always carefully ſmoothing it thereon with the bruiſer before the glaſs was applyed.

772. This method I am of opinion might be improved by moving the glaſs not by hand but by the pole and ſpring, ſomewhat after the method of Mr. *Huygens*, eſpecially if the pole were contrived not to move looſe in a round braſs hole above, but on a ſtrong point preſſed down by ſome ſpring; the length of the pole being equal to the radius of the tool, and the point where the ſpring preſſes the upper end of the pole, being truly perpendicular over the center of the tool, and exactly in the center of its ſphere.

773. Another method which the ſame author preſcribes for poliſhing in the tool is thus; he takes a ſheet of very fine paper and examining it very carefully by looking upon it and through it, he takes off with a fine penknife all the little lumps, hard parts, roughneſſes and inequalities that he can find; then he ſoaks it in clean water, then he takes it and dries it between two clean linen cloths, though not ſo much as to make it quite dry, but to leave it dampiſh; then with ſome very thin ſtarch or paſte he daubs equably all over the ſurface of his tool as thin as poſſible, but ſome every where; then he lays on his paper very gently and ſlowly, letting it touch and ſtick firſt at one ſide, and by degrees more and more towards the middle, and by degrees to cover the whole; and he does this ſlowly to let all the air get out; then with the palm of his hand he preſſes the center and every where round about it towards the circumference to make the paper ſtick every where; and this he does 3 or 4 times while it is drying to get out all the air; he lets it dry of its ſelf, then he reviſes it again with his knife as before; then he hath a very coarſe bruiſer of glaſs whoſe circumference is ſharply ground round and at right angles to its ſurface, which he had coarſely ground before in the ſame tool; with this and a very heavy handle he ſmooths and poliſhes and rubs off all the remaining inequalities of the paper, and when this is done he lays on a ſtreak of tripoly and poliſhes as in his other method.

774. At *CC* is repreſented a ſquare beam of wood a little longer than the diameter of the tool, and about $1\frac{1}{2}$ inch thick; the two extremities of it at *C* and *C* are bent downwards, and then are again directed parallel to the whole length, and ſerve for handles for the workman to lay hold upon. In the middle of this beam there is fixt an iron ſpike, ſo long that when the lower ſurfaces of the handles *C*, *C* are placed upon a plane, the point of the ſpike ſhall juſt touch the plane. This point preſſes upon the apex of the hollow cone, which deſcends through the hole in the ſlate, which by the interpoſition of a cloth was cemented to the glaſs *B* lying upon the tool *A*. To increaſe this preſſure a ſort of a bow *DED* is ſhaped out of a deal board, half an inch thick and 5 feet long, being 7 inches broad

Mr. *Huygens*'s deſcription of his machines for poliſhing. Fig. 563.

broad in the middle, and tapered narrower towards its extremities, ſo as to end almoſt in a ſharp point. The middle of the bow is fixt to the floor by an iron ſtaple at *E* driven croſs it; and is bent into an arch by a rope *FIIF*; to which two other ropes are tied at *I* and *I*; the interval *II* being equal to the length of the beam *CC*. One of theſe ropes *ICCG* goes over the back of the beam *CC*, paſſing through a hole in each handle at *C* and *C*, and then is lapped round a cylindrical pegg at *G*, that paſſes through two wooden chaps, to the bottom of which the other rope is tied that comes from the other *I*. So that by turning the pegg *G*, to lap the rope about it, the bow *DD* may be bent as much as you pleaſe. The tool *A* is placed upon a ſtrong ſquare board fixt to the table *O* on one ſide, and ſupported on the other ſide by the poſt *P*. Then the workman ſits down, and taking hold of the handles *CC*, he draws the glaſs to him and from him over the tool *A*, with a moderate motion. And after every 20 or 24 ſtrokes he turns the glaſs a little about its axis. This way of poliſhing took up two or three hours, and was very laborious as well as tedious, becauſe the glaſs being ſo much preſſed downwards was moved very ſlowly.

Fig. 564. 775. Inſtead of the bow *DD*, afterwards I invented another ſpring, by ſloping the flat ends of a couple of deal boards $\alpha\beta$, $\alpha\gamma$ and by nailing the flat ſlopes together very firmly that the boards might make an acute angle $\beta\alpha\gamma$. One of theſe boards ſo joined was laid upon the floor under the poliſhing table, the ends $\beta\gamma$ being under the middle of the tool *A*. So that they lay quite out of the way of the workman, who before was a little incommoded by the ends of the bow *DD*. The boards at the end α were 8 or 10 inches broad, and from thence went tapering almoſt to a point at β and γ. The board $\alpha\gamma$ lying upon the floor, the end β, of the upper board, was pulled downwards by a rope $\beta\epsilon\zeta$ that paſſed under a pully ϵ, fixt to the floor, and then was lapped round a ſtrong pegg ζ that turned ſtiff in a hole in the floor. Under the end γ the middle of a ſtrong ſtick $\delta\gamma\delta$ was fixt at right angles to the board $\alpha\gamma$, and cords were tied to each end of this ſtick at δ, δ, which went over the poliſhing beam *C*, *C* as in the former machine. This ſtick was lifted up but very little from the floor at the time of poliſhing; and by conſequence the ropes δC, δC were long enough to give liberty of motion to the poliſhing beam *CC*. Two iron pins θ, θ paſſing through the ends of the boards at α, were ſkrewed into the floor, but the heads of the pins ſtood up above the boards, to give them liberty to riſe up when the rope $\beta\epsilon\zeta$ was ſtretched.

Fig. 563. 776. To fecilitate the labour of moving the glaſs backwards and forwards in the tool, I made this addition to the former machine. At *M* is repreſented a ſtrong hand made of wood or iron, having a ſquare cavity cut through the bottom of it, for the poliſhing beam *CC* to paſs through, not tight but with ſome liberty. To one ſide of this hand *M* a long board *LL* is annexed, ſome way or other, by means of an iron bolt. The

breadth

breadth of the lower ſurface of this board *LL* is equal to the breadth of the hand *M*, being 2 inches and a half, its thickneſs is half an inch, and its length is equal to three ſemidiameters of the tool. The board *LL* muſt be drawn length ways backward and forward over a block *H* fixt firmly to a table *O*; the thickneſs of the block being ſuch that the board *LL* may lye an inch higher than the ſurface of the tool *A*. The wooden hooks at Π and the pins at Σ keep the motion of the board in the ſame direction by hindering it from ſlipping either ſideways or upwards. Over this board, at right angles to it, and over the middle of the block *H* there lyes a wooden roller, having a ſtrong iron axis which turns in the holes of two iron plates fixt to the ends of the block. The thickneſs of the roller is about an inch and a half. Through two holes bored through this roller and made wider at one end of them, two ſtrong cords are made to paſs with knots at one end of them, to be drawn into the wider parts of the holes, that they may neither ſlip through nor ſtand out from the roller. Then each cord is lapped round the cylinder ſeveral times; and one end of each is pegged firmly into the board *LL* at the end towards *M*, and the other ends of them are lapped round a pegg at *N*; which being turned round will ſtretch the cords as much as you pleaſe. At one end of the axis of this roller there is a handle *Q* which being turned round, backwards and forwards alternately, the board *LL* with the glaſs annexed to it is moved to and fro, ſo far, that about a third part of its diameter ſhoots both ways over the margin of the tool. The ſpike in the middle of the beam *CC* preſſes the glaſs a little obliquely, becauſe the hand *M* holds the beam *CC* not tight but ſomewhat looſely, to the end that the glaſs may paſs over the tool without trembling. Nevertheleſs this inclination of the ſpike muſt be but very ſmall; and may eaſily be increaſed or diminiſhed ſeveral ways. Two pins or ſtops muſt be fixt to the under ſurface of the board *LL*, to determine the length of the ſtroke. The tool *A*, or rather the ſtone to which it is cemented, is ſqueezed faſt between the block *H* and a ſtrong ſtop on the oppoſite ſide of the ſtone, by the interpoſition of a wedge. The workman ſits upon a round ſtool, and when one hand is tired with turning the roller, he applies the other; and therefore is not tired ſo ſoon as with the other machine, which required both hands, and alſo a reciprocating motion of his whole body. Sometime after, I cauſed a longer handle *Qx* to be made that turned at both ends, for the convenience of uſing both hands at once.

777. After every 20 or 24 ſtrokes it is neceſſary to give the glaſs a ſmall turn about its axis; which is eaſily done by laying hold of the ſlate, fixt to it, with one hand, while the other hand goes on with the poliſhing motion. The tool muſt alſo be moved a little after every 25 or 50 ſtrokes, by drawing it half a ſtraws breadth towards that part of it which the glaſs has left; and by drawing it back again after as many more ſtrokes. At the

the beginning of the work the tripoly will be gathered into little lumps in some places of the tool, but will be dispersed again in a little time; and then the area of the tool will become perfectly smooth.

778. If the tripoly does not appear to stick equally to the glass in all parts, and to be diffused over it in slender straight streaks, the frying-pan, with coals in it, must be held over the tool again; till you perceive the the area or coat of tripoly is not quite so cold as the other parts of the tool. Then let tripoly be rubbed upon the tool again, and let the glass be pressed over it with your hand, to try whether it sticks equally to the glass in every place. When it does you may proceed in the work of polishing. But after I began to make use of vitriol instead of verdigrease, all that I have said about warming the tool may be omitted. Because these coats always touch the glass as they should do, and stick better than before. Also let the tool be rubbed with tripoly over the coat without warming the tool, that the coat may be preserved more entire and that the glass may touch it better; which must always be repeated after 200 or 400 strokes in polishing. The glass should also be taken from the tool after 200 strokes, by withdrawing the bolt *L* which connects the hand *M* to the board *LL*; and by removing the beam *CC*. Then rub your finger upon the glass, or a clean rag or a bit of leather, to examine how much it is polished.

779. To save the trouble of counting the strokes, there is a wooden wheel ΔX, 7 or 8 inches broad, placed flat against a board, fixt to the side of a wall. It turns easily about an axis, and has 24 teeth, like those of a saw, which are pushed round by a bended wire *TYX*, in this manner. The wire turns about a center *Y*, and while one end of it is pulled by the string *TV*, tyed to the end of the board *LL*, the opposite end *YX* pushes back a long spring *RS* fixt to the board at *R*; which by pressing upon the wire at *S* causes the part *YX* to bend a little, and so the point *X*, in returning to the wheel (the string being relaxed) falls a little lower, into the next tooth, and pushes it forward into the position represented in the figure. There is a springing catch at Δ which stays the wheel, after every stroke at *X*. Lastly there is a pin fixt in the circumference of the wheel at *Z*, which by pressing the tail of a hammer and letting it go again, causes a bell to sound after every revolution of the wheel, and gives notice that the glass must be turned a little about its center. It is easy to understand that another piece of wheel work, having three or four indexes, whose revolutions are in decimal progression, may be fixt to the block *H* and impelled by the strokes of the board *LL*; by which means, without any trouble of counting, one may be informed how many strokes go to polish a glass.

780. A glass 5 or 6 inches broad requires about 3000 strokes upon each surface, to bring it to perfection. And you must carefully examine, the

middle of the glaſs oppoſite to the blacking, whether any place appears darkiſh or of an aſh-colour; or whether any ſmall ſpots appear by an oblique reflection of the light of a candle or of a ſmall beam of light let into a dark room. For the other parts of the glaſs will appear perfectly fine much ſooner than the middle.

781. After the glaſs has been ſufficiently poliſhed, let the ſtone, the cloth and the cement be warmed over a pan of charcoal, till the cement grows ſo ſoft that the glaſs may be ſeparated from it by a ſide motion. Then whatever cement remains upon the glaſs muſt be wiped off with a hot cloth dipt in oil or tallow, and laſt of all with cleaner cloths. Then if it does not appear perfectly poliſhed (for we are often deceived in this point) the work muſt be repeated again, by glewing the glaſs to the ſlate as before; then it muſt be wiped very clean and be made a little rough as we ſaid before. We may alſo lay a new fund or coat upon the tool if the old one be ſpoiled; provided no other glaſs has been poliſhed in the tool in the mean time. The old fund may be waſhed off from the tool with a little vinegar. Laſtly take care always to chooſe the thickeſt and the cleareſt pieces of glaſs, to avoid a great many difficulties that ariſe from the unequal preſſure in poliſhing.

CHAPTER II.

The method of caſting, grinding and poliſhing metals for reflecting teleſcopes, begun by the Honourable Samuel Molyneux *Eſquire, and continued by* John Hadley *Eſquire, Vice-Preſident of the* Royal Society.

It is chiefly to the invention of teleſcopes that we owe all the late diſcoveries and moſt of the preſent accuracy in Aſtronomy. An invention which certainly, in its firſt original, was put in practice by an Engliſhman *Frier Bacon*; although its firſt application to aſtronomical purpoſes may be juſtly attributed to *Galileo*. The greateſt improvement, which this invention has ever received, is indiſputably and ſingly owing to Sir *Iſaac Newton*: to whoſe extraordinary ſagacity, and very judicious experiments, the world firſt owes the diſcovery of the different refrangibility of the rays of light, and the inſuperable difficulties ariſing from thence in perfecting any refracting teleſcope. This led him to the practice of making teleſcopes by reflection; which having attempted with his own hands, he perfected ſome of ſix inches length about the year 1670. Whoſoever therefore would throughly underſtand the method of making theſe teleſcopes, will find it very proper in the firſt place, to peruſe and throughly conſider the account which the celebrated author of this invention hath himſelf given of it in his writings; which are to be met with in the Philoſophical

Hiſtorical preface.

losophical Transactions, N°. 80, 81, 82, 83, 88, 96, 97; and in his Treatise of Light and Colours, beginning at the 89th page of the english edition in 8°. It will be necessary here to note an error of the printer in this 89th page; where it is said "that the apertures of the object-glasses, and the charges or magnifying powers, ought to be as the cubes of the *square* roots of the lengths of the telescopes; the word *square* is false printed; it should be as the cubes of the square-square roots of the lengths.

782. As great an improvement as this was to telescopes, I do not find that it was ever effectually prosecuted from that time till about the year 1719 or 20; when a very ingenious gentleman Mr. *Hadley* of *Essex*, attempted it, and succeeded very well in making two instruments of this kind, of about 5 foot 3 inches long; one of which he has been pleased to give to the Royal Society. A very particular and curious description of this instrument, and of the apparatus for managing thereof, hath been given by Mr. *Hadley*, with a figure of it also, in the Philosophical Transactions for the months of March and April 1723 N°. 376. It will be necessary for any person that would attempt to make this instrument, to consider that account given by Mr. *Hadley*; for although he gives therein no account of the manner of casting, grinding and polishing the specula, yet as to the proportion and composition of the different parts of the instrument, and of the apparatus for moving it, the reader will there find several useful instructions. What is contained in the following papers is also chiefly owing to the communicative genius of that gentleman: and had he ever given himself the trouble to reduce to writing what he knows and hath practised in the abovementioned particulars, as to the construction of this instrument, the following account had been altogether unnecessary. Upon his encouragement and instructions, the Reverend Mr. *James Bradley* Professor of Astronomy in *Oxford*, attempted the same about three years ago; and having succeeded pretty well, would probably have perfected one of them, had he not been obliged suddenly to remove from the place where he then dwelt, and been since diverted from it by other avocations. Soon after this Mr. *Bradley* and I began our endeavours at *Kew* to perform the same, and our first attempt was to make them about 26 inches long. Notwithstanding Mr. *Bradley*'s former tryals and Mr. *Hadley*'s frequent instructions, we were a long while before we could tolerably succeed. The first good one that we finished was in May 1724, of the aforesaid length of 26 inches. I have since made a pretty good one of about 7 inches, and we are now about one of 8 foot.

Design. 783. The main drift of all our tryals hath been if possible to reduce the method of making these instruments to some degree of certainty and ease; to the intent that the difficulty in making them, and the danger in miscarrying, might no longer discourage any workman from attempting the same for publick sale; which no body but Mr. *Hauksbee* in *Crane* Court

Court hath ever ventured upon. He has made a good one of about 3 ½ foot, and is now about one of 6 foot and another of 12 foot, and deserves very well to be encouraged, being the first person who hath attempted it without the assistance of a fortune, which could well bear the disappointment. About the beginning of the last winter being pretty well satisfied as to most of the circumstances in this performance, and being desirous that these instruments might become cheap and of publick sale, we acquainted Mr. *Scarlet* near St. *Anne*'s Church, and Mr. *Hearne* a Mathematical Instrument-maker in *Dogwel Court*, *White Friers*, with the whole process of the operation as we had practised the same; and they have since succeeded in making these instruments. However as they are not yet become so common, so cheap and so universally made and used, as one would wish an instrument of this nature to be, we have been encouraged to give this following account, for the general information of all persons who would make the same for their own use or for sale.

784. Having in the first place considered of what length one would propose the instrument to be, and consequently what diameter it will be necessary to give to the large speculum, for which there are ample instructions by Sir *Isaac Newton*'s table in the Philosophical Transactions aforesaid*, allowing about an inch more than the aperture in the table for the false figure of the edges, which very often happens; I say having determined these things, take a long pole of fir deal or any wood, of a little more than double the length of the instrument intended, and strike through each end of it two small steel points, and by one of them hang up the same against a wall perpendicularly; then take two pieces of thin plate brass well hammered, a little thicker than a six-pence; these may be about an inch and a half broad, and let their length be in respect of the diameter of the speculum as 3 to 2; *viz.* if the speculum be 8 inches diameter, these may be about 12. Fix each of these strongly with rivets between two thin bits of wainscot, so that a little more than a quarter of an inch in the breadth, may stand out from between the boards. Then fix up these pieces horizontally against the wall under your pole, and therewith, as with a beam compass, strike an arch upon each of them; then file each of them with a smooth file to the arch struck, so as one may be a convex and the other a concave arch of the same circle. These brasses are the gages to keep the speculum, and the tools on which it is ground, always to the same sphere. And that they may be therefore perfectly true to each other, it is necessary to grind them with fine emery one against the other, laying them on a flat table for that purpose, and fixing one of them to the table. To make the gages.

785. When the gages are perfectly true, let a piece of wood be turned about 2 tenths of an inch broader than the intended speculum, and some- Patterns for the specula.

* See another table Art. 364. and also Art. 362.

what

what thicker, which it is beſt to caſt in no caſe leſs than 2 tenths of an inch thick, and for ſpecula of 6, 8, or 10 inches broad, this ſhould be at leaſt three or 4 tenths thick when finiſhed[a]. This board being turned, take ſome common pewter and mix with it about $\frac{1}{10}$ of regulus of antimony, and with that wooden pattern caſt one of this pewter, which will be conſiderably harder than common pewter. Let this pewter pattern be truly turned in a lathe, and examined by means of the gages aforeſaid, as a pattern for caſting the ſpecula themſelves; and take care when it is turned that it be at leaſt $\frac{1}{20}$ of an inch thicker, and about $\frac{1}{10}$ of an inch broader than the ſpeculum intended to be caſt therefrom.

To make the moulds for caſting the patterns and the ſpecula.

786. The manner of making the moulds for caſting is now to be explained, and will ſerve for a direction as well for caſting this pewter pattern, as afterwards for caſting thereby the ſpeculum it ſelf. The flaſks had beſt be of iron and muſt be at leaſt 2 inches wider every way than the ſpeculum intended. In each flaſk there ſhould be the thickneſs at leaſt, of one inch of ſand. The caſting ſand which the common founders uſe from *Highgate*, will do as well as any; and any ſand will do which is naturally mixt with a ſmall proportion of clay, to make it ſtick. The ſand ſhould be as little wet as may be, and well beaten but not too hard. The ingates ſhould be cut ſo as to let the metal flow in, in four or five ſtreams over the whole upper part of the mould; otherwiſe whatever pores happen in the metal, will not be ſo equally diſperſed as they ſhould be over the whole face of the metal; theſe pores generally falling near the ingate ſtreams. Let the flaſks dry in the ſun for ſome hours or near a very gentle fire, otherwiſe they will warp and give the ſpeculum, when caſt, a wrong figure. For beſides ſaving the trouble in grinding, it is beſt on many accounts to have the ſpeculum caſt of a true figure; and it is for this reaſon, that it is beſt to caſt it from a hard pewter pattern, and not from a wooden one as founders uſually caſt.

The compoſition of the metal and manner of caſting it.

787. The next point that is to be conſidered is the compoſition or ingredients of the metal it ſelf for the ſpecula. As to this it may be ſaid in general, that any hard white metal that will take a poliſh will do more or leſs well. We have made trials of above 150 different mixtures and found none of them entirely free from all faults. Three parts of copper and one part and a quarter of tin will make a very hard white metal, but it is very liable to be more porous than it ſhould be, eſpecially if the metal be too much heated in melting. Six parts of good ſhruff braſs[b] and one part of tin, will make a whiter and harder metal; but the fume of

[a] The metal of my reflecting teleſcope, made by that excellent workman Mr. *Hearne* in *Dogwel-Court, White Friers* by *Fleet-Street*, is 6 inches broad and between 6 and 7 tenths thick; beſides a braſs plate ſoldered to its back an inch and half broad, with 4 holes in it to fix a ſhort handle to it mentioned in art. 790; having alſo a ſocket in the middle to ſkrew in another handle when finiſhed. Its focal diſtance is five foot.

[b] Plate-braſs, caſt and milled; the beſt comes from *Hamburgh*.

the

the calamy ſtone in braſs, leaves very often ſtreaks of ſcabrous parts in the ſurface of the metal; which if many utterly ſpoil it. Take two parts of the former mixt metal of copper and tin, and one part of the latter braſs and tin, this alſo will make a good metal; let the copper and the braſs be firſt melted together, and keep them in fuſion for half an hour or more; then clear the pot and put in the proper quantity of tin for both; which will inſtantly melt; ſtir it well about and pour it off immediately. This mixture may be melted over and over again in caſe of neceſſity, provided always care be taken that the fire is not ſuffered to be too violent. A common bellows furnace has been found moſt convenient for governing the fire, and ſome metals have ſucceeded which have been caſt by a common braſs-founder in their ordinary way of caſting; the compoſition having been firſt made and melted together for the ſpeculum and delivered to them only for caſting. There hath been tried alſo another mixture and another manner of caſting, which ſucceeded better than any of the abovementioned: it was copper, ſilver, regulus of antimony, tin and arſenick; and the metal was caſt in very hot moulds of braſs; but as this method is very expenſive and will never become common, it need be no further inſiſted upon in this place.

788. The metal being duly caſt, grind the ſurface of it quite bright upon a common grindſtone; keeping it by means of your convex gage as near to your intended figure as may be. When all the outward ſurface and all the ſand holes and falſe parts and inequalities are ground off, then provide a good thick ſtone; a common ſmall grinding ſtone will do very well. Let its diameter be to the diameter of the ſpeculum as 6 to 5; with another courſe ſtone and ſharp ſand or courſe emery, rub this ſtone till it fits the concave gage; and then with water and courſe emery at firſt and afterwards with finer rub your ſpeculum upon this ſtone until it forms it ſelf into a true portion of a ſphere fitting your convex gage. A different manner of moving the metal upon the ſtone, will incline it to form it ſelf ſomewhat of a ſmaller or larger ſphere. If it be ſtruck round and round, after the manner of glaſs-grinders, the ſtone will wear off at the outſides, and the metal will form it ſelf into a portion of a leſs ſphere. If it be ſtruck croſs and croſs the middle, it will flat the ſtone and become ſomewhat of a larger ſphere. There ſhould be uſed but very little emery at a time, and often changed; otherwiſe the metal, will always be of a ſmaller ſphere than the ſtone and will hardly take a true figure, eſpecially at the outſide. For the better grinding the metal it is neceſſary that this ſtone ſhould be placed firm on a ſtrong round board fixed firmly on a poſt to the floor, as is uſual with glaſs-grinders, and the ſame table or pillar will ſerve for the further grinding and poliſhing the ſpeculum.

Of rough grinding the metal.

789 When the metal is caſt and rough figured (which ſhould be done with taking off as little of the ſurface of the metal as is poſſible, becauſe that

Mr. Hadley's continuation of this chapter.

that cruſt ſeems to be generally harder and more ſolid than the inner parts,) the ſides and back of it ſhould be ſmoothed and finiſhed; leaſt the doing that afterwards ſhould make the metal caſt and ſpoil the figure of the foreſide.

Provide a concave braſs tool.

790. A round braſs plate muſt be caſt of ſufficient breadth and thickneſs; (for a ſpeculum 6 inches diameter I uſed a plate 8 or 9 inches broad, and $\frac{1}{2}$ an inch thick.) Let one ſide be turned to the concavity you deſign your ſpeculum ſhould have; on the other ſide let it have ſuch a handle faſtened as may make it eaſily manageable. This handle ſhould be as ſhort as conveniently it can, and fixt to the plate's back rather by ſome other manner than either by ſkrewing it into a hole in the metal, or by a broad ſhoulder ſkrewed againſt the back of it, for fear of bending the plate.

And a convex marble tool covered with whetſtones.

791. Have ready a round marble about $\frac{1}{8}$ or $\frac{1}{10}$ part broader than the braſs plate, and an inch or inch and $\frac{1}{4}$ thick: let this be cut by a ſtone-cutter to the ſame convexity on one ſide as the concavity of the plate; and then grind it with the plate and emery till all the marks of the chiſſel are out. This marble is to be covered with pieces of the fineſt blue hone or whetſtone, chooſing thoſe that are neareſt of a breadth and thickneſs, but chiefly thoſe that when wetted appear moſt even and uniform in their colour and grain. They are to be cut into ſquare bits, and theſe, having each one ſide ground concave on the convex marble with emery or fine ſand, are to be fixt cloſe down on it with ſome tough and ſtrong cement in the manner of a pavement, leaving a ſpace of a ſmall ſtraws breadth between each; their grain being likewiſe placed in an alternate direction as repreſented in the figure. I chooſe rather to diſperſe the ſquares that come out of the ſame whetſtone to a diſtance from one another, than to place them together. They muſt then be reduced to one common convex ſurface to fit the braſs plate; and if the cement happen to riſe any where between them, ſo as to come up even with their ſurface, it muſt be dug out; and ſo from time to time as often as the hones wear down to it. Upon theſe ſquare pieces of whetſtone the laſt figure is to be given to the ſpeculum.

Fig. 565.

And a convex glaſs tool, or a marble one.

792. Beſide theſe there will be wanted for the laſt poliſh either a very thick round glaſs plate, (its diameter being about the middle ſize between that of the braſs tool and that of the ſpeculum it ſelf,) or if that cannot be procured of near $\frac{1}{2}$ an inch in thickneſs, I imagine a piece of true black marble of the eveneſt grain and freeſt from white veins or threads may do in its ſtead. This glaſs or marble muſt be figured on one ſide to the braſs tool likewiſe, and is to ſerve for the finiſhing the poliſh of the ſpeculum, when covered with ſarcenet as ſhall be directed.

And a concave bruiſer.

793. A ſmaller braſs or metal plate, of the ſame concavity with the larger will be uſeful, as well to help to reduce the figure of the hones when-

whenever it appears to be too convex, as to ſerve for a bruiſer to rub down any gritty matter happening to be amongſt your putty, before you put the ſpeculum on the poliſher, when you renew the powder. Any of the ſpeculums which prove bad in caſting will ſerve for this purpoſe.

794. When all is thus far ready, let the marble with the blue hones be fixt in ſuch manner, that it may be often waſhed during your work, by throwing on it about $\frac{1}{8}$ of a pint of water at a time, without inconvenience. Then place the braſs tool on the hone pavement, and rub it backwards and forwards with almoſt a direct motion; yet carrying it by turns a little to the right and left, ſo as to go a little over the edges of the pavement every way, regularly turning the tool on its own axis, and alſo changing the direction of the ſtroke on the hones. This continue, keeping them always very wet, till you have got out all the rings remaining in the plate from the turning, and the blackneſs from grinding the marble or glaſs in it; and towards the latter end often waſhing away the mud which comes from the whetſtones. Of grinding the braſs tool on the whetſtones.

795. When this is done lay the braſs tool down, and in it grind again with fine emery the glaſs or marble deſigned for the laſt poliſher, giving it as true a figure as you can; in order to which you may obſerve the directions of Mr. *Huygens* in his Comm. de form. & pol. vitr*. But give it no farther poliſh. Of grinding the glaſs tool on the braſs one.

*Art. 758. &c.

796. Chooſe a piece of fine ſarcenet as free from rows and great threads as you can. Let it be 3 or 4 inches broader than the glaſs or marble; and turn down the edges of the ſarcenet round the ſides of the glaſs &c. and ſtrain it by lacing it on the backſide as tight and ſmooth as you can, having firſt cleared it of all wrinkles and folds with a ſmooth-iron, and drawn out the knots and gouty threads. Then wet it all over as evenly as you can with a pretty ſtrong ſolution of common pitch in ſpirit of wine; and when the ſpirit is dried out repeat the ſame, and if any bubbles or bliſters appear under the ſarcenet, endeavour to let them out with the point of a needle. This muſt be repeated till the ſilk is not only ſtuck every where firmly down to the glaſs or marble, but is quite filled with the pitch. A large painters pencil, made of ſquirrels hair, is of uſe for the ſpreading this varniſh equally on the ſilk, eſpecially when it begins to be full. It muſt then be ſet by for ſome days, for the ſpirit to dry well out of it, and the pitch to harden, before any thing more be done to it. If you do not care to wait ſo long, the pitch may be melted into the ſilk without diſſolving it in ſpirits. In order to this, ſtrain a ſecond thin ſilk over the firſt, but you need not be curious in the choice of it, and having heated all together as hot as you think the ſilk or glaſs will ſafely bear, pour on it a little melted pitch (firſt ſtrained through a rag) ſo much as you judge ſufficient to fill both ſilks; it muſt be kept hot ſome time till the pitch ſeems to have ſpread it ſelf evenly all over. If you find you cannot Of covering the glaſs tool with pitch and ſarcenet, for a poliſher.

get it to ſink all into the upper ſilk, but it ſtands above it any where, it is a ſign there was too much pitch layed on, which ſhould be taken away in thoſe places while it remains liquid, with a hot rag preſſed down on it. When all is cold again, ſtrip off the outward ſilk and cut away the uſeleſs looſe edges of the inward. To take off the ſuperfluous pitch where it lyes too thick, and reduce the whole to a regular ſurface, it muſt be rubbed in the braſs tool with a little ſoap and water, till they are coloured of a pretty deep brown with the pitch: then waſh them away, and repeat the ſame with more ſoap and water till the weaving of the ſilk appears every where as equally as you can make it. As this work takes up ſome time, you may expedite it by putting a few drops of ſpirit of wine to the ſoap and water, (which will help them to diſſolve and wear away the pitch ſomething faſter) till it comes towards a concluſion; and if there are any places where the pitch lies very thick, you may ſcrape it away with a ſharp knife.

Of preſerving and repairing the poliſher.

797. This poliſher muſt be carefully kept from all duſt and grit, but particularly from emery and filings of hard metals; and therefore ſhould not be uſed in the place where the others come. After they have ſerved a good while they are more apt to ſleek the metals than at firſt; to prevent which their ſurfaces may be taken off by rubbing them with ſoap and water in the tool as before, and then ſtriking them once or twice over with the beforementioned ſolution of pitch with a pencil, proceeding as before, only that you muſt not now put any ſpirit to your ſoap and water, nor will you need to change them above once or twice.

Of grinding the metal ſmooth.

798. You may now begin to give the figure to your ſpeculum on the hones, rubbing it and the braſs tool on them by turns, till both are all over equally bright; having firſt fixt on to the middle of the back of your ſpeculum a ſmall and low handle with only pitch ſtrained through a rag. For of all cements that ſeems the leaſt apt to bend the metals in ſticking theſe handles &c. on to them.

Of figuring the metal to the curvity of the poliſher.

799. The poliſher being fixt likewiſe in a proper manner for your work, rub either the metal it ſelf, or rather the beforementioned bruiſer, being firſt alſo figured on the hones, with a little putty waſhed very fine and fair water; till it begins to ſhew ſome poliſh. Then if you find it takes the poliſh unequally, that is more or leſs about the edges than in the middle, tis a ſign the braſs tool and metal &c. are more or leſs concave than to anſwer the convexity of the poliſher; and muſt be reduced to the curvature of this, rather than to attempt an alteration in the figure of the poliſher; which would be a much more difficult as well as laborious work. If the ſpeculum appears too flat, the larger braſs tool muſt be workt on the hones for ſome time, keeping its center near their circumference, with a circular motion; but concluding for 4 or 5 minutes with ſuch a motion as was before deſcribed[a]. Then figure the metal anew on the

[a] Art. 794.

the hones and try it again on the poliſher as before. If the metal be too concave, the ſurface of the hones may be flatted by rubbing the ſmaller braſs plate, or the beforementioned ill caſt metal, on the middle of them; with a direct but ſhort ſtroke, ſo as but juſt to reach over their circumference with the edge of it. Then the larger braſs is to be workt on them in the ſame manner; and laſt of all the metal to be poliſhed. When you find the braſs tool and hones &c. anſwer the curvature of the poliſher, you may then examine the truth of the figure of the ſpeculum more ſtrictly, to avoid the loſs of time and labour in finiſhing its poliſh while the figure is imperfect.

800. Place the ſpeculum in a vertical poſture on a table about $3\frac{1}{2}$ or 4 feet from the floor. On another table ſet a candle whoſe flame ſhould be about the level of the middle of the ſpeculum, and very near the center of its concavity. About $\frac{1}{2}$ an inch before the flame place a flat tin, or thin braſs, plate about 3 inches broad but 4 or 5 high, having ſeveral holes about the middle of different ſhapes and ſizes; ſome of them as ſmall as the point of the ſharpeſt needle will make them, the biggeſt about the ſize of a large muſtard ſeed: darken the room and move this candle and plate about on the table, till the light from the brighteſt part of the flame, paſſing through ſome of the larger holes to the ſpeculum, is reflected back ſo as to form the images of thoſe holes, cloſe without one of the ſide edges of that thin plate. Thoſe largeſt images in this caſe will be viſible, (although the ſpeculum have no other poliſh than what the hones give it,) when received on a thick white card held cloſe to that edge of the plate, if the back of the card be either blacked or ſo ſhaded that the candle may not ſhine through it; and the eye be alſo ſkreened from the candle's direct light. If any difficulty happens in diſcerning them, the plate may be removed and the image of the whole flame will be eaſily ſeen. Have ready an eye-glaſs whoſe focal diſtance may be ſomething greater than the double of that of the eye-glaſs you intend for the inſtrument when finiſhed: you may try ſeveral at your diſcretion. Let this be ſupported by a ſmall ſtand moveable on the table, and capable of raiſing and ſinking it, as the height of the flame requires, and of turning it into any direction. By means of this ſtand bring the eye-glaſs into ſuch a poſition, that the light from ſome of the holes, after its reflection from the ſpeculum, may be received perpendicularly on its ſurface; and that its diſtance from the ſpeculum be ſuch, that the reflected images of the holes may be ſeen diſtinctly through it, near the edge of the thin plate, by the light coming immediately from the ſpeculum: guide the candle and thin plate with one hand, and the ſtand carrying the eye-glaſs with the other, till you have got them into ſuch ſituation, that you ſee diſtinctly at the ſame time, through the eye-glaſs, the edge of the thin plate, and the image of one of the holes cloſe to it. Meaſure the exact diſtance of the middle

To find the radius of the ſphere of the metal.
Fig. 566.

middle of the ſpeculum from the thin plate directly againſt the flame, and alſo from the edge cloſe to which you ſee the image of the hole. If theſe meaſures are the ſame, ſet it down as the exact radius of concavity of your ſpeculum, and proper curvature for any that are to be poliſhed on your poliſher, though that will allow ſome latitude: If the meaſures aforeſaid differ, take the mean between them.

To examine the figure of the metal.

801. You will now alſo judge of the perfection of the ſpherical figure of your metal by the diſtinctneſs with which you ſee the repreſentations of the holes, with their raggedneſs, duſts and ſmall hairs ſticking in them: and you will be able to judge of this more exactly and likewiſe to diſcover the particular defects of your ſpeculum, by placing the eye-glaſs ſo as to ſee one of the ſmalleſt holes in or near its axis; and then by turns ſhoving the eye-glaſs a very little forward towards the ſpeculum and pulling it away from it by turns, letting the candle and plate ſtand ſtill in the mean time. By this means you will obſerve in what manner the light from the metal comes to a point, to form the images, and opens again after it has paſt it. If the area of the light, juſt as it comes to or parts from the point, appears not round but oval, ſquariſh, or triangular &c. it is a ſign that the ſections of the ſpecular ſurface, through ſeveral diameters of it, have not the ſame curvature. If the light, juſt before it comes to a point, have a brighter circle round the circumference, and a greater darkneſs near the center, than after it has croſſed and is parting again; the ſurface is more curve towards the circumference and flatter about the center, like that of a prolate ſpheroid round the extremities of its axis; and the ill effects of this figure will be more ſenſible when it comes to be uſed in the teleſcope. But if the light appears more hazy and undefined near the edges, and brighter in the middle before its meeting than afterwards, the metal is then more curve at its center and leſs towards the circumference; and if it be in a proper degree, may probably come near the true parabolick figure. But the ſkill to judge well of this, muſt be acquired by obſervation.

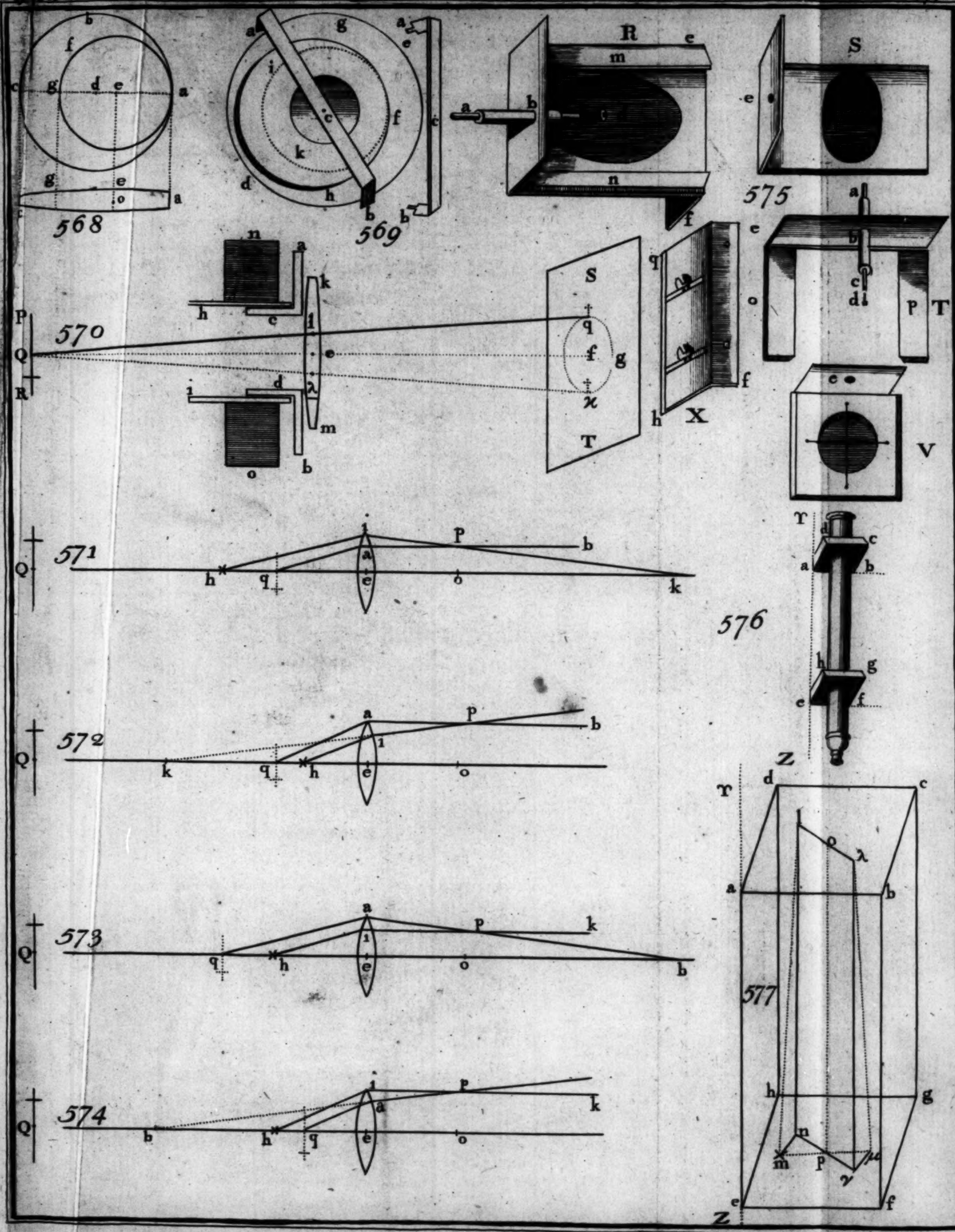
A caution to be obſerved.

802. In performing the foregoing examination, the image muſt be reflected back as near the hole it ſelf as the eye's approach to the candle will admit of; that the obliquity of the reflection may not occaſion any ſenſible errors: in order to which the eye ſhould be ſkreened from the candle; and the glaring light which may diſturb the obſervation may be ſtill more effectually ſhut out, by placing a plate, with a ſmall hole in it, in that focus of the eye-glaſs which is next the eye. *A* is the ſpeculum, *B* the candle and plate with the ſmall holes, *C* the cell with the eye-glaſs and plate behind it.

Fig. 566.

A circumſtance varied.

803. Inſtead of the flame of the candle and plate with ſmall holes, I ſometimes made uſe of a piece of glaſs thick ſtuck with globules of quickſilver, ſtrained through a leather and let to fall on it in a dew; placing this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this glaſs near a window and the ſpeculum at a diſtance on the ſide of the room, being it ſelf and every thing about it as much in the dark as can be. The light of the window reflected from the globules of mercury, appearing as ſo many ſtars, ſerves inſtead of the ſmall holes, with this advantage, that the reflection from the metal may be very near at right angles.

804. If the figure of the metal appears not ſatisfactory, the hones muſt be workt with the braſs tool and water for 2 or 3 minutes with the motion &c. firſt directed[a]; then work the metal on them with the like motion, and ſuch length of the ſtroke as may carry the edge of it about $\frac{1}{6}$ or $\frac{1}{4}$ of its diameter beyond that of the hone pavement each way; carry it likewiſe by turns to the right and left to about the ſame diſtance. Continue this about 5 minutes, not preſſing the metal down to the hones with any more than its own weight, and obſerve that the oftener the mud is waſhed away the more truly ſpherical the figure of the ſpeculum will generally be: but the leaving a little more of this mud on the ſtones has ſometimes ſeemed to give the metal a parabolick figure. I have likewiſe given it the ſame by concluding with a kind of ſpiral motion of the center of the metal, near the circumference of the hones, in the manner repreſented in the figure, for about half a minute.

To correct the figure of the metal.

a Art. 794.

Fig. 567.

805. If after ſeveral tryals the metal appears to have always the ſame kind of defect, and anſwering to the ſame particular part of the metal, it is a ſign of a different hardneſs in its ſeveral parts, which will make it very difficult to bring that ſpeculum to perfection. In working the tool or metals on the hones, there will often appear little ſpots in them, much blacker and harder than the reſt; theſe muſt be dug out as faſt as they appear.

Signs of a bad metal.

806. When the figure is to your mind, you may proceed to finiſh the poliſh on the ſarcenet with very little putty, and that diluted with a great deal of water. Before you put the putty and water on it, obſerve by holding it very obliquely between your eyes and the light, if it have any liſts or ſtripes acroſs it, which appear more gloſſy than the reſt. If it be ſo, let the motion of the metal in poliſhing be directly athwart theſe liſts and not along with them, nor ever circular. In other reſpects you may obſerve the ſame directions as were before given for its motion on the hones; not forgetting, after every 15 or 20 ſtrokes, to turn it on its axis about $\frac{1}{12}$ or $\frac{1}{16}$ of a revolution. As the poliſher grows dry you will find the metal ſtick to it more and move ſtiffer; at which time it both poliſhes faſter and with a better gloſs: only take care that it grows not ſo dry as for the metal to take hold of the ſarcenet and cut it up; or for the pitch and putty to fix in little knobs here and there on it: which if it happen will preſently ſpoil the figure. As faſt therefore as the ſarcenet appears to be growing dry at any of its edges, touch the place with the end of a feather dipt

To poliſh the metal.

dipt in clean water: you may uſe the ſame putty at leaſt half an hour. As often as you change it waſh the old clean away, and rub the new about firſt with your bruiſer, to ſee if there be any gritty or groſs particles in it, and rub them away for fear of ſcratching the metal; then laying down the edge of the ſpeculum a little way on the edge of the poliſher, where it is well covered with water, ſlide it on to the middle, and then proceed. The leſs putty you uſe at a time the ſlower the work will advance, but if you uſe too much, it will ſpoil a little of the figure round the edges. It will not want any conſiderable force to preſs it down; but if it be of 5 or 6 inches diameter or more it will be very laborious to go through the poliſh without ſome kind of machine. One ſomething like that deſcribed by Mr. *Huygens* in the aforenamed treatiſe*, may do very well; eſpecially if there be added to it ſome contrivance to hinder the metal's turning irregularly on its axis, and to give the workman a command of it in regard to the lateral motion.

* Art. 774.

CHAPTER III.

How to center an object-glaſs.

807. A CIRCULAR object-glaſs is ſaid to be truly centered, when the center of its circumference is ſituated in the axis of the glaſs[a], and to be ill centered when the center of its circumference lies beſide the axis. Thus let *d* be the center of the circumference of an object-glaſs *abc*; and ſuppoſe *e* to be the point where its axis cuts its upper ſurface. If the points *d* and *e* do not coincide, the glaſs is ill centered. Let *afg* be the greateſt circle that can be deſcribed about the center *e*, and by grinding away all the margin without this circle, the glaſs will become truly centered. Now the center *e* which lyes in the axis of the glaſs, may be found by ſeveral methods, but I prefer this that follows.

a Art. 38. Fig. 568.

808. Let a couple of ſhort cylindrical tubes be turned in wood or braſs, and let the convexity of the narrower be ſo exactly fitted to the concavity of the wider as juſt to turn round in it, with eaſe but without waddling; and let the planes of the baſes of the tubes be exactly perpendicular to their ſides. Place the baſe of the narrower tube upon a ſmooth braſs plate or a wooden board of an equal thickneſs, and with any ſharp pointed tool, deſcribe a true circle upon the board round the outward circumference of the baſe; and upon the center of this circle, to be found when the tube is removed, deſcribe a larger circle upon the board. Theſe two circles ſhould be ſo proportioned, that the one may be ſomewhat greater and the other ſomewhat ſmaller than any of the glaſſes intended to be centered by them. Then having cleared out all the wood within the inner circle, put the end of the tube into this hole, and there faſten it with glew,

Fig. 569.

glew, so that the base of the tube may lye in the surface of the board: then having fixt the wider tube very firmly in a hole made in a window-shutter, and having darkened the room, lay the glass to be centered upon the board fixt to the narrower tube; and having placed the center of it as near as you can guess over the center of the hole, fix it to the board with two or three lumps of pitch, or soft cement, placed at its circumference. Then put the narrower tube into the wider as far as it can go, and fix up a smooth skreen of white paper to receive the pictures of objects that lye before the window; and when they appear distinct upon the skreen, let the inner tube be turned round about its axis; and if the center of the glass happens to be in this axis, the picture will be perfectly at rest upon the skreen; if not, every point of it will describe a circle. With a pencil mark the highest and the lowest places of any one circle, described by some remarkable point in that part of the picture which appears most distinct; and when this point of the picture is brought to the highest mark, stop the circular motion of the tube, and keeping it in that position depress the object-glass till the point aforesaid falls exactly in the middle between the two marks. Then turn the tube round again, and the point of the picture will either rest there or will describe a much smaller circle than before, which must be reduced to a quiescent point by repeating the same operation. Then I say the center (of refraction [a]) of the glass will lye in the axis of the tube, and by consequence will be equidistant from the circumference of the large circle described upon the board fixt to it. Now to describe a circle upon the glass *fgb* about its center of refraction, let a long slender brass plate *acb* be bent square at each end, as represented in the figure, leaving a piece in the middle equal in length to the diameter of the large circle *adbe* that was described upon the board; and let the square ends of the plate be filed away so as to leave a little round pin in the middle of each. Then having laid it over the glass, along any diameter of the large circle *adbe*, make two holes in the board to receive the pins *a*, *b*; and find the center of this circle upon the long plate; and with this center *c* describe as large a circle as you can, upon the glass underneath, with a diamond-pointed compass; and grind away all the margine as far as this circle *fik*, in a deep tool for grinding eye-glasses; and then the glass will be truly centered. If the pitch or cement be too soft to keep the glass from slipping, while the circle is describing, it may be fixt firmer with wax or harder cement.

a Art. 228.

Fig. 569.

809. To shew the reason of this practice, the 570th figure represents a section of the object-glass *klm*, of the board *ab*, and of the tubes *cd* and *hi*, and of the window-shutter *no*. Imagine the plane of this section, or of the scheme, to pass through *e*, a point in the glass which keeps its place while the rest are turning round it, by the motion of the tube; let it also pass through *l* the center of refraction in the glass, and cut an object in

The reason of this practice. Fig. 570.

 the

the line PQR; then let a pencil of rays flowing from any Q be collected to the focus q upon the ſkreen ST; and the points Q, l, q will be in a ſtraight line deſcribed by the axis, or principal ray, of the pencil[a]. Draw Qef cutting the ſkreen in f; and while the tube is turning round, the line Qlq will deſcribe a conical ſurface whoſe axis is the fixt line Qef; and therefore the focus q or image of the point Q will deſcribe a circle qgx about f, to be found upon the ſkreen by biſecting the interval qx between the higheſt and loweſt points of the circle. Now as f is the center of this circle, ſo e is the center of another circle deſcribed by l; therefore by depreſſing the glaſs kl along the ſurface of the board ab till the image q falls upon the mark f, the point l will be depreſſed to e the center of motion, and then it will be in the axis of the tube, and conſequently equidiſtant from the circumference of the circle deſcribed upon the board ab; and here it is plain that the image q will be at reſt in the point f.

a Art. 228.

810. It is not neceſſary to the accuracy of the practice that the point Q ſhould be in the axis of the glaſs. For in fig. 181, if the glaſs KLM be turned about its axis QLq, the image p of any collateral point P will remain at reſt; becauſe the points P, L are at reſt, and the axis PLp of the oblique pencil is a ſtraight line.

811. The chief advantage of having a glaſs well centered is this, that the rays coming through any given hole or aperture whoſe center coincides with the axis of the glaſs, will form a diſtincter image, than if that center lay beſide the axis; becauſe the aberrations of the rays from the geometical focus of the pencil, are as the diſtances of their points of incidence from the center of refractions in the glaſs[b].

b Art. 325.

To examine how well an object glaſs is centered.

812. If the picture be received upon the unpoliſhed ſide of a piece of plane glaſs, inſtead of the paper ST, its motion may be diſcerned more accurately by viewing it from behind through a convex eye-glaſs; as in a teleſcope where croſs-hairs are uſually ſtrained over a hole put into the place of the rough glaſs. Therefore as object-glaſſes are commonly included in cells that ſkrew upon the end of the tube, one may examine whether they be pretty well centered, by fixing the tube, and by obſerving while the cell is unſkrewed whether the hairs keep fixt upon the ſame lines of an object ſeen through the teleſcope.

To place the croſs hairs in the focus of a teleſcope.

813. In the application of teleſcopes to aſtronomical inſtruments and many other purpoſes, it is abſolutely neceſſary to fix the plane of the croſs hairs exactly upon the plane of the picture of an object; which may eaſily be done from a knowledge of the following properties. Firſt let the interval between the two convex glaſſes of the teleſcope be adjuſted to ſhew an object diſtinctly; and if the hairs appear confuſed, they will ſeem to dance upon the object while the eye moves ſideways; and in dancing if they ſeem to move the ſame way as the eye does, they lye behind the picture of the object, but if they move the contrary way, they

lye

lye before it; and muſt be removed accordingly till they appear diſtinct; and then they will alſo ſeem fixt upon the object notwithſtanding the motion of the eye. Secondly let the interval between the hairs and the eye-glaſs be firſt adjuſted, till the hairs appear diſtinct; then if the object appears confuſed, it will alſo appear to dance while the eye moves ſideways; and in dancing if it moves the ſame way as the eye does, its picture is behind the hairs; if the contrary way, its picture is before them: and to bring it to the hairs, either the object-glaſs muſt be moved, or elſe the hairs and eye-glaſs both together. In both theſe caſes it is the confuſed object (for the hairs may alſo be called ſo) that ſeems to move, and the diſtinct one to ſtand ſtill; as in viſion with the naked eye. For to a perſon in motion, ſuppoſe he be walking, any object appears fixt that he fixes his eyes upon and ſees diſtinctly, while the reſt that are nearer or farther off, appear confuſed and in motion; the reaſon of it is too obvious to need an explanation. But to ſhew it in the teleſcope, let h be the interſection of the croſs-hairs, and hik a pencil of rays flowing from it; which after refraction through the eye-glaſs eai, belong to the focus k, either at a finite or an infinite diſtance. Draw he, the axis of this pencil, cutting the object in Q and its picture in q; and let the emergent rays of the pencil qab, flowing from q, cut the emergent rays of the former pencil in the points p, and belong to the focus b, either at a finite or an infinite diſtance. Now if the eye be placed at any point o in the common axis of theſe pencils, the points h, Q will both appear in the ſame direction oe; but if the eye be moved ſideways from o to p, the point Q will appear in the direction pa*, and the point h in the direction pi. And from hence the reaſon of the foregoing caſes will be ſufficiently manifeſt, by attending to the figures. Laſtly while the focuſes h, q are disjoined, the mutual inclination of the emergent rays in one pencil, muſt be different from the mutual inclination of the emergent rays in the other; and ſo the humours of the eye cannot be adapted to collect the rays in both pencils to two diſtinct points. If one be diſtinct the other will be confuſed and in a different part of the retina; (except when the eye is in the axis;) but when the focuſes h, q are united, the focuſes k, b of the emergent rays will alſo be united; and conſequently the coinciding rays of both pencils will be united in the ſame point of the retina, wherever the pupil of the eye be placed; and therefore the correſponding points of the object and croſs-hairs will appear fixt together without any parallax.

Fig. 571 to 574.

* Art. 102.

814. When the place of the hairs is thus determined, it may be of uſe to meaſure their diſtance from the object-glaſs; which is the exacteſt way of finding its focal diſtance if the object be very remote. And to keep this diſtance always the ſame whenever the teleſcope is uſed, it is convenient to have marks or ſtops at the end of each joint of the tube. For then whatever eye-glaſs be applyed, the object and hairs will both appear diſtinct

The focal diſtance thereby determined.

ſtinct at the ſame time, and without parallax. Inſtead of hairs the fineſt ſilver wires are now made uſe of, but are ſtill called hairs.

The line of ſight what. a Art. 228.

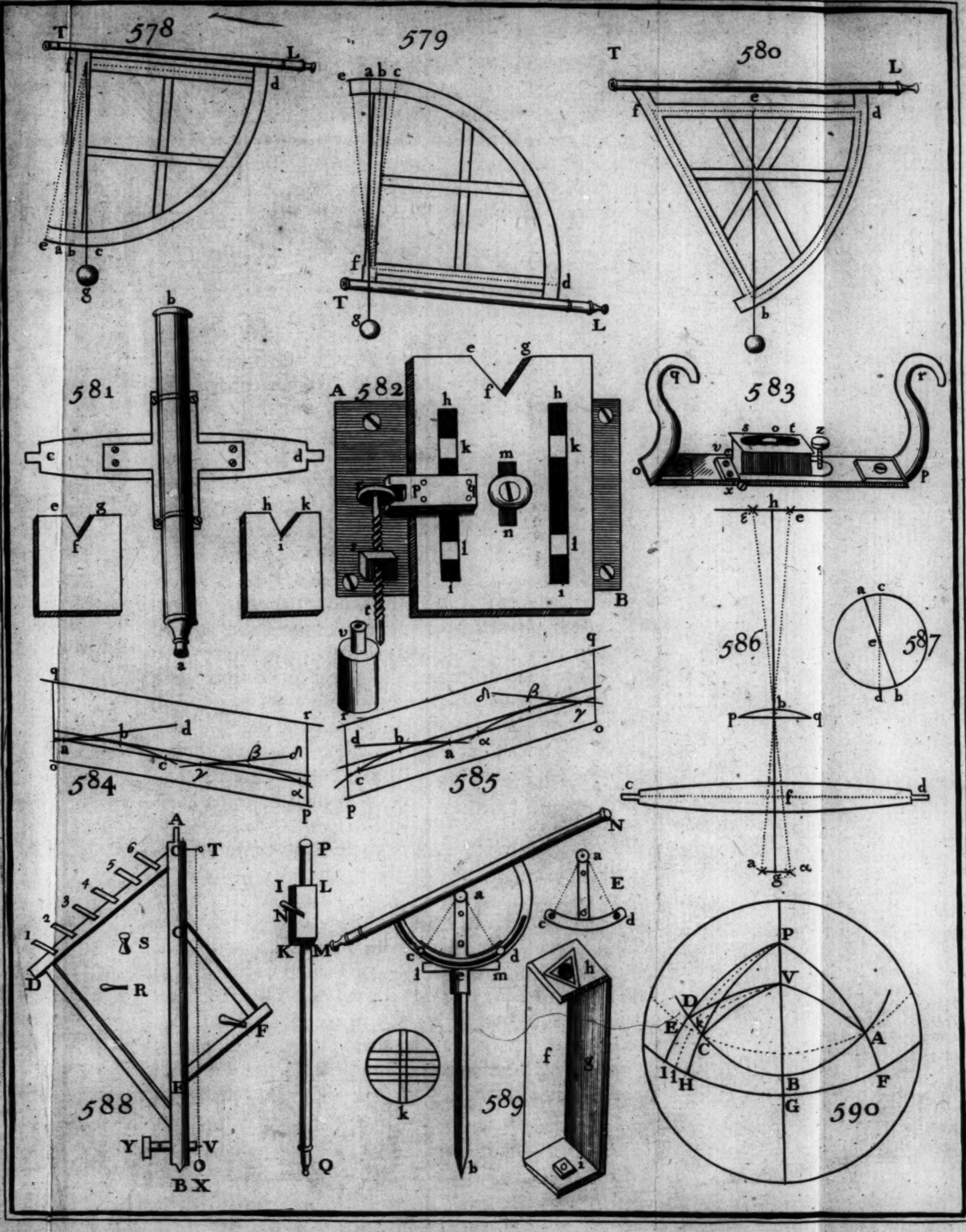
815. A line drawn from the interſection of the hairs through the center of refractions [a] in the object-glaſs, whether it coincides with the axis of the glaſs or is inclined to it, is called the line of Collimation or line of ſight. Becauſe this line produced falls upon the object in that point, whoſe image falls upon the interſection of the hairs: and therefore the ſtraight ray that deſcribes this line, anſwers to the viſual ray by which we take aim at an object with plain ſights. Hence when the object-glaſs and hairs are firmly fixed in a ſtrong tube or to a ſtraight ruler, it is manifeſt that the line of ſight is as immutable with reſpect to the tube, as if two little holes or plain ſights were ſubſtituted in the places of the interſection of the hairs and of the center of refractions in the object-glaſs.

The mechaniſm for moving the croſs-hairs.

816. In order to ſet the line of ſight parallel to a given line upon the plane of an inſtrument, the object-glaſs muſt be firmly fixt, and the ring or plate that carries the croſs hairs muſt have two gradual motions in its own plane by two ſkrews at right angles to each other; for by this means the interſection of the hairs may be moved to any given point in that plane. Theſe motions are effected by 3 braſs plates laid over one another. The uppermoſt, having a circular hole in it, over which the hairs are ſtrained, ſlides over the middlemoſt in the direction of an oblong hole cut in it, whoſe breadth is ſomewhat greater than that of the hole above it; and theſe two together ſlide ſideways over the undermoſt plate in which there is a larger oval hole. I ſhall deſcribe theſe plates more particularly in a contrary order. On each ſide of the oval hole in the middle of the plate *R* laſt mentioned, two braſs ledges *m*, *n* are firmly riveted to receive the dove-tailed ſides of the plate *S*; and the contiguous ends of both theſe plates are turned up ſquare at *b* and *e*; and through a hole *b*, in the middle of the part turned up in the larger plate *R*, there works a pretty thick ſkrew *a b c*, whoſe fore end *c* being filed to a neck, goes through a hole *e* in the lip of the other plate *S*; and in the end of the neck *c* there is made a ſmall ſkrew-hole to receive a ſkrew-pin *d*; ſo that by turning the ſkrew *a b c* with a kind of a watch-key, the plate *S* is moved backwards or forwards between the ledges *m*, *n*. The figure *T* repreſents two more ledges *o*, *p* that are to be riveted upon the plate *S*; theſe ledges are part of the plate *T* turned up at right angles to them, in which part there is the like contrivance of a ſkrew *a b c d* to move a third plate *V* between the ledges *o*, *p*, at right angles to the former motion. The ſilver wires are ſtrained over the hole in the plate *V* by four ſmall peggs, that fix them in four little holes. The other end of the plate *R*, oppoſite to the part *b* that carries the ſkrew, is bent ſquare the contrary way to the part *b*; or which anſwers the ſame purpoſe, one ledge *e f* of the plate *X* bent ſquare, is riveted to the backſide of the plate *R* at the end oppoſite to the ſkrew *b*;

Fig. 575.

and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and its other ledge gh is ſkrewed to the ſide of the tube of the teleſcope; and the necks of the ſkrews go through long ſlits in this ledge, to give liberty of placing it accurately at the due diſtance from the object-glaſs: and for the purpoſe of letting this braſs work into the tube, two large ſlits muſt be cut in two contiguous ſides of it: one of which may beſt be covered with a thin piece of horn, to admit the light of a candle upon the hairs in obſerving ſmall ſtars in the night time.

CHAPTER IV.

Rectification of teleſcopick ſights.

817. To make the line of ſight through a moveable teleſcope parallel to a given line YZ upon a fixt plane; let the ends of the tube of the teleſcope, whether ſquare or cylindrical, be put through two holes in two ſquare plates $abcd$, and $efgh$, made exactly equal to each other, and ſo fixt to the tube that the ſides of the one may be exactly parallel to the ſides of the other; which is eaſily done by applying their corners a, e to the given line YZ and by drawing two lines ai, ek perpendicular to it, upon the given plane, and by making all the correſponding ſides as ab, ef coincide with theſe perpendiculars. Then obſerve what point of a remote object is covered by the interſection of the hairs when the corners a, e touch the given line YZ, and likewiſe what other point is covered by them when the oppoſite corners c, g touch the ſame line in the ſame places, that is when the teleſcope is turned upſide down or half round. Then conceiving theſe two points of the object to be connected by a ſtraight line, move the croſs-hairs by the two ſkrews till you judge their interſection biſects that line; and by repeating the ſame practice you may ſoon bring the interſection of the hairs to cover one and the ſame point of the object, when the oppoſite corners of the ſquares are ſucceſſively applyed to the line YZ; and then the line of ſight will be parallel to it. To rectify the line of ſight. Fig. 576.

To ſhew the reaſon of this practice, we may ſuppoſe the center of refraction in the object-glaſs to be any point l of the ſquare $abcd$, and the interſection of the hairs to be any point m of the ſquare $efgh$. Upon the plane of the firſt ſquare, and through its center o, draw $lo\lambda$, and take $o\lambda$ equal to ol; alſo upon the plane of the ſecond ſquare, and through its center p, draw $mp\mu$, and take $p\mu$ equal to pm. Joyn lm and $\lambda\mu$, and ſuppoſing ln and $\lambda\nu$ parallel to the axis op, join mn, $n\nu$, $\nu\mu$. Then becauſe the reſpective ſides about the equal angles mpn, $\mu p\nu$ are made equal, the lines mn, $\mu\nu$ oppoſite to them are alſo equal and parallel. Now the parallel lines nl, po, $\nu\lambda$ produced will fall upon a remote object in 3 points ſo cloſe together as to appear like a ſingle point through the teleſcope; Fig. 577.

and

and consequently the planes of the parallel triangles lmn, $\lambda\mu\nu$ produced will cut the same object in two parallel lines so close together as to appear but one line through the telescope; and since the angles mln, $\mu\lambda\nu$ are equal, the intersection of the hairs, now at m and then turned half round to μ, will cover two points in that line equidistant from the point abovementioned, and on opposite sides of it; therefore by removing the intersection from m to n, it will appear to bisect the interval between those two points; and then the line of sight nl will be parallel to the axis po, and to the sides of the parallelopiped, and also to the given line YZ.

The application.

818. A telescope thus prepared may be useful upon several occasions; as if it be required to rectify the hairs in a telescope fixt to any instrument, so as to set the line of sight parallel to a given line upon the plane of the instrument. Apply the corners of the squares of the telescope abovementioned to the given line, and observing what point of a remote object is covered by its cross hairs, move the cross hairs of the fixt telescope till they cover the same point of the object, and the business is done.

Rectification of a quadrant.

819. But the telescopick sights of quadrants and sectants, whose planes may be readily placed in any given posture, may be rectified by a plumb-line. I will here transcribe an account of these rectifications from Mr. *Molyneux*'s Dioptricks p. 238. "I come now to the rectification of these sights on quadrants and sextants, for taking angles. This may be done either before or after the division into degrees, &c. are made on the limb of the quadrant. If it be done before, then we suppose the telescope TL fixt to the quadrant, which we suppose continued a little farther than the fourth part of a circle. Choosing then an object pretty near the horizon; let us look through the telescope, in the usual posture of observation, and observe the point in the object marked by the cross-hairs; and at the same time we are to note most nicely the point c, which the plumb-line fcg, hung from the center f of the quadrant, cuts on the limb. Then we are to invert the quadrant into the posture of fig. 579, (which is easily done by the usual contrivances for managing great quadrants, by toothed semicircles and endless skrews) keeping still the telescope TL nighly upon the same height from the ground as before, unless the object we look at, be so far distant, that the breadth of the quadrant subtends but an insensible angle. But yet for certainty, it is better to keep the telescope, as it is said, upon the same height from the floor; then direct the telescope TL, that the cross-hairs may cover exactly the same point in the object, as before in the posture of fig. 578. And hanging now the plumb-line afg on the limb of the quadrant; let us remove it to and fro, till we find out the exact point a, from which the plumb-line being hung, shall most nicely hang over the center of the quadrant f. Then carefully marking the point a, let us divide the arch ca into two equal parts in b; and drawing bf, the point b is the point from which we are to begin the divisions of

Fig. 578, 579.

of the quadrant: and the line of collimation through the teleſcopick ſight, ſtands exactly at right angles to the line *bf*. So that the quadrant *bfd* being compleated and divided, the ſaid line of ſight through the teleſcope runs exquiſitely parallel to the line *fd*.

820. In the next place, ſuppoſing the quadrant *bfd* truly compleated and divided; and that we deſigned to fix thereto the teleſcopick ſight *TL*; ſo that the line of ſight may run exactly at right angles to the line *bf*, or parallel to the line *df*. We are to do as in the foregoing praxis. And if in dividing the arch *ac*, we find its half exactly coincident with the point *b*, we have our deſire. But if it differs from the point *b*, and falls between *b* and *d*, then the line of collimation through the teleſcope ſtands at an obtuſe angle with the line *bf*; and the inſtrument errs in exceſs: if this half arch fall without *b* and *d*, then the line of collimation makes an acute angle with the line *bf*; and the inſtrument errs in defect. And by often tryals, we are to remove the croſs-hairs within the tube, ſo much, as is requiſite to correct this error. And when we have thus rectified them to their due place, there they are to be ſtrongly fixt. Or elſe, in obſervations taken by this inſtrument, we are to make allowance for this error; by ſubtracting from (if it be in exceſs) or by adding to (if it be in defect), each obſervation ſo much, as we find the error to be.

821. The reaſon of this rectification is moſt plain; for it is manifeſt, that *cfd* wants of a full quadrant, as much as *afd* exceeds a quadrant. So the difference of the two arches in the two poſtures being *ac*; half this difference *bc* added in fig. 578, or *ab* ſubtracted in fig. 579, makes *bd* a compleat quadrant.

822. If we find our inſtrument errs in taking angles, and we deſire to know the error more nicely, than perhaps the diviſions of the inſtrument it ſelf will ſhew it, we are to do thus. Suppoſing the quadrant *bfd* already accurately divided, and that the plumb-line plays over the point *c*; and upon the inverſion of the inſtrument, we find that before we can get it to play exactly over the center *f*, we muſt hang it over the point *e*; ſo that the arch *eb* exceeds *bc* by the arch *ea*; it is plain that the angle *efa* is the error of the inſtrument: for had the plumb-line hung over *a*, and over the center *f* in this latter poſture, the inſtrument had been exact; becauſe *a* is as much on one ſide *b*, as *c* is on the other ſide *b*. Wherefore *efa* being the angle, by which our inſtrument errs in obſervation, let us turn the inſtrument into the uſual poſture of obſervation, as in fig. 578 and hanging the plumb-line on the center *f*; let us bring it to play nicely on the point *e*, and obſerve what diſtant object is covered by the croſs-hairs: then let us bring it to play exactly on the point *a*, and obſerve likewiſe what diſtant object is pointed at by the teleſcope-hairs. Laſtly, by a large teleſcope and micrometer, let us meaſure the angle between theſe two objects, and we ſhall have the angle of error much more nicely, than

in

it is possible the angle efa should be given by the divisions on the limb of the quadrant ea. And thus much for adjusting a quadrant.

Rectification of a sextant. Fig. 580.

823. A sextant is rectified in like manner; if we consider that if from the center f to the beginning of the divisions d there be drawn the radius fd; and it be divided equally in c; and from c there be supended the plumb-line cb: when the plumb-line hangs over the 60th degree at b; then the line fd lies horizontal: and consequently, if the line of collimation through the tube be parallel to fd, this line also lies horizontal. To try which, whilst the sextant stands in this posture, observe the object marked by the cross hairs; then invert the sextant; and over the point b hang the plumb-line; and when from the point b the plumb-line hangs over the middle point c, then again is the line fd horizontal in this posture. Mark then, whether the cross hairs cover the same object as before; if they do, then the line of collimation is parallel to fd; if they do not, but the point in the object marked in this latter posture be higher than the point marked in the first posture, the instrument errs in excess; if it be lower, the instrument errs in defect. And either we are to remove the cross hairs, till we bring all to rights, and there fix them; or by the methods before laid down in the rectification of the quadrant, we are to find the quantity of this erroneous angle, and to allow for it in observation.

Rectification of a moveable sight.

824. In instruments furnished with two pair of telescopick sights, one on a fixt arm, and the other on a moveable arm (by the ancients termed an *Alidade*); it is easy rectifying the sights on the moveable arm thus. After the sights on the fixt arm are rectified by what foregoes; bring the index of the moveable arm to the beginning of the divisions on the limb of the instrument, be it quadrant or sextant, &c. it is then manifest, that the line of collimation through the moveable telescope (if it be right) should lye parallel to the line of collimation through the fixt telescope. Observe therefore, whether the cross hairs in both telescopes do at the same time cut the same star, or fall on the same point in an object distant three or four miles. If they do, then the moveable telescope agreeing with the fixt, and the fixt being supposed rectified to the divisions on the instrument, the moveable is right likewise. But if the hairs in the moveable telescope do not agree in marking the same point with the cross hairs in the fixt telescope; then the hairs in this moveable telescope are to be removed (by whatever contrivance there is for that purpose) and brought to rights, and there fixt.

825. There are other methods propounded for rectifying telescopick-sights on other sorts of instruments, by means of observations towards the zenith, as our former methods have been imployed towards the horizon. But it is sufficient here to lay down only what foregoes, as being of the greatest and most frequent use: referring for the others to M. *Picard*'s Trea-

Treatise of the *measure of a degree of a great circle of the Earth*, published at the end of the *Memoirs of a Natural History of Animals* &c. by the Royal Academy at *Paris*; translated into English and printed at *London* 1688. fol.

Chapter V.

A meridian telescope and its uses.

826. A telescope *ab* fixt at right angles to an axis *cd*, and turned about it in the plane of the meridian, is called a meridian telescope. The vulgar use of it is to correct the motion of a clock or watch, by observing day after day the exact time when the sun or a star comes to the meridian. It serves also for other uses to be mentioned hereafter. A meridian telescope what. Fig. 581.

827. The axis *cd* consists of a strong brass plate, broader in the middle than towards the ends, having another long plate placed edgeways and soldered along the middle of the backside of it from one end to the other. This plate is also broader in the middle than towards the ends to prevent its springing. To the ends of these plates two solid pieces of brass are firmly jointed and soldered; and the whole being placed in a turning lathe, the solid ends are turned truly cylindrical, to an equal thickness, and are also well smoothed and polished with oyl and proper powders. Flat upon the broader plate of the axis, there lyes another brass plate, in the shape of a cross; one bar of it being parallel, and firmly skrewed, to the axis. The ends of the other bar, being turned up at right angles, have semicircular notches filed in them, to receive the cylindrical tube of the telescope; which is made of brass to prevent its warping; and is firmly fixt in these notches by two half collars of brass, which go over the tube and are skrewed to the ends of the semicircular notches; as represented in the figure. The cylindrical ends of the transverse axis *cd*, are to be placed upon two angular notches, *efg* and *hik*, filed in two thick brass plates, and well planed and polished, for the axis to turn very smoothly upon them. In order to bring the axis *cd* exactly perpendicular to the plane of the meridian, each of the notched plates are contrived to be moveable by a skrew, one upwards or downwards and the other sideways; by sliding them upon the surfaces of two other brass plates immoveably fixt to two posts or freestone pillars. The telescope and transverse axis described.

828. In order to move the notched plate *efg* up or down, two long upright slits *hi* are cut through it; in each of which, at *k* and *l*, are two square studs of brass, firmly riveted to the fixt back-plate *AB*; being exactly as broad as the slits that slide by them. In the middle between them there is also a shorter parallel slit *mn*, through which a broad headed skrew- How the supporters are moved. Fig. 582.

 pin

pin is ſkrewed into the back-plate, having a round ſpringing plate under the head of it; thus the two plates are preſſed together or relaxed, according as the notched plate is to be fixt or moved. To perform this motion gradually, *p q* repreſents a ſmall piece of another braſs plate, laid upon the fore-plate and riveted to the middle of one of its upright ſides. In the end *p*, which projects over this ſide, there is a ſmall ſquare notch, which receives the circumference of a ſmall circular plate *r*, like a ſkrew-head, fixt perpendicular to a long ſkrew *rst*, placed parallel to this upright ſide of the notched plate. This ſkrew works round in a concave ſkrew in the middle of a thick braſs ſtud at *s*, firmly riveted to the back-plate; and being turned by a hollow key *v*, put upon the ſquare end of it, it gradually raiſes or depreſſes the notched plate *efg*. Which is fixt more firmly at its proper height by turning the middle ſkrew at *o* abovementioned.

The angular notch that ſupports the other end of the tranſverſe axis is moveable ſideways by the ſame contrivance: as will appear by ſuppoſing the notch *efg* to be cut in the ſide oppoſite to the ſkrew *rst*, and this ſide to be turned upwards.

Spirit level. Fig. 581, 583.

829. The tranſverſe axis is to be placed horizontal upon the angular notches by a ſpirit level; conſiſting of a hollow glaſs tube *st*, 5 or 6 inches long; not quite ſtraight, but as little incurvated as poſſible; and filled with ſpirit of wine (or any liquor that will not freeze or grow foul) all but a bubble of air; which by the laws of hydroſtaticks will always move to the higheſt place in the cavity of the tube. The tube with its convexity upwards, is placed in a long ſlender braſs box, made fit to receive it, and is faſtened in it by very hard cement. This box is laid lengthways upon a long ruler *op*; and through the bottom plate of the box, a little produced, there paſſes a fine ſkrew-pin *z*, to be turned by the fingers; whoſe end, by working againſt the ſurfaces of the ruler below, gradually raiſes or depreſſes this end of the box, while it turns upon a ſmall axis *vx* near the bottom of the other end of it.

Upon the ends of the ruler *op* two braſs plates *oq*, *pr* are fixed, whoſe planes are perpendicular to the length of the ruler; theſe plates being equal in length have their upper parts formed into circular hooks *q*, *r*, to be hung upon the cylindrical ends of the tranſverſe axis *ab*; the interval between the hooks being of a proper length to interpoſe them between the ſhoulders of the cylinders at the end of the axis and the braſs notches they reſt upon.

How to level the tranſverſe axis.

830. The frame *poqr* being thus ſuſpended upon the tranſverſe axis, raiſe or depreſs the tube of the level by twiſting the knob of the ſkrew *z*, till you bring either end of the air bubble to reſt at any point *o*, towards the middle of the tube, and obſerve any mark upon the ſide of the tube that is oppoſite to it. Then take off the frame *poqr*, and turning the ends of it contrary ways, hang it again upon the cylinders. And if the air bubble reſts

exactly

exactly at the same mark as before, the transverse axis is truly horizontal: if not, depress that end of the axis which lyes on the same side of the mark as the bubble does, or raise the opposite end, till the bubble returns about half way towards the mark. Then having made a new mark where it rests, transpose the frame again, and repeat the same practice till the bubble rests exactly at the same point in both positions of the frame. Instead of making marks, over against one end of the bubble, it is better to have a fine pointer, sliding upon a wire along the top of the tube.

831. Let a plane passing through the axis qr and the mark a, where the bubble resteth, cut the tube of the level in the curve abc; and let a line ac drawn parallel to the axis qr, cut off the arch abc, and after the frame of the level has been transposed, the chord ac, of the arch abc will still be situated some where in the same parallel to qr as before, suppose at $\alpha\gamma$. Now since the ends of the chords ac, $\alpha\gamma$ are turned contrary ways in the same line, it is plain that the mark a, which was the highest point in the arch acb, being now transposed to α, is become the lowest point of the same arch in the situation $\alpha\beta\gamma$. Therefore the bubble cannot now remain at the mark α, but will ascend to the highest point of the arch $\alpha\beta\gamma$; which will be γ, if the arch be circular. For then the tangents ad, $\gamma\delta$ to the points a and γ will be parallel to each other; and ad being horizontal, $\gamma\delta$ will be so too; and consequently γ will be the highest point. Let $b\beta$, drawn parallel to the axis or to $a\gamma$, touch the arch $\alpha\beta\gamma$ in β; and it is evident the point β will be lower than γ. Therefore by depressing that end of the axis qr, which lyes on the same side of the mark α as the bubble does at γ, the point γ will also be depressed; and so the bubble will recede from γ towards the mark α till it comes to β, when β becomes the highest point; that is when the tangent $b\beta$ and the axis qr are parallel to the horizon: and then if the frame $qopr$ be transposed back again, the bubble will rest at the same point β as before. Now if the arches abc, $\alpha\beta\gamma$ be circular, b and β are exactly the middle points of the arches. But if the arches be not circular, the points b and β will not be in the middle, yet being at the greatest distance from the chords ac, $\alpha\gamma$, they will be the points where the bubble will rest when the axis is horizontal. The reason of this practice. Fig. 584, 585.

832. It must be observed of these levels so suspended, that if you gently turn the frame about the axis of suspension, the air bubble will frequently change its place though the supporters of the axis keep fixt; either because the plane of the curvity of the tube is not parallel to the axis, or because the cavity of it is not cylindrical, or upon both accounts. It is therefore necessary to reduce the plane of the frame as nearly to the same vertical position as possible, which may be done either by a plumbet hanging down by the side of a perpendicular ruler fixt to the horizontal one op in fig. 583.; or by another tube of a spirit level fixt upon the ruler op at A caution to be observed.

 right

right angles to it. But if the frame *poqr* vibrates freely upon the axis, this additional apparatus will be needless.

How to stay the telescope.

833. If the far end of the telescope be just made to preponderate, the elevation of it, for observing the appulses of stars to the cross hairs, may be gradually altered by a string, tied to its near end, and wound about a pegg fixt below the telescope. The pegg may be made to turn with a proper degree of stifness between two semicircular notches pressed gently together by a skrew-pin passing through both the pieces. Or the same design may be executed several other ways.

How to rectify the cross hairs.

834. If after the telescope is turned upside down, and the contrary ends of the transverse axis are placed upon the same notches, you perceive that the same point of the object is covered by the intersection of the hairs in the focus of the telescope; it is certain that the line of sight[a] is perpendicular to the transverse axis. But if the intersection covers a different point of the object, the hairs must be moved by the key abovementioned, till they appear to bisect the line that joined those two points of the object as near as you can judge; then by reverting the telescope to its former position you will find whether the hairs be exactly adjusted.

a Art. 815.

The reason of it. Fig. 586.

835. For let the line *cd* be the transverse axis which the telescope turns about, *a* the intersection of the cross hairs, *b* the center of the refractions in the object-glass, and *ab* the line of sight. Let *ab* produced meet a remote object in *e*; and the point *e* will appear in the telescope to be covered by the intersection *a*, because the image of it falls upon *a*. Imagine a line *gh* drawn through the center *b* to cut the axis *cd* at right angles in *f*, the hair *aα* in *g* and the object *eε* in *h*; and when the ends of the axis *cd* are transposed by reverting the telescope, the perpendicular *bg* will keep its position, (or be parallel to it,) but *ba* will be transposed into the position *bα* equally inclined to the contrary side of the perpendicular *bfg*. Produce *αb* till it meets the remote object in *ε*, and the cross hairs at *α* will now appear to cover the point *ε*. But the perpendicular *gbh* bisects the interval *eε* in *h*. Therefore by moving the intersection of the cross hairs from *α* towards *g*, till you perceive the middle point *h* is covered by it, it is manifest that the line of sight is now perpendicular to the transverse axis *cd*; and consequently upon reverting the telescope the same point *h* will be covered as before. Hitherto I have supposed the telescope to be turned about an axis *cd* lying in the same plane with the line of sight; but if it turns about any other line parallel to *cd*, it is easy to understand that the motion of the line *gbh* will still describe a plane perpendicular to this transverse axis. It is scarce necessary to be observed that the perpendicularity of the line of sight to the transverse axis, has no dependence upon its passing through the middle of the aperture of the object-glass, nor of being parallel to the sides of the tube; nor is it required to coincide with the axis of the object-glass[b].

b Art. 810.

836.

836. To adjuſt the period of a clock, it is not neceſſary that the line of ſight through the teleſcope ſhould deſcribe the meridian. If it always deſcribes the ſame vertical circle at every tranſit of a ſtar, it is ſufficient. For the intervals of time between the ſucceſſive appulſes of a fixt ſtar to the ſame vertical, are equal to the intervals between its appulſes to the meridian. But leaſt the uncertainty of the air's refraction ſhould cauſe any difference in the times, it is better to have the vertical circle near the meridian; and to obſerve a ſtar near the equator, becauſe its tranſit is quicker. Now the line of ſight may be kept in the ſame vertical circle at every obſervation, by examining the level of the tranſverſe axis every time; and by obſerving alſo whether the croſs hairs always cover the ſame mark upon a diſtant object, which in the night time muſt be illuminated by a lantern. For then the line of ſight will deſcribe the ſame vertical, notwithſtanding any warping of the materials or even of the building to which it is fixt. Now we learn from aſtronomy that the period of the clock will be exactly equal to a ſolar day of a mean length, if the ſame fixt ſtar comes to the croſs hairs 3 minutes 56 ſeconds ſooner every ſubſequent night than the night before it. For the interval of time between the apparent appulſes of a fixt ſtar to the ſame point of the heavens, is the ſame as the period of the earth's motion about its axis, which is called a ſidereal day and is 3′. 56″ ſhorter than a ſolar day of a mean length; I ſay of a mean length, becauſe ſolar days, or the intervals between the ſun's appulſes to the meridian, are unequal. But the error in the clock's period will be diſcovered more exactly by comparing two remote obſervations rather than two ſucceſſive ones. For example ſuppoſe a ſtar comes to the croſs hairs at 9h. 30′. 18″ by the clock, and ſeven days after at 8h. 50′. 24″ by the clock; the difference of theſe times is 39′. 54″, and a ſeventh part of it is 5′. 42″; which ſhews that the diurnal period of the clock is 5′. 42″ longer than a ſidereal day; whereas a ſolar day of a mean length is but 3′. 56″ longer; and by conſequence the diurnal period of the clock is 1′. 46″ too long. To rectify the period of a clock.

837. Mr. *Huygens* preſcribes a way of obſerving theſe tranſits of ſtars even without a teleſcope; by chooſing a proper place from whence you may ſee ſeveral of them inſtantly diſappear in paſſing behind high buildings; and by fixing a plate with a hole in it, to look through, of the bigneſs of the pupil; in order to bring it always to the ſame point[a]. Without a teleſcope.

[a] Horol. Oſcil. p. 13.

838. If the meridian teleſcope be placed without doors, or ſo near the roof of a houſe, as to have a proſpect both to the north and ſouth part of the meridian, through a ſlit cut in the roof; it may be brought exactly into the plane of the meridian by obſerving the tranſits of the circumpolar ſtars both above and below the pole. For the tranſverſe axis being placed horizontal as before, if the vertical circle deſcribed by the line of ſight coincides with the meridian, it will biſect the polar circles deſcribed by the ſtars; if not, it will divide them into two unequal parts. Having To find the meridian.

then

then obſerved by the clock the times of a ſtar's appulſes to the under and upper parts of the vertical deſcribed by the line of ſight; according as the time which the ſtar takes in deſcribing the eaſtern part of its orbit, is found to be longer or ſhorter than the time of deſcribing the weſtern part, it is plain that the teleſcope points weſtward from the meridian in the former caſe, and eaſtward in the latter; and muſt be altered accordingly till thoſe times come out equal. When the meridian is thus found a mark muſt be made upon a point of a diſtant object then covered by the croſs hairs. And at all times after, when any obſervation is to be made of a tranſit over the meridian, the line of ſight muſt be examined by this mark, and the croſs hairs muſt be brought to cover it; the tranſverſe axis being firſt examined and corrected by the level.

To find the apparent time.

839. The meridian line being thus determined, the apparent time of the day is ſoon found, by obſerving the ſun's tranſit over the meridian; that is by noting the times by the clock when the precedent and the ſubſequent edges of his body come to the croſs hairs. For then the middle time between them is known by the clock; and its diſtance from XII ſhews how much the clock is faſter or ſlower than the apparent time for that day. The apparent time being found, the true time will alſo be known, by conſulting an Equation Table.

And differences of right aſcenſion and declination.

840. By the ſame inſtrument the aſcenſional differences of any objects in the heavens may be found moſt accurately, by taking the times of their tranſits over the croſs hairs, and by converting the intervals of time into the correſponding arches of the equator. The differences of declination of two ſuch objects as will paſs over the aperture of the teleſcope when fixt, may alſo be found by the oblique hairs or by the inclined plates hereafter mentioned: and for theſe purpoſes it is more expedtious to uſe a clock whoſe period is adjuſted to a ſidereal day.

For night-obſervations it is neceſſary to illuminate the croſs hairs by the light of a candle tranſmitted through a piece of horn fixt in a hole made in the ſide of the teleſcope. And to give the teleſcope a proper elevotion in the day time for taking a known ſtar into its aperture, it is convenient to have a ſmall ſemicircle, fixt in the plane of the meridian, to one of the pillars that ſupports the tranſverſe axis; having its center in a point of the axis, and a radius fixt to the axis, or ſome index fixt to the teleſcope, that ſhall ſhew the degrees of its elevation pretty nearly.

Dimenſions of the inſtrument.

841. Our great aſtronomer Dr. *Halley*, before he was provided with a Mural Arch, hereafter deſcribed, made all his obſervations of the moon's right aſcenſions with an inſtrument of this ſort. His teleſcope is $5\frac{1}{2}$ foot long; and the tranſverſe axis is about an ell long; being ſurrounded with light braces of braſs, to keep it from ſpringing, and ſupported upon two freeſtone pillars, erected upon a very large ſtone laid deep in the ground. Nevertheleſs a good teleſcope only $2\frac{1}{2}$ or 3 foot long is ſufficient for obſerv-

observing transits to the exactness of a second of time; and the transverse axis need not exceed 2 foot, especially if the spirit level be accurate.

842. If any one desires to know to what degree of exactness the transverse axis is placed horizontal by the level, let him lay the bottom of the ruler op, which supports the level, along the top of the telescope; and tye them fast together with strings; then let him gradually move the telescope about the transverse axis, by turning the pegg abovementioned[a], till one end of the bubble is situated exactly over against a fine point near the middle of the tube; then let him observe through the telescope what mark upon a remote object is covered by the cross hairs. This being noted let him return to the air bubble, and turn the pegg again, but no more than is just necessary to cause the least sensible remove of the bubble from the mark; then let him observe through the telescope what other mark on the object is now covered by the cross hairs; and let the angle subtended at the object-glass, by the interval of the two marks, be computed; or be measured by a micrometer hereafter described; and this will be the least sensible error of the level, and consequently of the transverse axis levelled by it.

To examine the accuracy of the level. Fig. 581, 583.

a Art. 833.

843. By the same method one may determine what part of a long tube is the best for making a spirit level; by fixing the tube along the side of the telescope, first with one side uppermost and then another; till you find in what side and in what part of that side the motion of the air bubble is the greatest, with respect to a given motion of the telescope observed as above by the cross hairs If the air bubble has no adhesion to the tube, but moves as freely as the ball of a plumbet, it is easy to understand that a spirit level will distinguish a small inclination to the same accuracy as a plumb line, whose length is equal to the radius of the curvity of the tube. For if $acbd$ be a sphere filled with water, and an air bubble be supposed at the upper end a of any diameter ab, and a round globule of lead be supposed at its lower end; the bubble and the globule will describe equal arches about the center e in their passage from the oblique diameter ab to the vertical one cd: and for a given inclination of ab to cd, the arches ac, bd are proportionable to their radius ae or eb.

How to choose a tube for a level.

Fig. 587.

CHAPTER VI.

Telescopick Instruments for finding time, by observing when the sun or any star has equal altitudes on each side of the meridian.

844. WHEN Sir *Isaac Newton* made a present, to our Observatory at *Trinity* College in *Cambridge*, of an excellent pendulum clock; that great mathematician Mr. *Roger Cotes*, my worthy predecessor, contrived an instrument of small expence, but very accurate, to set it

it by, and to adjust its motion; by finding the times by the clock it self when the sun or any star has equal altitudes before and after its passage over the meridian. He also sent the following description of his instrument to Sir *Isaac Newton*. "*AB* is a strong wooden axis about six feet in length; *CD* and *DE* on one side, *EF* and *FG* on the other, are pieces framed to each other and to the axis as firmly as was possible. Into the piece *CD*, and at the angle *F*, were fixed strong wooden pins nearly parallel to each other, and perpendicular to the plane *CDEFG*. *PQ* is the cylindrical brass tube of a five foot telescope; (belonging to our sextant;) this was well fastened with iron staples and skrews to the piece of wood *IKML*, whose under plane surface is here represented as objected to view. Into this surface there was perpendicularly fixed a strong wooden pin *N*, which was designed to hang the upper end of the telescope upon any of the pins in *CD*, whilst its lower end rested upon the pin *F*. Now that the telescope might be taken off, and yet afterwards be again placed accurately in the same position, I ordered the edges *IK* and *CD* which touched each other, to be rounded like the surface of a cylinder, as also the edge *EF* into which the pin *F* was fixed, and against which the cylindrical tube of the telescope rested; so that the contact in both places might be made in a point. Upon the same account the pins in *CD* were made a little hollow as is represented at *R*; and the pin *F* was a frustrum of a cone, that thereby the telescope might more surely touch the edges *CD* and *EF*. Into the two ends of the wooden axis were strongly fixed two pieces of well tempered steel, that at the upper end *A* was a cylinder well turned, which moved in a collar whose cavity, represented by *S*, was figured like two equal hollow and inverted frustrums of cones joined together: the lower at *B* was a cone moving in a conical socket of a somewhat larger angle. This socket had liberty to be moved horizontally, and to be fixed in any position by two skrews, which pressed against it sideways at right angles to each other. The instrument being thus prepared, I fixed a needle *V*, at the lower end of the wooden axis, whose point stood out from it about an inch; then suspending a fine plumb-line *TVX* from the upper end of the same axis, I altered the position of the instrument by the skrews, until the plumb-line came to beat against the point of the needle in the whole revolution of the instrument, and there I fixed it as prepared for use.

Mr. *Cotes*'s instrument. Fig. 588.

845. So far Mr. *Cotes*. The plumb-line or fine wire *TVX* was suspended by a loop *T* upon a brass pin that skrewed into the top of the axis *AB*; a nick being filed round the pin to stay the loop from sliding out of it. Then by skrewing the pin in or out, the plumb-line was brought to the same distance from the axis *AB* as the point of the needle is at *V*: which was fixt in the end of a thick wooden pin *Y*, not in the axis of it, but towards one side; so that by turning the pin round it self, in a hole

bored through the axis *AB*, the needle's point might deſcribe a ſmall circle and be brought to touch the plumb-line when parallel to the axis of motion of the wood *AB*.

846. The Right Honourable the Earl of *Ilay*, in his noble collection of instruments, has a very excellent one of this kind, for taking equal altitudes, made all of braſs except a ſquare ſteel axis *ab* 30 inches in length. To one ſide of the upper part of this axis, there is fixt a ſmall ſextantal arch *cd* repreſented ſeparately at *E*; its center *a* being at the top of the axis *ab*. The teleſcope *MN* is alſo 30 inches long, and is fixt along the diameter of a ſemicircle of the ſame radius as the ſextant and concentrick to it. The teleſcope with the ſemicircle being moveable about this center upon the plane of the fixt ſextant *acd*, may be faſtened to it in any elevation by two nuts and ſkrews *c*, *d*, fixt in the ends of the ſextantal arch; a circular ſlit being made all along the limb of the ſemicircle for theſe ſkrew pins to ſlide in. Cloſe under theſe arches, the axis *ab* is ſurrounded by a ſhort cylinder *e*, about an inch in diameter, well turned and poliſhed. The lower end of the axis is formed into a fine conical point *b*. The frame in which the axis turns, is a long hollow parallelopiped wanting two ſides. Its other ſides *f*, *g* are two braſs plates, equal in length to the part *be* of the axis, and are ſkrewed together edgeways. It has for its baſes two equal plates *h*, *i*, 4 inches ſquare. In the middle of the upper ſquare there is a round hole, large enough to receive the cylinder *e* without touching it; and over this hole is fixt a triangular hole in another plate, one of whoſe ſides is moveable by a ſkrew, to make all the ſides of the triangle touch the cylinder. Upon the lower ſquare there lyes a ſmaller plate *i*, with a fine center hole in it, to receive the point *b* of the axis. This center plate is moveable ſideways by two ſkrews at right angles to each other, which, when the frame is firmly fixt into a nitch of a free-ſtone pillar, will bring the axis *ab* exactly perpendicular to the horizon. This poſition is known by a ſpirit level *lm* fixt at right angles to the axis, above the cylinder, upon the ſide oppoſite to the ſemicircle. Along the top of the level there is a ſliding pointer, to be ſet to the end of the air-bubble; and when the poſition of the axis is ſo adjuſted by the ſkrews below, that the air-bubble keeps to the pointer for a whole revolution of the inſtrument, the axis *ab* is certainly perpendicular to the horizon; and then the line of ſight through the teleſcope deſcribes a circle of equal altitudes in the heavens. There are ſeveral of theſe circles deſcribed in the heavens even when the teleſcope is fixt to the ſextantal arch. For the round hole in its focus, has five wires parallel to the horizon, at equal intervals from one another as at *k*; and they are croſſed at right angles in the middle by two other upright wires at a ſmall diſtance from each other. The deſign of ſo many wires, is to obſerve when the ſame ſtar is ſucceſſively covered by every one of the 5, both in the eaſt and

The Earl of *Ilay*'s inſtrument.
Fig. 589.

west; so that the time of its passage over the meridian may be had more accurately, by taking a medium among all the observations. The distances between the five wires need be no greater than to afford time enough to write down the several observations, which must be taken when the star is between the perpendicular wires.

Uses of these instruments.

847. Time shewn by a clock may be called mechanical, to distinguish it from solar and sidereal time.

848. By observing when a star has equal altitudes before and after its culmination or appulse to the meridian, we have the mechanical time of its culmination. Then by substracting the sun's right ascension computed to this mechanical time, from the stars right ascension determined for the same time, we have the solar time of the stars culmination, and consequently the difference between the mechanical and solar times.

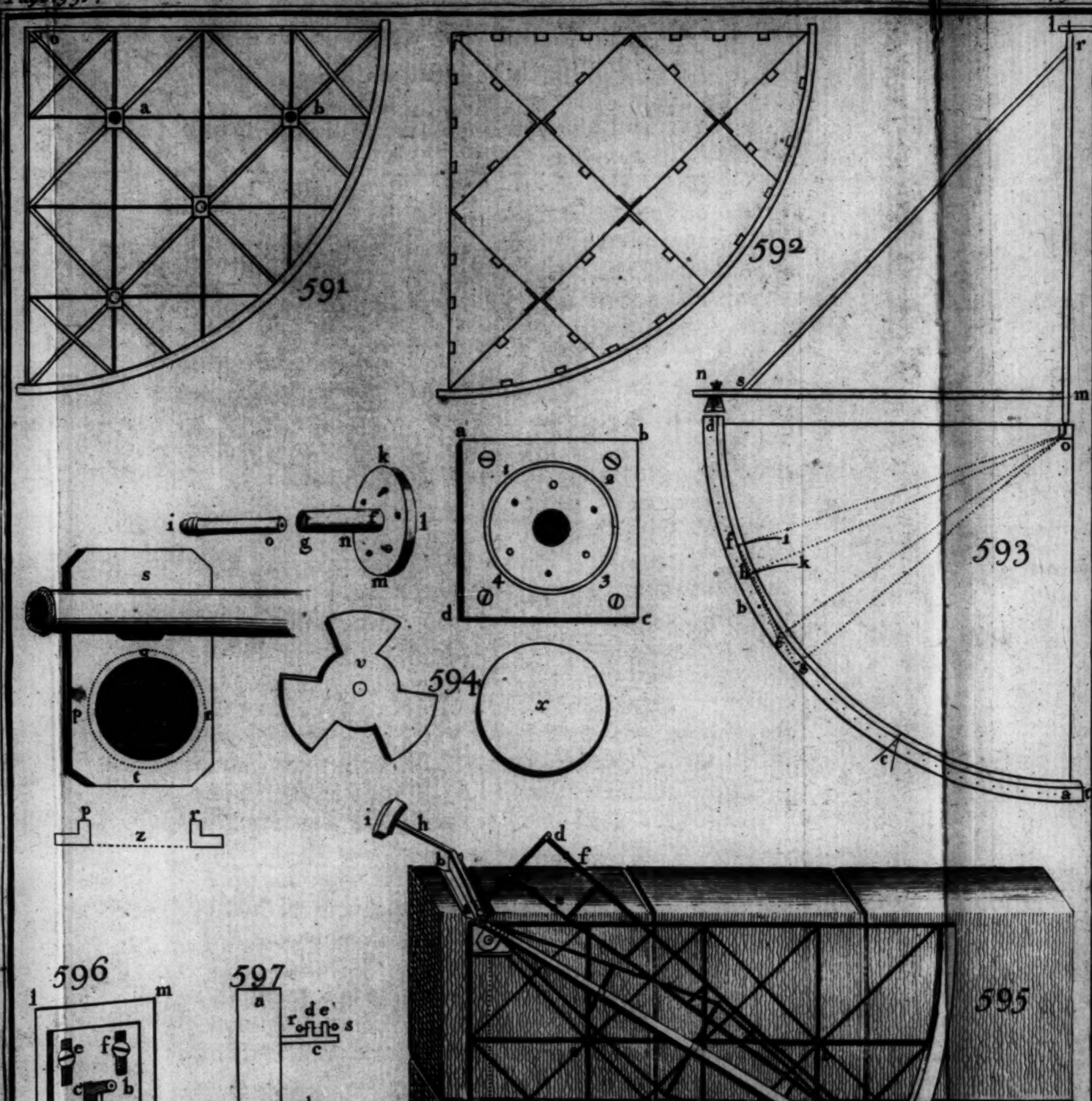
849. Thus by finding the mechanical times, when the same star culminates any two nights, rather at a distance from each other than successive, we have the difference between a sidereal day and a mechanical day; and consequently between a mechanical day and a solar day of a mean length.

Hence any number of mechanical minutes may be converted into solar, or into sidereal minutes by the rule of three.

These observations will answer the purpose the more exactly as the star is nearer to the prime vertical; because the variation of its altitude is here greater in a given time, than if it were situated in any other vertical oblique to the meridian. In the latitude of 50 degrees an error of one minute in altitude, at any point of the prime vertical, will cause an error of $6\frac{2}{9}$ seconds in time; and in the latitude of 55 degrees it will cause an error of near 7 seconds; as that excellent geometer Mr. *Cotes* has shewn, in his Treatise concerning the *Estimation and Limits of Errors* in mixt mathematicks, which I published at the end of his admirable Book called *Harmonia Mensurarum*. It is also the safest to choose a star as high as possible, least a different state of the atmosphere should cause a different refraction of the visual rays, and consequently an error in the times of observation.

850. The solar time may also be found by observing when the sun himself has equal altitudes in the morning and evening; if we correct the time of the latter observation by a just allowance for the variation of the sun's declination, as follows. Upon a celestial globe let the pole be at P; the vertex of the observer's place at V; the complement of its latitude PV; its meridian $PVBG$; a circle of equal altitudes $ABCD$, described about its pole V, and passing through the sun's center at A at the time of the morning observation, and through it at D at the time of the evening observation; two circles of declination, PAF, PDI, cutting the equator $FGHI$, in F and I; a parallel of declination ACE, cutting the

the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the circle PDI in E, and ABD in C; three equal vertical arches VA, VC, VD; and lastly a third circle of declination PCH cutting the equator in H. Now had not the sun varied his declination, from A or E to D; in the evening he would have had the same altitude at C as in reality he has at D. And then as the angles VPC, VPA would have been equal, so the times of the evening and morning observations would also have been equidistant from noon; being measured by those angles or by the arches GF, GH. Therefore the angle CPD or the arch IH, which measures it, is also the measure of a portion of time to be substracted from the evening observation, if the sun's declination varies northwards, otherwise to be added, to give the time sought equidistant from noon. Let another circle of declination Pkl bisect the small angle HPI, and consequently the small arches CD and HI in k and l. Draw the vertical arch kV; and in the triangle kPV, we have given, PV the complement of the latitude, and Pk half the sum of the given complements, PC or PA and PD, of the sun's declinations at the times of the two observations, and lastly the included angle kPV, by converting half the interval of time between the observations into degrees and minutes. Hence by trigonometry we have the angle PkV, of an intermediate magnitude between PCV and PDV, and therefore fitter to be used instead of either of them. Hence also we shall have the arch IH; by taking it in proportion to DE the difference of the declinations, as the co-tangent of the angle PCV, to the sine of the arch PC or Pk. For IH is to DE in a ratio compounded of IH to CE and of CE to DE, that is of the radius to the sine of the arch PC, and of the co-tangent of the angle DCE or PCV to the radius: as appears by taking away the common angle DCP from the right angles ECP and DCV; and by considering the small triangle DCE, right angled at E, as if it was rectilinear. The calculation supposes the sun's center has equal altitudes at A and D; which is agreeable to the observations that determine when his upper or his under limb has equal altitudes. I have drawn out this method of computation from Theorem. 23. of Mr. *Cotes*'s Treatise upon *the Estimation of Errors*.

851. These are the best sort of instruments and the best methods I know of for determining time; however those that are desirous of knowing a greater variety of them and of fuller instructions in relation to them, may receive satisfaction by consulting Mr. *William Molyneux*'s little book upon his *Telescopick Dial*, printed at *London* in 4to. 1700.

CHAPTER VII.

The Mural Arch in the Royal Observatory at Greenwich *described.*

Preface.

852. SINCE the measure of time by pendulum-clocks, and consequently of the apparent diurnal motion of the heavens, has of late been brought to the utmost perfection, Astronomers are agreed that a large quadrant (with a telescopick sight) fixt in the plane of the meridian to a free-stone wall, and thence called a Mural Arch, is by far the most accurate, expeditious and commodious instrument of all others for the chief purposes in astronomy. For by observing the times by the clock of the appulses of any objects in the heavens to the plane of the meridian, we have their right ascensions; and by their meridian altitudes observed at the same times, (the latitude of the place being once determined,) we have also their declinations; and by consequence their places in the heavens. Thus by a good instrument of this kind, a Catalogue of the places of the fixt stars, may be made in less than a tenth part of the time, and with much greater certainty and exactness than by the best moveable quadrant or sextant, as might easily be shewn; not to mention the saving an immense labour in trigonometrical calculations. I think the noble *Tycho Brahe* was the first Astronomer that used a mural arch, for taking meridian altitudes; but he could not determine right ascensions so accurately as now, for want of the late improvements in pendulum-clocks. After him, *Hevelius*, *Flamsteed* and others made use of the like instruments, whose descriptions may be seen in their printed works; but I pass them all over as being far inferior to this at *Greenwich:* the expence of which was defrayed by the bounty of his late Majesty King *GEORGE* the first; and the particular accuracy, whereby it excels all others, is owing to the extraordinary skill and contrivance of Mr. *George Graham* Watch-maker in *Fleet Street* and *F. R. S.*; who besides the direction of the whole design, and inspection of the inferior workmen, was pleased to perform the divisions of the arch and all the nicer parts of the work with his own hands; and to him I am highly obliged for informing me in the methods he took to perform it. How far the tables of the moon's motion are corrected by an incredible number of observations made with this noble instrument by our Royal Astronomer Dr. *Halley*, and how near they are brought to sufficient exactness for finding the Longitude at Sea, shall be related in the philosophical discourses of the next book. At present I proceed to a description of the instrument.

Disposition of the iron bars. Fig. 591, 592.

853. Excepting the circular limb, the quadrant is chiefly composed of straight iron bars, joined together, as represented in the 591st and

592d

592d figures. The breadth of every bar is 2 inches and 9 tenths and its thickneſs 1 tenth and $\frac{3}{4}$ nearly. In ſpeaking of the diſpoſition of theſe bars, thoſe whoſe planes compoſe the plane of the quadrant, I ſhall call flat bars; and thoſe, whoſe planes are perpendicular to the former, I ſhall call perpendicular bars. The lines in fig. 591 repreſent the diſpoſition of all the flat bars, and thoſe in fig. 592 repreſent the diſpoſition of all the perpendicular bars, placed behind the flat ones; and are only to be ſeen on the back ſide of the quadrant. The chief deſign in this diſpoſition of both ſorts of bars, is to ſecure the figure and plane of the quadrant from any alterations that may be cauſed, either by the weight of the materials, or by their ſwelling or ſhrinking by the weather, or by the motion of the teleſcope about the center of the quadrant, or by any accident whatever. The whole fabrick is farther ſtrengthened by a great number of ſhort iron plates or pieces of the like bars, bent to a right angle, and placed behind the quadrant in the angles made by the flat and perpendicular bars, and riveted to them both. Their number and places, where they are riveted, are repreſented in the 592d figure, by the ſmall parallelograms adjoining to the lines: and to make more room for the rivets, the edge of each perpendicular bar does not divide the breadth of the flat bar in the middle, but in the ratio of 2 to 1; and the little plates are riveted on the broader ſide. The black thickenings of the lines at their interſections in the 592d figure, repreſent little plates of iron, bent in right angles, and riveted in the angles made by the interſections of the perpendicular bars. At the circumference of the quadrant there is alſo a perpendicular bar, bent circular, and faſtened all along the middle of the breadth of the limb or flat arch of the quadrant, by a ſufficient number of the little plates we have been ſpeaking of.

854. The limb of the quadrant is compoſed of two quadrantal arches, of the ſame length, breadth, and thickneſs; one of iron, the other of braſs laid over it. The breadth of each is 3 inches and 4 tenths, and the common part of their breadths, where they lye doubled one over the other and are riveted together, is 2 inches and 2 tenths, the braſs limb being remoter from the center than the iron one by an inch and 2 tenths. The limb was reduced to a true plane as follows. In the 593d figure, *abdo* repreſents the quadrant, placed very firm upon a level plane, with its braſs limb lying upwards; and *lm* repreſents an axis made of iron, placed perpendicular to the plane of the quadrant, and pointing to its center *o*; *mn* is an arm of iron, equal in length to the radius of the quadrant, and fixt at right angles to the bottom of the axis *lm*; to the end of this arm an iron ſcraper *np* was fixt directly over the braſs limb; and being firmly ſupported by the arm and its braces, was turned about the axis *lm*, till by ſcraping the braſs, it reduced its ſurface to a perfect plane;

Structure of the limb.

Fig. 593.

care

care being taken that the edge of the ſcraper was exactly perpendicular to the axis of its motion.

Diviſion of the limb.

855. There are two arches ſtruck upon the braſs limb; one with a radius of 8 foot, or more exactly of 96, 85 inches; and the other, with a radius of 95, 8 inches. This inner arch is divided into degrees, and 12th parts of a degree; and the outward arch, into 96 equal parts, which are ſeverally ſubdivided into 16 equal parts. The beam of the compaſs which ſtruck theſe arches, was ſecured from bending by ſeveral braces faſtened to it; and when an arch was ſtruck, 60 degrees of it was determined by placing one point of the compaſs at *a* and by making a ſtroke with the other at *b*. This arch *ab*, was biſected in *c* by drawing two ſmall arches upon the centers *a* and *b*, with ſuch a radius as to croſs the arch *acb*, in two points as near together as poſſible without touching each other; then the ſmall interval between them was biſected at *c*, by eſtimation of the eye, aſſiſted by a magnifying glaſs. After this, the interval between the points *a* and *c* or *c* and *b*, was taken with the beam-compaſs, and was transferred from *b* to *d*, which determined the length of the quadrantal arch *acbd*. Every one of the three arches being biſected in the ſame manner, the quadrant became divided into 6 equal parts, containing 15 degrees apiece; and every one of theſe was divided into three equal parts as follows. To avoid making any falſe or ſuperfluous points in the quadrantal arch, with its radius unaltered, but upon any other center, there was ſtruck another faint arch; upon which the chord of 15 degrees already found was transferred from the quadrantal arch; and the third part of 15 degrees, being determined by trials upon the faint arch, was transferred back again upon the quadrantal arch; which then was divided into 18 equal parts containing 5 degrees apiece; and the 5th part of theſe was found by trials, as before in dividing a ſeparate arch, drawn upon a new center for this purpoſe only. The ſub-diviſions of the degrees into 12 equal parts were made by biſections and triſections as before. Thus was the whole quadrant divided without any falſe or ſuperfluous points.

856. The outward quadrantal arch was divided into 96 equal parts, by no other method than that of biſection, till 60 degrees or two thirds of the quadrant became divided into 64, and the remaining third into 32, equal parts; which make 96 in the whole. And every one of theſe were alſo divided into 16 equal parts by continual biſections. Theſe two ſorts of diviſions are a check upon each other, being in effect two different quadrants; and the diviſions in one being reduced into the diviſions of the other, by a table made for that purpoſe, they are never found to differ above five or ſix ſeconds in any place of the limb: and when they do, the preference ought to be given to the biſected diviſions, as being determined by a ſimpler operation.

857. The divisions hitherto mentioned being only very fine points in a fine arch *abd*, scarce discernible by the naked eye; it was necessary, as usual, to strike lines perpendicular to the arch, through every one of them. But since it is very difficult, and tedious too, to draw lines exactly through every point by the edge of a ruler, the following method was judged more accurate and expeditious. It was proposed then to divide any other concentrick arch, *fht*, by cross strokes, into similar parts to those in the given arch *acgebd*. Take a small beam compass, and having once fixt its points at any convenient interval; upon the centers *e*, *g*, &c. being the given points of the divided arch, strike the small arches *fi*, *hk*, &c. cutting the undivided arch in *f*, *h*, &c: then will the intercepted arches as *fh*, &c. be similar to the arches *eg*, &c. that is, they will subtend the same angles at their common center *o*. For joining *ef*, *gh*, and also *of*, *oh*, *oe*, *og*, the triangles *eof*, *goh*, will be similar and equal to each other; every side in one being respectively equal to every side in the other. Therefore by taking away the common angle *eoh*, from the equal angles, *eof*, *goh*, the angles, *eog*, *foh*, that remain, will also be equal. Fig. 593.

If the triangles *efo*, *gho*, &c, be right angled at *f* and *h*; the dividing strokes *fi*, *hk*, &c. will cut the quadrantal arch *fht*, at right angles also, at *f* and *h*, &c.

858. In the 594th figure *abcd* represents a square piece of brass (with several steady pins in it) skrewed to the flat bars, at the center of the quadrant, the skrew holes being so large as not to touch the skrews; and *klmn* represents a thick circular plate of brass, with a hollow pipe *fg* fixt perpendicular to the middle of it; this plate was turned exactly circular in a lathe upon a brass arbor *oi*, turned tapering and a little hollow in the middle, so as to fit the cavity of the pipe *fg*, and to bear against it chiefly at both ends. When the hollow pipe *fg* is put through the hole (exactly fit for it) in the middle of the square *abcd*, the brass circle *klmn*, is fixt to the plane of the square *abcd*, with skrews and steady pins. The point *o*, in the pole of the arbor *oi*, is not only the center of the circular neck *klmn*, about which the telescope must turn, but also the very center upon which the divided arches were struck upon the limb of the quadrant. The end of the telescope that holds the object-glass lies cross one end of an oblong plate of brass *st*, at right angles to its sides, and is held to the plate by a brace that may be widened and straightened by a skrew. Towards the other end of the plate *st*, there is a round hole, lined with a steel collar *pqr* to be put over the brass neck *klmn*, and to turn round upon it. The section of this collar, made perpendicular to the plane of it, is represented at *z*; the broader of the two rings being under the plate *st* and contiguous to the square plate *abcd*. Over this neck and collar there is fixt a brass spring represented at *v*, and skrewed to the neck *klmn*, to keep the collar from slipping from it; and over all these is skrewed a cap represented

The center work. Fig. 594.

sented

ſented at x, to cover the center work, and to keep off the duſt; which is alſo prevented from coming between the plates $abcd$ and st to the neck and collar, by means of a braſs hoop ſurrounding the broad rim or baſe of the collar pr, and ſkrewed to the backſide of the plate st; which hoop is received into a circular groove 1, 2, 3, 4 made in the ſquare plate $abcd$, without touching any part of it.

The chief excellency of the preſent center work conſiſts in preſerving the place of the central point of the quadrantal arch in the pole of the arbor oi. For whenever the neck-plate $klmn$ ſhall be wore ſo much as to cauſe an unſteady motion of the teleſcope about the center of the quadrant; a new neck-plate and pipe may be caſt, well hammered and turned upon the poles of the ſame arbor oi, to fit the hole and collar, and then it will carry the teleſcope about the center of the limb, as exactly as when all was new.

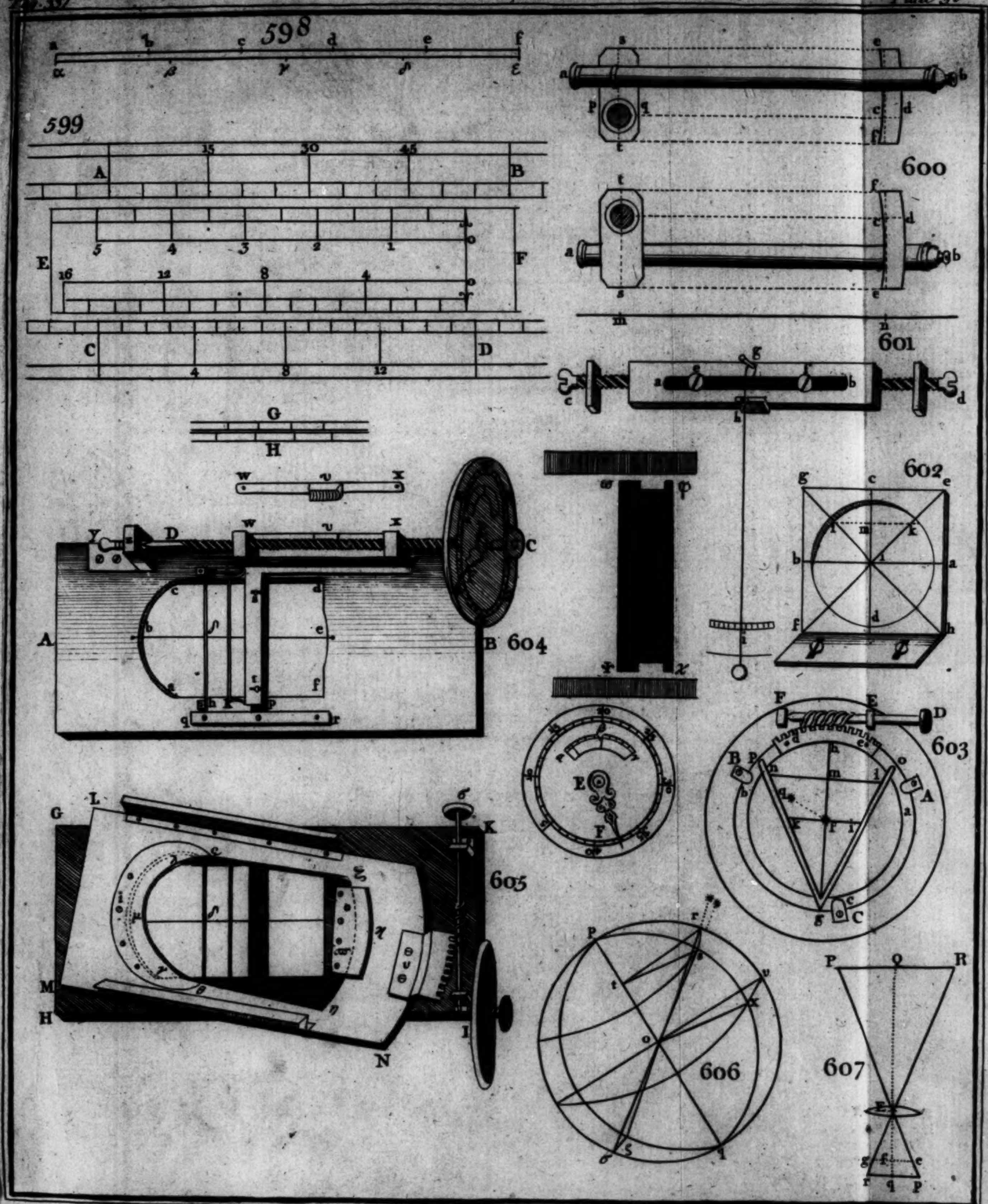
How the quadrant is fixt to a free-ſtone wall.
Fig. 595.

859. The 595th figure gives a view of the quadrant fixt to the eaſtern ſide of a free-ſtone wall, built for that purpoſe in the plane of the meridian. The whole weight of the quadrant is ſupported by two ſtrong iron pins fixt to the wall (as hereafter deſcribed) and projecting through two holes made in two ſquare plates of iron riveted to the quadrant at a and b, in the 591ſt figure. The pin at a, which bears the greateſt part of the weight, is immoveably fixt in the wall, but the pin at b is moveable up or down by a ſtrong ſkrew, in order to bring one ſide of the quadrant to an horizontal, and the other to a vertical poſition.

Fig. 596.

The contrivance for the motion of the pin b is this. In the 596th figure $lmno$ repreſents an oblong plate of iron, let into the free-ſtone wall, and faſtened to it by bolts of iron, which paſs through the wall and through another plate let into the oppoſite ſide of it; the bottom of each plate being bent ſquare and bedded in the ſtone. And e, f, g, h, are the heads of 4 iron ſkrews, whoſe ſhanks going through 4 long ſlits, made in another iron plate, repreſented by the ſmaller parallelogram, are ſkrewed into the fixt plate $lmno$. The moveable pin bc is fixt to this leſſer plate, which is raiſed, or depreſſed by means of a long ſkrew ki, working againſt the bottom of the pin bc at d, being turned round in a ſtrong concave ſkrew, fixt to the bottom of the larger plate at pq. The key for turning the long ſkrew ki, is a ſector of a circular plate repreſented at rst; the ſquare hole in its center t being put upon the ſhank k. The radius of the key is juſt ſo big as to move in the ſpace between the wall and the bars of the quadrant; and a chiſel v is inſerted into the teeth upon the arch of the key, to give power to the hand that moves it.

The weight of the quadrant being thus ſupported by the pins a, b, the plane of it is fixt to the wall and adjuſted in any poſition, by much the ſame number of hold-faſts as there are little ſquares round about the quadrant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drant in the 592d figure. Each hold-faſt conſiſts of two ſeparate parts; one of them is fixt to the wall, and the other to the quadrant.

In the 597th figure, *ab* repreſents the wall ſeen endways, and *c*, *c*, ſeveral hold-faſts fixt into it. Between the chaps of each hold-faſt, repreſented at *d*, *e*, there paſſes one end of a ſmall plate of braſs, whoſe plane is parallel to the plane of the quadrant, the other end being bent to a right angle and riveted to the perpendicular bars of the quadrant; and each plate is pinched by two oppoſite ſkrews *r*, *s*, that work through the chaps *d*, *e*; which are made pretty wide for adjuſting the poſition of the plane of the quadrant. The intent of the ſkrews in the chaps of the hold-faſt, was alſo that if the wall or quadrant ſhould ſwell or ſhrink, ſo as to alter their proportions, the braſs plates might ſlide without diſtending the inſtrument. The hold-faſts are not faſtened in the wall with lead, which is apt to yield, but with a compoſition made of ſtone-duſt, pitch, and brimſtone, or roſin; ſuch as ſtone-cutters uſe for cementing broken ſtones. Fig. 597.

860. The continuance of the exactneſs of the quadrant depends in a great meaſure upon a free and eaſy motion of the teleſcope round the center of it; which will be obtained by counterpoiſing the weight of the teleſcope, and by eaſing the center of the quadrant of as much of thoſe weights as poſſible. For this purpoſe in fig 595, *ab* repreſents an iron axis laid croſs the top of the wall; having two braſs plates fixt perpendicular to the ends of it, with notches or holes cut in them for this axis to turn in, which points to the center of the quadrant at right angles to its plane. To that end of this axis next the quadrant, an iron arm *cd* is fixt, having two braſs plates *ce*, *df* almoſt perpendicular to it; to them are riveted two ſlender ſlips of fir, whoſe other ends meet at *g*, near the eye-glaſs; being held together in a braſs cap or ſocket. Through a ſmall plate fixt to one ſide of a collar, embracing this lower end of the teleſcope, there paſſes a ſkrew-pin at *g* parallel to the teleſcope; which pin being ſkrewed into the cap at the end of the ſlips, holds up the teleſcope tight againſt the center-work. The ſlips are ſtrengthened by 5 or 6 croſs braces of the ſame wood, as repreſented in the figure. To the other end of the axis *ab*, another arm *bi* is fixt parallel to the teleſcope, and in a contrary direction, carrying a weight *i* to counterpoiſe the weight of the teleſcope, and make it reſt in any poſition. And for greater eaſe and freedom of its motion, two ſmall braſs rollers are fixt to each ſide of it, at *k* and *l*, which are held tight to the plane of the limb by a plate ſpringing againſt its backſide, which plate has alſo a roller at each end of it. How the teleſcope is balanced. Fig. 595.

When the teleſcope is pretty nearly directed to an object whoſe altitude is to be taken, a plate *mn*, which is carried by the teleſcope along the limb, and lies croſs it, may be fixt to it by a ſkrew, not here repreſented. Then by twiſting the head *o* of a long ſkrew *op*, which is parallel to the

 limb

limb and which works through a female ſkrew, annexed to the plate mn, and whoſe neck at p turns round in a collar annexed to the teleſcope; a very gradual motion is given to the teleſcope for bringing the croſs hairs exactly to cover the object.

The ſub-diviſions how made and numbered. Fig. 598.

861. To avoid the trouble of ſub-dividing the quadrantal arch into ſmaller parts, the teleſcope carries a ſmall braſs plate, which ſlides upon the limb, and is called a *Nonius*, from the name of its inventer. To underſtand the reaſon and uſe of this plate, it is convenient to premiſe the following Theorem. If a line af, be divided into any number of equal parts, ab, bc, cd, de; and an equal line $\alpha\varepsilon$ be divided into other equal parts, $\alpha\beta$, $\beta\gamma$, $\gamma\delta$, $\delta\varepsilon$, whoſe number is one leſs than the number of parts in af; I ſay that $\alpha\beta$, $\alpha\gamma$, $\alpha\delta$, $\alpha\varepsilon$, will exceed ab, ac, ad, ae, reſpectively by one, two, three, four parts of ab, whoſe denominator is the number of parts in ae or in $\alpha\varepsilon$. For let the lines af, $\alpha\varepsilon$ be coincident at both ends, and ſince any equimultiples of two quantities ab, $\alpha\beta$, are in the ſame ratio as the quantities themſelves[a], it will be as $ab : \alpha\beta :: ac : \alpha\gamma :: ad : \alpha\delta :: ae : \alpha\varepsilon$ or af, and disjointly as $ab : b\beta :: ac : c\gamma :: ad : d\delta :: ae : e\varepsilon$ or ef. The conſequents, $b\beta$, $c\gamma$, $d\delta$, $e\varepsilon$, are therefore in the ſame arithmetical progreſſion as the antecedents ab, ac, ad, ae; and the firſt of the conſequents $b\beta$ is the ſame part of its antecedent ab, as the laſt conſequent ef is of its antecedent ae, or as $\alpha\beta$ is of $\alpha\varepsilon$, the number of parts in ae and $\alpha\varepsilon$ being equal by the firſt ſuppoſition. And it is manifeſt that any two equal and coincident arches of a circle have the ſame property.

a Eucl. V, 15.

Fig. 599.

862. The upper arch AB repreſents a degree divided into 12 equal parts, containing 5 minutes in each; and the under arch CD a 96th part of the quadrant, divided into 16 equal parts; and EF the Nonius, or ſubdividing plate fixt to the teleſcope, and ſliding with it in the ſpace between the arches AB, CD. The degrees and minutes, and alſo thoſe 96 parts of the quadrant, are numbered from the left hand to the right, beginning from the interſections of the vertical radius, in order to meaſure the diſtances of objects from the zenith; but the parts upon the Nonius are numbered the contrary way, beginning from the line oo, called the index; which is drawn perpendicular to the ſides of the Nonius at the end next the right hand; and the line of ſight through the teleſcope is ſo adjuſted by the croſs hairs in its focus, as to be parallel to the index oo produced through the center of the quadrant. In the ſcheme, the Nonius EF is ſo ſituated, that the upper end of the index oo is not oppoſite to any one ſtroke upon the adjoining arch, but to ſome unknown point of a 12th part of a degree, intercepted between 50 and 55 minutes. To find the overplus above 50, I obſerve by looking back from the index, that a ſtroke of the Nonius, which lies between the numbers 3 and 4 is directly oppoſite to a ſtroke upon the adjoining arch; which ſhews that 3 minutes and a half is to be added to the 50 minutes aforeſaid.

For

For ſince a degree is divided into 12 equal parts, containing 5 minutes in each; and ſince the length of the Nonius is made equal to 11 of thoſe parts, and is divided into 10 equal parts; it appears by the theorem[a], in counting back again from the coincident ſtrokes to the index, that the firſt part of the Nonius exceeds the firſt upon the limb by $\frac{1}{10}$ of this latter part, that is by $\frac{1}{10}$ of 5 minutes, which is half a minute; and by conſequence that 7 parts of the Nonius, from the coincident ſtrokes to the index, exceed the 7 correſponding parts of the arch, by 7 half minutes, or $3'.30''$. a Art. 861.

863. When it happens that no one ſtroke upon the limb is directly oppoſite to a ſtroke upon the Nonius, then look for that ſingle part of the limb, which is ſo oppoſed to a ſingle part upon the Nonius, as to be exceeded by it at both ends; as repreſented in the parts *G* and *H*. Then if by eſtimation of the eye, this part of the Nonius exceeds the part of the limb equally at each end, allow $15''$ more than if they had coincided at their ends next the index; and according as the exceſs next the index is judged to be one third, one half, double or treble of the other exceſs, allow $7\frac{1}{2}''$, $10''$, $20''$, $22\frac{1}{2}''$, reſpectively. For ſince the ſum of the two exceſſes is always the ſame, and anſwers to $30''$, (as is plain when one of them is diminiſhed to nothing,) the number of ſeconds to be added will always be to $30''$, as the exceſs next the index, is to the ſum of the two exceſſes. Fig. 599.

864. The lower arch of the Nonius is divided into 16 equal parts, and is equal in length to 17 equal parts upon the oppoſite arch, and conſequently will determine 16th parts of any one of them, by the theorem and the method abovementioned. In the preſent ſcheme the oppoſite ſtrokes of the Nonius and the lower arch, are ſuppoſed to coincide at the end of the 9th part upon the Nonius, which ſhews that the index cuts off 9 ſixteenths of the oppoſite part of the arch. And ſo the length of the arch, from the beginning of a 96th part of the quadrant, is thus denoted, 15, 9, the lower pointer being paſt the 15th ſtroke.

865. This way of ſub-dividing by a Nonius is preferable to the common method of drawing diagonals; both becauſe the trouble of drawing ſo many diagonals is intirely avoided; and alſo becauſe they cannot be drawn ſo exactly by the edge of a ruler, as the lines upon the Nonius; and laſtly becauſe the interſection of theſe diagonals with the index or fiducial edge, (as they call it,) by reaſon of their great obliquity to each other, cannot be determined ſo exactly by the eye, as the coincidence of two ſtrokes in the Nonius and the arch, which ſtand directly oppoſite to one another.

866. The object-glaſs being firmly and immutably fixt in the teleſcope, the Nonius-plate *cd* and the collar-plate *st*, were both ſkrewed faſt to the teleſcope when taken off from the quadrant; and then the Rectification of the line of ſight. Fig. 600.

 line

line of sight was brought to be parallel to the line *co*, drawn through *o* the center of the collar *pq*, to *c* the beginning of the divisions on the Nonius, in this manner. The lines *sot* and *ecf* being drawn upon these plates both perpendicular to *oc*, any distances *ot* and *cf* were taken equal to each other on one side of *oc*; and any other distances *os* and *ce*, (long enough to go beyond the telescope) were also taken equal to each other on the opposite side of *oc*. Through the four points *e*, *s*, *t*, *f*, the ends of the two plates were filed exactly parallel to *oc*. Then placing the points *t*, *f* upon two points *m*, *n* of an horizontal line drawn upon a firm plane, a point of a remote object covered by the cross hairs was marked. And the telescope being turned half round its axis *ab*, and the opposite points *e*, *s* of the plates being placed upon the same points *m*, *n*, another point of a remote object now covered by the cross-hairs was also marked; and the telescope remaining fixt, the cross-hairs were moved in its focus, till after several repetitions of this practice, the same point of the object was covered by them in both positions of the telescope; and then the line of sight was exactly parallel to the line *oc* *, supposing the object was very remote. But because smaller marks upon a nearer object are better discerned, the hairs were so adjusted till in each position of the telescope they covered a separate mark, the interval of the marks being taken equal to the difference of the heights of the axis of the telescope above the fixt line *mn*, as near as could be measured.

* Art. 834.

How the quadrant was adjusted for use.

867. The object-glass being well centered by the method in art. 808, the line of sight was first of all made parallel to the plane of the quadrant, as near as it need be, by the measures of the brass-work annext to the telescope; and then the plane described by the line of sight, turned about the center of the quadrant, was brought into the plane of the meridian, by observing whether the fixt stars passed over the cross hairs at the same instant of time, as they passed over a meridian telescope, adjusted as above described [a], and placed so near the quadrant, that the two observers could hear each other calling out at the times of the transits. And by the coincidence of these observations upon stars at various altitudes, it appeared that the plane of the quadrant was wrought very true. For it is certain that the meridian plane described by the meridian telescope, as turning upon a transverse axis, must be truer than that described by the quadrantal telescope, as guided by the rollers upon the limb.

a Art. 838.

Fig. 594, 595.

868. When the quadrant was thus reduced into the plane of the meridian by the hold-fasts above described, that radius of it, which terminates 90 degrees, was placed exactly vertical (by the movement abovementioned) with a plumb-line of very fine silver wire; so suspended as to play exactly over the middle of the central point *o* (in the pole of the arbor *oi*) and also over the stroke at 90 degrees upon the limb below. This position of the quadrant being once found, another plumb-line was suspended

ſuſpended by the ſide of the quadrant, quite clear of the center-work; ſo as to play exactly over the middle of a fine point made in the limb below; in order to examine afterwards with more expedition whether the quadrant has kept its place. For this purpoſe an oblong piece of braſs *ab*, laid flat upon the ſquare plate at the center of the quadrant, was gradually moved to the right or left, by two ſkrews *c*, *d*, working againſt the ends of it: a ſlit *ab* being cut lengthways through the plate to ſlide along two other ſkrew-pins *e*, *f*, fixt in the back plate. The wire of the plumbet was hung by a loop upon a pin *g*, and lay in a very fine angular nick filed in the edge of a little plate *h*, which projected a little farther than the loop, for the wire to bear upon it. This plate *h* and the pin *g* were both fixt to the oblong plate *ab*; by whoſe gradual motion above deſcribed, the wire *hi* was brought to play exactly over the middle of the hole *i* in the limb; and then the plate *ab* was preſſed to the quadrant by the ſkrews *e*, *f*. Fig. 601.

869. When this quadrant was fixt upon the eaſtern ſide of the wall, ſome proviſion was made for fixing ſuch another upon the weſtern ſide; to obſerve the altitudes and tranſits of ſtars over the northern half of the meridian; and as ſoon as the Government ſhall think proper to allow the expence of ſo uſeful an inſtrument, we may expect in a few years time to be furniſhed with a more exact Catalogue of all the ſtars viſible in our hemiſpere, than can poſſibly be had from all the inſtruments and obſervations yet extant. And ſuch a Catalogue being the very foundation of all the deſired accuracy in Aſtronomy, Geography and Navigation, I need not dilate upon the uſefulneſs of it. Proviſion for another quadrant.

870. As to the Catalogue of the ſouthern conſtellations, partly inviſible in our latitude, no doubt it will ſoon receive an accurate correction and increaſe, by an excellent Mural Arch of 4 foot radius, made by Mr. *Siſſons* in the *Strand*, exactly after the model of this at *Greenwich*; excepting that the limb of it is continued about 12 degrees towards the ſouth of the plumb-line, for obſerving the places of the moon and planets in paſſing towards the north from the zenith of *Jamaica*; where this quadrant is to be fixt, for the uſe of *Colin Campbell* Eſquire; a young gentleman of whom the learned in the Sciences have great expectations: not only from his own genius and application, but alſo from the advantages of his ſituation, his fortune and education; in which laſt he has had the happineſs to be directed by no leſs a judge than the Right Honourable the Earl of *Ilay*; to whom he has alſo the honour of being related. Mr *Campbell's* Mural Arch at *Jamaica*.

CHAP.

CHAPTER VIII.

Of meaſuring ſmall angles with a teleſcope.

A micrometer what and how applyed.

871. A MICROMETER is a ſmall piece of mechaniſm, contrived for moving a fine wire parallel to it ſelf, in the plane of the picture of an object formed in the focus of a teleſcope, and with great exactneſs to meaſure its perpendicular diſtance from a fixt wire in the ſame plane: and the uſe of this inſtrument is to meaſure ſmall angles ſubtended by remote objects at the naked eye. Let a planet, for example, be viewed through the teleſcope; and when the parallel wires are opened to ſuch a diſtance as to appear exactly to touch two oppoſite points in the circumference of the planet, it is evident that the perpendicular diſtance between the wires is then equal to the diameter of the picture of the planet, formed in the focus of the object-glaſs Let this diſtance, whoſe meaſure is given by the mechaniſm of the micrometer hereafter deſcribed, be repreſented by the line pq in fig. 181; then ſince the meaſure of the focal diſtance qL is alſo known by the method abovementioned[a], the ratio of qL to qp, that is of the radius to the tangent of the angle qLp, will give the angle it ſelf, by a table of ſines and tangents; and this angle is equal to the oppoſite angle PLQ*, which the real diameter of the planet ſubtends at L or at the naked eye. But the ſame angle may be meaſured ſtill more exactly and independently upon the abſolute meaſures of qL and qp, their proportion being ſufficient, as ſhall be ſhewn hereafter.

a Art. 814.

* Art. 43.

Mr. *Huygens*'s micrometer.

b Syſtema Saturn. p. 82.

872. Mr. *Huygens* uſed to meaſure the apparent diameters of the planets or any ſmall angles by a very eaſy contrivance which he thus deſcribes. "There is a certain place, ſays he[b], within the tube of a teleſcope about as far before the convex eye-glaſs as the eye is behind it, wherein the fineſt and ſlendereſt things being placed, appear through the eye-glaſs exceeding plain and diſtinct. And by conſequence will intercept the view of a certain diſtinct part of an object ſeen through the teleſcope. Here if you place a round hole, (made in a braſs plate,) whoſe diameter is a little leſs than that of the eye-glaſs, it will moſt neatly and diſtinctly circumſcribe all the ſpace in the heavens viſible at once through a fixt teleſcope. We muſt firſt determine ſome way or other how many minutes of an arch are the meaſure of the apparent diameter of this ſpace. The beſt way is by meaſuring the time of a ſtar's paſſage over the diameter of it, either by a ſimple pendulum or by a pendulum-clock. For it is well known that a very ſmall matter above one degree of the heavens paſſes over a fixt hour circle in 4 minutes of time. So that for example if a fixt ſtar, ſituated in the equator, or very near it, appears to paſs over the diameter of the ſaid ſpace in 69 ſeconds, the teleſcope being fixt, it follows that the apparent diameter

diameter of that ſpace in the heavens ſubtends an angle of 17 $\frac{1}{4}$ minutes at the naked eye; and this is the meaſure of the ſpace taken in by my teleſcope whoſe length is 23 feet. This being determined, let two or three long and ſlender braſs plates be prepared of various breadths, whoſe ſides are very ſtraight and converge very gradually. Then while a planet is viewed through the teleſcope, ſlide one of them through two ſlits in the oppoſite ſides of the tube, ſo that the plane of the long plate may touch the plane of the round hole; then take notice in what place of the plate the breadth of it can juſt cover the whole planet; for by taking this breadth between the points of a fine pair of compaſſes, and by comparing it with the diameter of the hole, the apparent diameter of the planet may be eaſily collected. Thus far Mr. *Huygens*.

873. But the diameters of planets thus meaſured, as Sir *Iſaac Newton* has obſerved, are ſomewhat bigger than they ſhould be; as is evident by comparing Mr. *Huygens*'s meaſures with others taken by parallel hairs; and alſo by comparing the diameter of Mercury when obſerved in the ſun, with his diameter obſerved out of the ſun; which latter is ſenſibly bigger than the former. All lucid objects ſeen upon dark ones appear bigger, and dark ones ſeen upon bright ones appear leſs, than they would do if the lights of both objects were equally ſtrong. Becauſe the circumference of the brighter image upon the retina diffuſes it ſelf into the contiguous darker image; and the bright image of the planet being intercepted by Mr. *Huygens*'s plate, that faint diffuſed light becomes more ſenſible, and is miſtaken for the edge of the planet. But this error may be avoided by uſing long tapering ſlits, cut in braſs plates, inſtead of tapering plates; or elſe very fine converging wires inſtead of the ſlits.

874. The place of any ſtar, ſo ſituated with reſpect to a known ſtar, as by its diurnal motion to follow it or go before it over the aperture of a fixt teleſcope, may be found very accurately without a micrometer of any ſort; only by obſerving the differences of their right aſcenſions and declinations; which differences being added to, or taken from, the given right aſcenſion and declination of one of the ſtars, give thoſe of the other, and conſequently its place. For this purpoſe it is requiſite to have four croſs hairs in the focus of the teleſcope; of which *ab* and *cd* croſs at right angles, as alſo *ef* and *gh*; which are inclined to the two former in half right angles; and all croſs one another at the point *i*. Then having directed the teleſcope to make the preceding ſtar appear upon the hair *ab*, and having turned the teleſcope about its axis till the ſtar moves along *ab*, let it remain fixt; and obſerve by a pendulum-clock the time of this ſtar's appulſe to the center *i*; and alſo the time of the ſubſequent ſtar's appulſe to the perpendicular hair *cd*. The interval of time between theſe appulſes to the hair *cd*, which coincides with part of an hour circle, being turned into degrees and minutes, gives the number of them contained in the correſpond-

Relative places of ſtars how obſerved by a teleſcope.

Fig. 602.

responding arch of the equator that passed over the said hour circle; which arch is the difference of the right ascensions of the stars. To find the difference of their declinations, the times of the subsequent star's appulses at *k* and *l* to the oblique hairs *ef*, *gh* must also be observed; half of which is the time it spends in describing half the line *kl*, that is *mk* or *ml*. This half being converted into minutes of an arch, gives the number of them contained in the corresponding arch of the equator that passed by the fixt hour circle *cd* in that time; and this number being diminished in the ratio of the radius to the sine complement of the given stars declination, gives the difference of their declinations, or the number of minutes in the arch *mi*. This rule shall be demonstrated by and by, and is sufficiently accurate though the sine complement of the given star's declination is used instead of the sine complement of the unknown star's declination, because their difference is but small in comparison to the whole.

875. Signor *Cassini* who first introduced this useful and accurate method of observing by 4 hairs, has shewn us how he applies it to determine all the particulars observable in Eclipses. See Phil. Trans. N°. 236. It has one great advantage, that the observations are not liable to the uncertainty of the air's refraction. For the telescope being fixt, all objects that appear in it, having nearly the same altitudes, are equally refracted very nearly; it is therefore the best method of observing the places of Mercury and Venus or any of the planets and comets when near the sun, and by consequence but little elevated in the night time, as Dr. *Halley* has observed, Phil. Trans. N°. 366. Where speaking of the accuracy of this way of observing, he tells us, "that he himself was present when Dr. *Pound* and his Nephew Mr. *Bradley* did this way demonstrate the extream minuteness of the sun's parallax so exactly, that upon many repeated trials it was not more than 12″ nor less than 9″. These observations may be seen in the Phil. Trans. N°. 363; from which Dr. *Halley* calculated the parallax by *Cassini*'s method (described in his book upon the comet in 1680.) which is very well explained in the 7th of Mr. *Whiston*'s Astronomical Lectures.

This method made easier.

876, The only trouble in this way of observing, lyes in turning the telescope about its axis into such a position, that the stars shall move parallel to one of the hairs. But this difficulty is intirely removed, though the telescope be never so long, by a very pretty contrivance, invented by that ingenious and accurate Astronomer the Reverend Mr. *James Bradley*, *Savilian* Professor at *Oxford*; which he has also applyed to the micrometer hereafter described. As near as I can remember it is this. Let *ABC* represent a flat ring of brass fixt in the focus of the telescope; and *abc* a smaller concentrick ring lodged in a circular groove turned within the larger, and kept in the groove by three small plates of brass fixt to the

Fig. 603.

the outward ring and extended over the edge of the inner one. Upon the inner ring is fixt a concentrick arch of a wheel *de*, having teeth cut in its convexity, which are driven round by the threads of an endleſs ſkrew, whoſe axis *DEF* turns in a collar at *E* and upon a point at *F*, both fixt to the outward ring. The hairs *gh*, *ik* croſs at right angles in *f* the center of the rings; and when the teleſcope is ſo fixt that the image of a ſtar falls upon *f*, let it move along any line *fq*, and by turning the ſkrew *DEF* and by conſequence the hair *fk* about the fixt point *f* till it touches the ſtar at *q*, it will then coincide with the tract of the ſtar's motion; and then all other ſtars will move parallel to it as was required.

To find the difference of declination of two ſtars, he obſerves the times of their appulſes to the edges of two ſlender braſs bars *gio*, *gkp*, fixt to the inner ring, and equally inclined to its diameter *gh* in ſuch angles that the perpendiculars *fi*, *fk*, on each ſide of *fg*, ſhall be ſeverally equal to half *fg*; and conſequently that the whole baſe *ik*, of the equicrural triangle *igk*, ſhall be equal to its perpendicular height *fg*; and by conſequence that the difference of any two baſes *ifk*, *lmn* ſhall be equal to *fm* the difference of their heights; ſo that the difference of the times of the tranſits of two ſtars over theſe baſes, may give the difference of their declinations as before explained.

877. A micrometer of the beſt ſort is made in this manner. In the middle of an oblong plate of braſs *AB*, there is cut an oblong hole *abcdef*, (to be placed in the focus of a teleſcope,) having a fine wire *be* extended length ways over the middle of it, at right angles to two ſlender braſs bars or ſights *gh*, *ik* lying croſs the hole; of which *gh* is fixt to the plate *AB* by ſkrews at *g* and *h*, but *ik* is moved parallel to *gh* by twiſting a round knob *C* fixt upon one end of a long iron ſkrew *DE*; which turns upon a tapering point at its end *D*, while its other end turns round in a hole at *E* in the center of an index-plate *EF* fixt at right angles to the main plate *AB*. The long ſkrew *DE* works through two hollow ſkrews in two cubical blocks of braſs fixt behind the plate *lm*, bent ſquare to the plate *no*, that ſlides upon the main plate *AB*, either backward or forward, and carries a perpendicular arm *op* extended over the hole *be*; while *p* the extremity of the arm *op* ſlides under a braſs ledge *qr* ſkrewed to the main plate *AB* along the ſide of the hole. One ſide *st*, of the moveable ſight-plate *ikst*, lyes over the arm *op*, being fixt to it by flat headed ſkrews at *s* and *t*, the holes in the plate *st* being oblong or larger than the ſhanks of the ſkrews, to give liberty for placing the edge *ik* coincident with *gh*, when carried up to it by turning the ſkrew *DE* by the knob *C*; the part *ik*, which projects over the arm *op*, being hammered down to lye flat upon the main plate *AB*. The edge *ih*, after this adjuſtment, will always move parallel to the edge *gh*; its inclination to the ſkrew *DE* being every where the ſame, provided the ſkrew be

Deſcription of a micrometer. Fig. 604.

ſtraight, and the interval of the concave ſkrews behind lm be ſufficiently greatand their motion ſteady. For this purpoſe about a quarter-round of a third concave ſkrew preſſes upon the long ſkrew DE at v, the block of it being fixt to the middle of a ſpringing plate wvx; whoſe extremities lying behind the blocks at l, m, not ſo near as to touch them, are preſſed towards them with ſkrews at w and x; which occaſions the block at v to ſpring upon the ſkrew DE, and to hold it tight to the oppoſite ſides of the concave ſkrews at l, m. To prevent any motion lengthways in the ſkrew DE, its tapering point D turns in a hollow point at the end of an oppoſite ſkrew y, which working through a fixt block at z, holds up the ſhoulder of the long ſkrew DE againſt the back of the index-plate, where its neck is inſerted.

The two indexes upon the plate EF ſhew the number of revolutions and parts of a revolution of the ſkrew DE, anſwering to the interval of the ſights gh, ik. In the outward plate there is a circular ſlit $\alpha\beta\gamma$, which diſcovers part of the diviſions upon the circumference of an inner plate, turned about a center by two wheels and pinions within: ſo that for every revolution of the ſkrew and index EF, which ſhews the parts of it, one diviſion upon the plate $\alpha\beta\gamma$, paſſes by a fixt pointer at k; which ſhews the number of revolutions anſwering to the interval of the ſights gh, ik.

An improvement of it.

878. This micrometer has lately received a very great improvement by an ingenious contrivance of the Reverend Mr. *James Bradley* Profeſſor of Aſtronomy at *Oxford*, for turning it in its own plane about the interſection δ of the fixt ſight gh and of the tranſverſe wire $b\delta e$, without ſtirring the teleſcope; which is thus executed. Upon the backſide of the main-plate turned upwards, and here repreſented by the parallelogram

Fig. 605.

$GHIK$, there is laid ſuch another plate $LMNO$, of the ſame breadth and thickneſs but ſomewhat ſhorter; in the middle of which there is an oblong hole, anſwering to that other in the lower plate, but ſomewhat larger; being terminated at its ſides by the ſtraight lines $\epsilon\zeta$, $\eta\theta$, and at its ends by the concave arches $\theta\iota\epsilon$, $\zeta\kappa\eta$, whoſe common center is the point δ abovementioned. The concave arch $\epsilon\iota\theta$ ſlides round this center againſt a concentrick convex arch of an annular plate $\lambda\mu\nu$; ſomewhat longer than the concave arch, of the ſame thickneſs as the upper parallelogram, and ſtrongly ſkrewed to the under one, round that end of the hole which is neareſt to the center δ: and at the ſame time the other concave arch $\zeta\kappa\eta$ ſlides alſo againſt another concentrick convex arch $o\varpi$, of another annular plate juſt as thick as the upper parallelogram, and ſtrongly ſkrewed to the under one. This convex arch $o\varpi$ is ſhorter than the contiguous concave one $\zeta\kappa\eta$, to give room for the circular motion of the plates; which are held together by two annular plates ſimilar to $\lambda\mu\nu$ and $o\varpi$, but ſomewhat broader, to cover the coincident arches, when laid over them, and ſkrewed down to the reſpective annular plates underneath. The circular

cular motion upon theſe arches about their center δ, is gradually given to the upper parallelogram by an endleſs ſkrew at ϱ, having an axis $\sigma\tau$ laid croſs the end of the under parallelogram, and turning upon a point at one end and in a collar at the other, both fixt to the under plate; while the ſpiral thread ϱ moves the teeth of a braſs arch fixt at v to the end of the upper parallelogram.

To hold the micrometer in the tube of a teleſcope, along each ſide of the upper parallelogram there is fixt a long braſs plate about an inch broad; having its oppoſite ſides bent contrary ways, ſo as to form two oppoſite ledges, about $\frac{1}{8}$ of an inch broad, at right angles to the intermediate part of the plate, as repreſented in the figure. One of the ledges of each plate is placed inwards along the ſides of the upper parallelogram, and is firmly fixt to it by ſeveral ſkrews. The figure $\varphi\chi\psi\omega$ repreſents one of the equal and oppoſite holes cut in the ſides of a ſquare tube, through which the micrometer is put; the notches $\varphi\chi$ being made to receive the ledges of the ſide-plates, to keep the plane of the micrometer perpendicular to the tube at a juſt diſtance from the object-glaſs. Which diſtance being once determined by trials, as above explained, muſt be kept invariable in all obſervations, by ſtops or pins, if the tube conſiſts of two or more joints that draw in and out.

The meaſures of the Micrometer.

	Inches
The length of the plate AB	8, 0
Its breadth MN	3, 6
Its thickneſs	0, $\frac{1}{2}$
Length of the hole be	3, 5
Its breadth $gb = \delta e$	2, 2
Breadth of the hole in the other plate at $\zeta\eta$	2, 6
Length of the ſkrew DE	5, 5
Its thickneſs	0, 3
The line Ab	1, 6
The interval $lm = wx$	3, 0
Length of the ſide cheeks	4, 5
Their breadth	0, 8
Their ledges	0, 2
Diameter of the index-plate	3, 1
Its thickneſs (being double with two wheels within)	0, 3
The greateſt opening of the ſights gb, $ik = \delta e$	2, 2

Threads of the ſkrew in an inch, 40

The inch is divided by the index-plates into 40 times 40 or 1600 equal parts. Inſtead of the braſs ſights gb, ik, two others with parallel wires may be ſkrewed on at pleaſure.

To find the angles answering to the revolutions of the skrew.

879. When the sight-plates are made to coincide, the two indexes of the revolutions and their parts must be set to the beginning of the numerations upon the index-plates. Then as the sights are opened, it is evident from the make of a skrew, that the numbers of revolutions will be as the intervals of the sights, and consequently as the angles subtended by them at the center of the object-glass; the intervals being insensibly different from the arches that measure these small angles. Therefore when any one angle corresponding to a given number of revolutions, is determined by experiment, an angle corresponding to any other number of revolutions may be found by the rule of three. And thus may tables be made to shew by inspection the number of minutes and seconds in an angle answering to any given number of revolutions and parts.

880. To determine some one angle, the larger the better, because the same error in the determination will be proportionably smaller in a given angle deduced from it; fix the telescope upon any known star in the equator or very near it, and open the sights to their utmost limit and note the number of revolutions of the skrew. Then by a pendulum-clock observe the interval of time in the star's transit over the given interval of the sights, and having turned it into minutes and seconds of an arch, they are the measure of the angle required. But if the star be remote from the equator, the number of minutes and seconds thus found, must be diminished in the ratio of the radius to the sine of the star's distance from the pole.

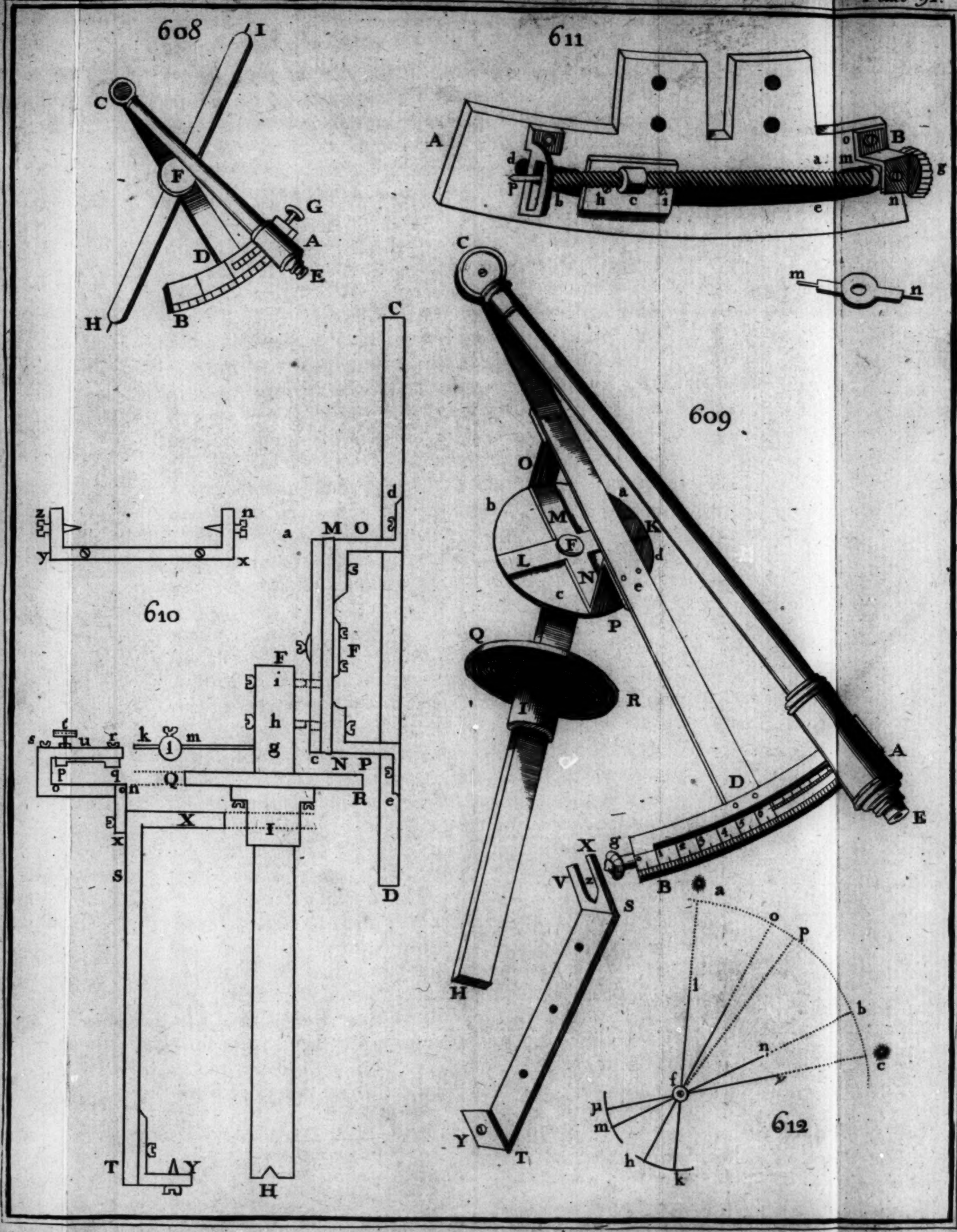
Fig. 606.

881. For let o be the center and poq the axis of a sphere, parallel to the earth's axis; $prvq$ and $psxq$ two hour circles cutting the equator of the sphere in v and x, and the circle parallel to it, described by the star's diurnal motion, in r and s. Join ov and ox, and from t the center of the parallel draw tr and ts. Then if you please we may suppose o to be the center of the object-glass, and the diameter $ro\varrho$ to be equal to twice the length of the telescope, and the little arch rs, or the equal and opposite arch $\varrho\sigma$, to be the interval of the sights; and then the angle ros or $\varrho o\sigma$ is the angle required. Now since the lines $ro\varrho$, $so\sigma$ are the visual rays at the times of the star's appulses to the sights, the interval of time, turned into minutes and seconds of an arch or angle, gives the number of them contained in the angle vox or rts, in which the planes of the hour circles are inclined to each other. But the angle rts is to the angle ros, subtended by the same perpendicular rs, as ro to rt*, that is as the radius to the sine of the star's distance from the pole p. Or thus, the triangles or sectors vox, rts being similar, the radius vo is to the sine tr (as the arch vx to the arch rs or) as the angle vox to the angle ros, taking rs for an arch of a great circle.

* Art. 60.

By Molyneux's method. Dioptr. p. 249.

882. Another way of finding the angle answering to one of the greatest openings of a micrometer, is this. Having fixt the micrometer at its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due distance Ef from the object-glass, requisite for viewing the most remote object without parallax[a]; on the side of a wall or house far distant, mark out two conspicuous objects as P, R that may both at once be received into the fixt telescope. Measure nicely their distance PR from each other, and also their distance EQ from the object-glass E, (the axis of the telescope QEq being perpendicular to the middle of the line PR) and by trigonometry calculate the angle PER. Then looking through the telescope open the micrometer, till its two edges exactly meet with and embrace the two objects P, R; and observe how many revolutions, and parts of a revolution are performed in this opening. For so many compleat the angle before calculated. So far Mr. *Molyneux*. Fig. 607. a Art. 813.

883. Upon an even piece of ground where a long distance may be measured exactly by a pole, this experiment may be made with very great accuracy, only by fixing up a rail or horizontal board, at right angles to QE, with a black line drawn lengthways upon it, and marked with two cross strokes P, R of a sufficient thickness to be visible through the telescope. And although it should happen for want of even ground enough, that the object is too near to be seen distinctly and without parallax, unless the telescope be lengthened, yet the experiment may be performed with sufficient accuracy by making the following correction. An improvement of it.

Draw back the micrometer to the distance Eq requisite for viewing the present object most distinctly, and without parallax; and measure Eq as exactly as you can, for great exactness is not necessary; then having observed the number of revolutions and parts, answering to the apparent diameter of the object or the computed angle PEQ; diminish this number in the given ratio of QE to Qq; and you have the number of revolutions answering to the same angle QER when the place of the micrometer is adapted for viewing the remotest objects. For supposing pqr the image of the near object PQR and efg the image of the most remote object subtending the same angle as PQR does; these two images would be terminated by the same lines if both were distinct. And consequently the number of revolutions that would answer to the image efg, is to the number of revolutions answering to the image pqr, as the line efg to the line pqr. But supposing the rays to go back again from q to Q; since f is the focus of parallel rays coming the contrary way, we have $qQ : qE :: qE : qf$* and disjointly $QE : Qq :: (Ef : Eq :: eg : pr ::)$ as the number of revolutions in eg, to the number in pr. * Art. 239.

884. A micrometer is the more useful for taking in a larger angle, and for this purpose the broadest and thickest eye-glass and one that magnifies the most is the best. Through such an eye-glass the hairs of the micrometer will appear crooked indeed[b], but provided they appear sufficiently distinct, the accuracy of the measure of an angle is not at all disturbed thereby. Because the right lines upon the object that are cover- Apparent curvity of the hairs no cause of error. b Art. 154. 605.

ed

due diſtance Ef from the object-glaſs, requiſite for viewing the moſt remote object without parallax[a]; on the ſide of a wall or houſe far diſtant, mark out two conſpicuous objects as P, R that may both at once be received into the fixt teleſcope. Meaſure nicely their diſtance PR from each other, and alſo their diſtance EQ from the object-glaſs E, (the axis of the teleſcope QEq being perpendicular to the middle of the line PR) and by trigonometry calculate the angle PER. Then looking through the teleſcope open the micrometer, till its two edges exactly meet with and embrace the two objects P, R; and obſerve how many revolutions, and parts of a revolution are performed in this opening. For ſo many compleat the angle before calculated. So far Mr. *Molyneux*. Fig. 607. a Art. 813.

883. Upon an even piece of ground where a long diſtance may be meaſured exactly by a pole, this experiment may be made with very great accuracy, only by fixing up a rail or horizontal board, at right angles to QE, with a black line drawn lengthways upon it, and marked with two croſs ſtrokes P, R of a ſufficient thickneſs to be viſible through the teleſcope. And although it ſhould happen for want of even ground enough, that the object is too near to be ſeen diſtinctly and without parallax, unleſs the teleſcope be lengthened, yet the experiment may be performed with ſufficient accuracy by making the following correction. An improvement of it.

Draw back the micrometer to the diſtance Eq requiſite for viewing the preſent object moſt diſtinctly, and without parallax; and meaſure Eq as exactly as you can, for great exactneſs is not neceſſary; then having obſerved the number of revolutions and parts, anſwering to the apparent diameter of the object or the computed angle PEQ; diminiſh this number in the given ratio of QE to Qq; and you have the number of revolutions anſwering to the ſame angle QER when the place of the micrometer is adapted for viewing the remoteſt objects. For ſuppoſing pqr the image of the near object PQR and efg the image of the moſt remote object ſubtending the ſame angle as PQR does; theſe two images would be terminated by the ſame lines if both were diſtinct. And conſequently the number of revolutions that would anſwer to the image efg, is to the number of revolutions anſwering to the image pqr, as the line efg to the line pqr. But ſuppoſing the rays to go back again from q to Q; ſince f is the focus of parallel rays coming the contrary way, we have $qQ : qE :: qE : qf$* and disjointly $QE : Qq :: (Ef : Eq :: eg : pr ::)$ as the number of revolutions in eg, to the number in pr. * Art. 239.

884. A micrometer is the more uſeful for taking in a larger angle, and for this purpoſe the broadeſt and thickeſt eye-glaſs and one that magnifies the moſt is the beſt. Through ſuch an eye-glaſs the hairs of the micrometer will appear crooked indeed[b], but provided they appear ſufficiently diſtinct, the accuracy of the meaſure of an angle is not at all diſturbed thereby. Becauſe the right lines upon the object that are cover- Apparent curvity of the hairs no cauſe of error. b Art. 154. 605.

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ed by the hairs, appear to be juſt as much bent as the hairs themſelves. If a hair be ſtretched upon a page of a book, and be viewed through the edge of a very convex glaſs, the hair though diſtorted will appear to cover the very ſame letters as when viewed by the naked eye. And the caſe is the ſame in the focus of a teleſcope; becauſe the rays that come from the points of the object, diverge upon the eye-glaſs from points contiguous to the hairs.

CHAPTER IX.

Mr. Graham's *Aſtronomical ſector.*

Deſign of this inſtrument. 885. IT is allowed that a micrometer is the moſt accurate and convenient inſtrument for obſerving the place of a planet or comet, when it happens to be near enough to any known ſtar; by taking the differences of its right aſcenſion and declination from thoſe of the ſtar, as explained above. But this being frequently impracticable, by reaſon that many large places in the heavens are void of ſtars whoſe places are known; it is neceſſary to have recourſe to moveable quadrants or ſextants, furniſhed with teleſcopick ſights for taking larger diſtances. But beſides the difficulty and charge of procuring good inſtruments of this ſort, the great trouble and uncertainties in obſerving with them are very notorious, ariſing chiefly from the difficulty the obſervers find in making their obſervations, at each teleſcope, correſpond together at the ſame inſtant, while the inſtrument is following the diurnal motion of the heavens. The lovers of Aſtronomy are therefore much obliged to the ingenious Mr. *George Graham* F. R. S. not only for many uſeful improvements in the mechaniſm of ſeveral aſtronomical inſtruments, but alſo for contriving a very commodious and accurate one for the purpoſe aforeſaid; that is for taking ſuch differences of right aſcenſion and declination as are too large to be obſerved through a fixt teleſcope; and yet with equal facility, and exactneſs too in proportion to the radius of the inſtrument. I will firſt give an idea of it and then deſcribe the particulars of the mechaniſm.

An idea of the inſtrument. Fig. 608. 886. Let *AB* repreſent an arch of a circle containing 10 or 12 degrees well divided, having a long plate *CD* for its radius, fixt to the middle of the arch at *D*. Let this radius be applied to the ſide of an axis *HFI*, and be moveable about a joint fixt to it at *F*, ſo that the plane of the ſector may be always parallel to the axis *HI*; which being parallel to the axis of the earth, the plane of the ſector will always be parallel to the plane of ſome hour circle. Let a teleſcope *CE* be moveable about the center *C* of the arch *AB*, from one end of it to the other, by turning a ſkrew at *G*; and let the line of ſight be parallel to the plane of the ſector. Now

Now by turning the whole inſtrument about the axis *HI*, till the plane of it be ſucceſſively directed firſt to one of the ſtars and then to another, it is eaſy to move the ſector about the joint *F* into ſuch a poſition, that the arch *AB* when fixt, ſhall take in both the ſtars in their paſſage by the plane of it; provided the difference of their declinations does not exceed the arch *AB*. Then having fixt the plane of the ſector a little to the weſtward of both the ſtars, move the teleſcope *CE* by the ſkrew *G*, and obſerve by a clock the time of each tranſit over the croſs hairs and alſo the degrees and minutes upon the arch *AB* cut by the index at each tranſit: then is the difference of the arches the difference of the declinations, and by the difference of the times we have the difference of the right aſcenſions of the ſtars.

Deſcription of the parts. Fig. 609.

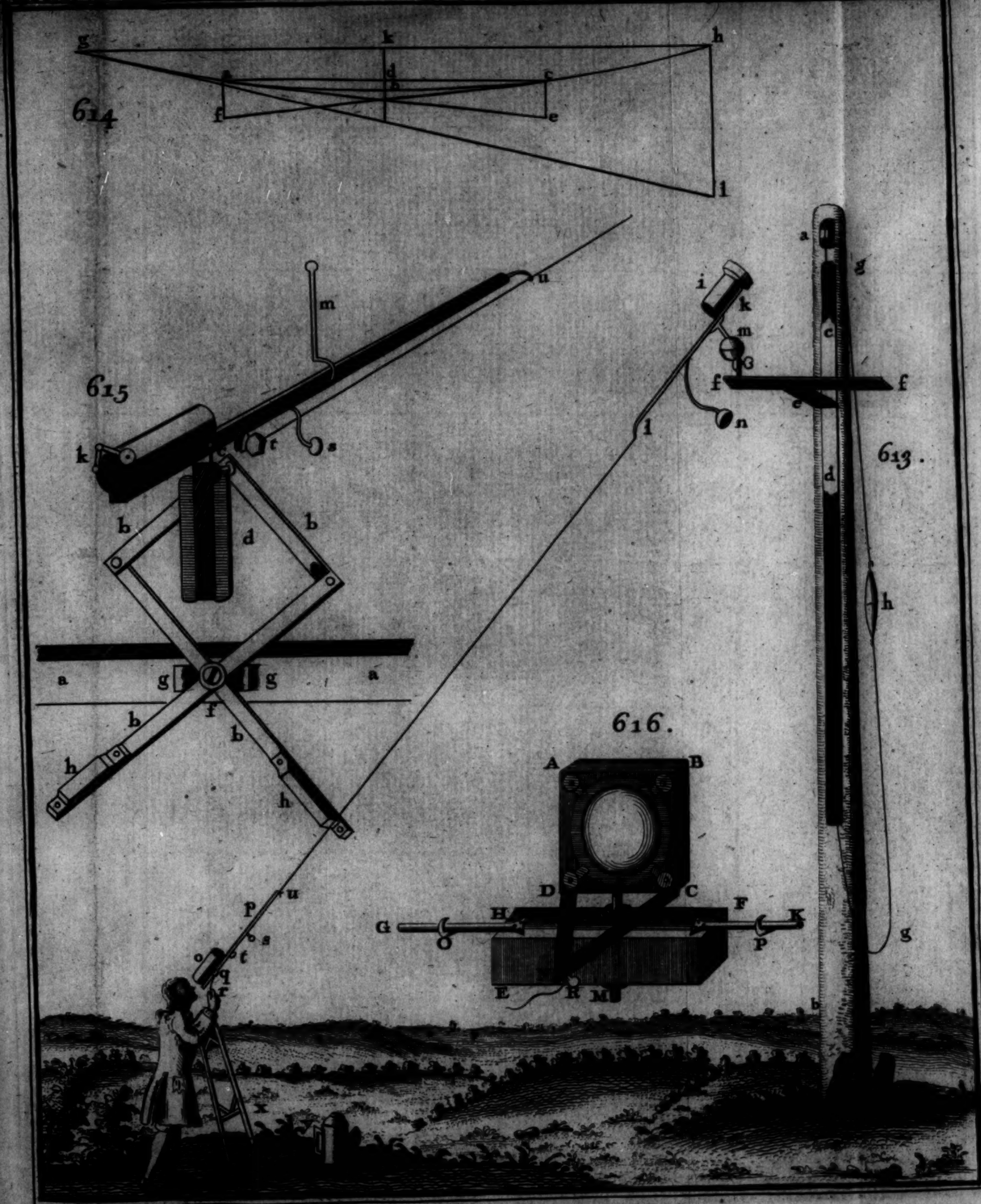
887. The mechaniſm of the principal parts of the inſtrument is this. Upon the ſide of an iron axis *HIF* wrought ſquare, and near the top of it, there is fixt a broad circular plate *abc* of ſolid braſs; upon which there lies a braſs croſs *KLMN*, which turns about a joint at the center *F*. At the ends of the croſs plate *MN*, are erected two equal perpendiculat arms *O* and *P*, whoſe extremities are fixt by the ſkrews *d*, *e* to the backſide of the radius *CD*; which is ſtrengthened by a long braſs rib on its backſide, placed edgeways from one end to the other as repreſented in fig. 610. The arms *O*, *P* are no longer than is neceſſary for the ſector *ABC* to turn about the joint *F* quite clear of another circular plate *QR*; which is fixt to the upper baſis of a braſs cylinder *I*, the iron axis *HF* being put through a ſquare hole in the middle of them both, and immoveably fixt in it. *ST* repreſents a long ſubſtantial plate of braſs, having two ſhort plates *VX* and *YT* fixt perpendicular to the ends of it. Let us ſuppoſe the length of the plate *ST* to be parallel to the earth's axis, and to be firmly fixt in this poſition upon a pedeſtal, or otherwiſe, with its flat ſides facing the north and ſouth. Remove the axis *HI* and place the conical hole made in the end *H* upon a conical point of a ſkrew-pin at *Y*; and the cylinder *I* into the ſlit *VZX*; whoſe parallel ſides *V*, *X* embrace it, while it reſts againſt two points of an angular notch in the bottom of the ſlit at *Z*. By this means the whole inſtrument will always turn true about one and the ſame imaginary line. When the ſector is turned about the joint *F* till the radius *CD* becomes parallel to the axis *HI*, the 610th figure repreſents a ſection of the whole inſtrument, made by a plane paſſing at right angles through the radius *CD*, and through the rib on the backſide of it, and through the axis *HI* and through the ſupporter *ST*. The ſeveral parts of the inſtrument are here denoted by the ſame letters in both figures. The arms *O* and *P* have two ſlits through the middle of their ends to receive the edge of the rib *CD*. The circular plate *ac* is fixt to the axis by the ſkrews *h*, *i*. The braſs rod *gk* is ſkrewed into the axis *HI*, and carries a braſs ball *lm* that ſlides along it, and is fixt by a ſkrew *m* at a proper place Fig. 609, 610.

place for ballancing the weight of the sector and telescope placed on the opposite side of the axis. At the top of the supporter *ST* there is a hold-fast *nopqrstv*, whose cavity *nopq* receives the circular plate *QR*. The end *q* of a springing plate *pq* is fixt by a skrew *r* to the inside of the upper plate *rs*, while its other end *p* may be pressed down upon the circle *Q* by twisting the knob of a skrew *t* which works in a socket *v*. And to prevent this pressure from dislocating the plane of the circle *QR* and consequently the position of the axis *HI*, the hold-fast *nopq* has liberty of yielding or turning upon the ends of two skrew-pins, that go into two conical holes in the opposite edges of the under plate *no*; one of these skrews is seen at *n*; and the fixt piece that they skrew into, is represented separately in full view at *nxyz*; *n*, *z* being the points that the hold-fast turns upon. By this means the same skrew at *t* causes the upper and under plates of the hold-fast *nopq* to compress the circle *Q* with equal forces. To the arm *O* there is fastened such another hold-fast, which so compresses the circle *ac* and the cross plate *MN*, as to stay the sector and telescope in any given position, from turning about the joint *F*. This joint is nothing but a cylindrical pin passing through the plates *MN*, *ac*. The flat head of the pin is fixt by 3 small skrews to the plate *MN*; and to the opposite end of this pin a circular springing plate is fixt by a skrew, that skrews into the end of the pin. And the joint *C* at the center of the sector *ABC* is made in the same manner.

Fig. 611. 888. The 611th figure represents the skrew-work upon the backside of the limb *AB*; contrived for moving the telescope by twisting the knob *g*. Here *gab* is a long straight skrew, which works through a skrew-hole in a brass head *c*; whose neck is moved in a long circular slit *de* cut through the limb; the other end of this neck turns round in a hole in the Nonius-plate, (one end of which is fixt to the telescope,) and draws it along the limb: *h* and *i* are the heads of two skrews, whose shanks go through a springing plate, to make the motion steady, and through the slit *de*, and are skrewed into the Nonius-plate on the other side of the limb. Since the length of the skrew *ab* must have a small angular motion while it carries the head *c* along the arch *de*; the shank of it near the end *a* turns round in a hole made in a short flat axis *mn*, placed perpendicular to the limb and held so by an arm *no*, while the other end *b* moves in a slit *p* parallel to the limb, being cut in a small plate fixt perpendicular to the limb. The long skrew *gab* is kept from slipping backwards and forwards through the hole in the axis *mn*, by shoulders or nuts fixt on each side.

Dimensions of the instrument.

889. The dimensions of this instrument are these. The length of the telescope, or the radius of the sector is $2\frac{1}{2}$ feet; the breadth of the radius near the end *C* is $1\frac{1}{2}$ inch; and at the end *D* 2 inches. The breadth of the limb *AB* is $1\frac{1}{2}$ inch, and its length 6 inches, containing 10 degrees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grees divided into quarters and numbered from either end to the other. The telescope carries a *Nonius* or subdividing plate [a]; whose length, being equal to 16 quarters of a degree, is divided into 15 equal parts; which in effect divides the limb into minutes, and by estimation into smaller parts. The length of the square axis *HIF* is 18 inches, and of the part *HI* 12 inches, and its thickness is about $\frac{1}{4}$ inch; the diameters of the circles *QR* and *abc* are each 5 inches. The thickness of the plates and the other measures may be taken at the discretion of a workman. a Art. 861.

Rectification of the instrument.

890. This instrument may be rectified, for making observations, in this manner. By placing the intersection of the cross hairs at the same distance from the plane of the sector, as the center of the object-glass found by art. 808, the plane described by the line of sight, during the circular motion of the telescope upon the limb, will be sufficiently true or free from conical curvity: which may be examined by suspending a long plumb-line at a convenient distance from the instrument, and by fixing the plane of the sector in a vertical position, and then by observing, while the telescope is moved by the skrew along the limb, whether the cross hairs appear to move along the plumb-line.

891. The iron axis *bfo* may be elevated nearly parallel to the axis of the earth by means of a small common quadrant; and its error may be corrected by making the line of sight follow the circular motion of any of the circum-polar stars, while the whole instrument is moved about its axis *bfo*, the telescope being fixt to the limb. For this purpose let the telescope *kl* be directed to the star *a* when it passes over the highest point of its diurnal circle, and let the division cut by the Nonius upon the limb be then noted. Then after 12 hours, when the star comes to the lowest point of its circle, having turned the instrument half round its axis, to bring the telescope into the position *mn*; if the cross hairs cover the same star supposed at *b*, the elevation of the axis *bfo* is exactly right: but if it be necessary to move the telescope into the position $\mu\nu$ in order to point to this star at *c*, the arch $m\mu$ which measures the angle $mf\mu$ or *bfc*, will be known; and then the axis *bfo* must be depressed half the quantity of this given angle if the star passed below *b*; or must be raised so much higher if above it: and then the trial must be repeated till the true elevation of the axis be obtained. By making the like observations upon the same star on each side the pole, in the six a clock hour-circle, the error of the axis towards the east or west may also be found and corrected, till the cross hairs follow the star quite round the pole. For supposing *aopbc* to be an arch of the meridian, (or in the 2d practice, of the six a clock hour-circle,) make the angle *afp* equal to half the angle *afc*; and the line *fp* will point to the pole, and the angle *ofp*, which is the error of the axis, will be equal to half the angle *bfc* or $mf\mu$, found by the observation; because the difference of Fig. 612.

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the two angles *afb*, *afc* is double the difference of their halves *afo* and *afp*. Unless the star be very near the pole, allowance must be made for refractions by the Table in the next book.

CHAPTER X.

To manage long telescopes without tubes.

Preface. 892. It has has been shewn in the seventh chapter of the second book, that the only way to improve dioptrick telescopes is to increase their lengths, in no less a ratio of the proposed increase of the apparent diameter of an object, than what is duplicate of it. So that to magnify twice as much as before with the same light and distinctness, the telescope must be lengthened four times; and to magnify thrice as much, nine times; and so on. It was shewn at the same time, that the necessity of using such great lengths proceeds from the imperfections of the picture of the object formed by the object-glass; which imperfections mathematicians formerly attributed to the unfitness of spherical surfaces to produce an exact union of the rays in each pencil; and therefore they prescribed to give their glasses the figures of conical sections turned about their axes. But the great Sir *Isaac Newton* discovered their mistake, by shewing that conical figures would only remove an inconsiderable cause of these imperfections of the picture, while the different refrangibility of the rays, which is the principal cause, would unavoidably remain as before. This led him to the invention of his reflecting telescope about the year 1670, but before it was lately brought into common use by the ingenious Mr. *Hadley*, mathematicians still despaired of any other improvements of telescopes but by lengthening their tubes, and contriving proper machines to move and manage them. A great variety of these machines may be seen in *Hevelius*'s *Machina Cœlestis*; but astronomy at last was very happily freed from this expensive lumber by the ingenuity of the great *Hugenius*: who placing the object-glass upon a long upright pole, contrived to direct its axis towards any object by a fine silk line, coming down from the glass above to the eye-glass below. This invention was successfully practiced both by himself and others; particularly with us by the late Dr. *Pound* and by his nephew Mr. *James Bradley* Professor of Astronomy at *Oxford*, with an object-glass of 123 feet focal distance, and an apparatus belonging to it, made and presented by *Hugenius* to the Royal Society, and described in his *Astroscopia Compendiaria tubi optici molimine liberata*, printed at the *Hague* in 1684; which treatise I have here translated. For although Sir *Isaac Newton*'s reflecting telescopes are now made in so great perfection, that one of five feet is equivalent to the *Hugenian* glass of 123 feet; yet there is one considerable advantage in these dioptrick telescopes

which

which the catoptrick ones have not, to wit in applying a micrometer. For the diameters of the pictures of objects in both telescopes being as their lengths, the same micrometer will measure the larger image more exactly than the smaller, in the same proportion of the lengths of the telescopes. It is chiefly for this reason that I give Mr. *Huygens*'s account of his aerial telescopes; and also of a small variation in their contrivance by Mr. *de la Hire*.

893. "In a large area every way open to the view of the heavens, let a long pole or mast be fixt upright in the earth. That which we first used was 50 feet high; and was sufficient for a telescope 70 feet long or more, though not for the greatest altitudes of the stars, for which the pole should be almost as long as the telescope. Before the pole is erected let one side of it be wrought plane, to which let two long ledges be nailed parallel to each other, at the distance of an inch and a half; so as to make a long channel a little broader within than without, reaching from the top of the mast within three feet of the bottom. And through the top of the mast, over against this long channel, let a slit or mortise be made to receive a pully, over which let a rope be put as long again as the mast, and near half an inch thick. And for a person to ascend to the top upon occasion, let wooden triangles be fixt to the mast at equal intervals, to serve for steps. Then let the bottom of it be daubed over with hot pitch, so far as it is to be fixt in the ground, and let sand be put into the hole round about it to keep it from rotting. The use of the mast is to raise the object-glass to a proper height as follows. Fig. 615.

894. Let a board of two foot long be sloped on each side, so as to move freely up and down in the channel abovementioned; and to the middle of this board let a wooden arm be fixt and extended a foot in length from the mast; and let the middle of another board, a foot and an half long, be laid horizontal and at right angles over the end of that arm, and be fixt to it. The object-glass must be placed upon one end of this transverse board; and the whole must be lifted up and down by the rope abovementioned; the ends of it being tyed to the top and bottom of the upright board that slides in the channel; and the whole must be counterpoised by a leaden weight fixt to the rope on the other side of the pully, in such a place that the weight may be at the top when the object-glass is at the bottom, and on the contrary. The ends of the weight must be made of a conical figure to prevent its catching and sticking at the wooden triangles abovementioned.

895. The manner of fixing the object-glass is this. First let it be fixt within a tube four inches long, made of sheet-tin or brass; and to the outside of this tube (or rather to a hoop that surrounds it) let a straight

ſtick be fixt, about an inch thick, and extended about 8 inches or a foot from the end of the tube. Then to ſupport the whole let a braſs-ball as big as a filbert-nut, be fixt to the ſaid ſtick by a ſhort neck, and be lodged in a hollow ſocket, in which it may play very freely without danger of dropping out. Let the ſocket and its cylindrical pedeſtal be ſlit in two halfs, and be held together by a ſkrew paſſing through both, but not ſo cloſe as to pinch the ball. By this means the object-glaſs and the ſtick annexed, are moveable every way; and to keep them in equilibrio, an equal counterpoiſe of lead is fixt to the under part of the ſtick by a ſtiff braſs wire; ſo that by bending this wire to and fro, the common center of gravity of the weight, the lens and parts annexed may be eaſily placed in the center of the braſs ball; and then the whole compound will be moveable with the leaſt touch, and will reſt in any given poſition: and in this lies the judgement of the whole invention. Having ſtuck the pedeſtal of the ball and ſocket into a hole in the end of the tranſverſe board above deſcribed; to the tail of the ſtick, annext to the object-glaſs, let a ſilk line be tied, and let the length of it rather exceed the length of the intended teleſcope, that the other end of it may be brought to the eye-glaſs. Hence when the object-glaſs is raiſed towards the top of the maſt, by gently drawing this thread while you are moving round the maſt, the object-glaſs will readily obey its motion, and be directly oppoſed to what ſtar you pleaſe; which could never be performed without placing it in the ſtate of libration above deſcribed. Now ſince it is abſolutely neceſſary that the ſtick, annexed to the object-glaſs, ſhould be parallel to the extended line; for this purpoſe a ſhort braſs wire is fixt into the tail of the ſtick, and is bent downwards ſo far till the end of it, where the ſtring is tied, be as much below the ſtick as the center of the ball and ſocket is. The reaſon for uſing a flexible ſpringing wire ſhall be given hereafter.

896. Let us now deſcribe the poſition of the eye-glaſs and its connection with the object-glaſs; which will take up but few words, ſince the mechaniſm below is almoſt ſimilar to that above. Here alſo the eye-glaſs is included in a ſhort tube, connected to a ſtick, which may alſo have a ball to reſt upon or rather a little tranſverſe axis, and a weight below the ſtick to ballance the tube and eye-glaſs. The obſerver takes hold of a handle fixt to the tranſverſe axis, and holds the lower ſtick directed towards the upper one, by means of the line that connects them and winds about a pegg fixt in the lower ſtick; ſo that by pulling gently, to extend the line, it is evident the two glaſſes will become parallel to each other. The lower part of the ſtring comes through a ſmall hole made with a wire at the far end of the lower ſtick; and the obſerver by turning the pegg, like that of a muſical inſtrument, ſhortens or lengthens the line at pleaſure; till he brings the interval between the glaſſes to a juſt length, requiſite for diſtinct viſion.

897. But in order to keep the eye-glaſs ſteady, as it ought to be, it is convenient for the obſerver, whether ſtanding or ſitting, to ſupport his arms upon a reſt; conſiſting of a ſlender tranſverſe beam ſupported by two feet, made of any light ſtuff, while he holds the eye-glaſs in one of his hands: which is a readier and more commodious way than to fix the eye-glaſs upon a reſt ſupported by three feet.

898. In order to find a ſtar in the teleſcope, when the nights are dark, we make uſe of a lantern; which collects the light into a ſtream either by tranſmiſſion through a convex lens, or by reflection from a concave ſpeculum. For by directing this ſtream of light till it falls upon the object-glaſs and makes it viſible, it is eaſy for the obſerver to change his place, till he finds the ſtar is covered by the middle of the object-glaſs; and then to apply his eye-glaſs. Which is ſooner done than with a teleſcope conſiſting of a long tube. By moon-light the object-glaſs is viſible without the help of the lantern. But in viewing the moon through the teleſcope, it is neceſſary to fix an umbrella made of thin paſt-board about the object-glaſs, of ſuch a diameter as ſhall cover a ſpace in the ſky above twice as broad as the moon; to intercept that light from coming to the eye which would paſs by the ſides of the object-glaſs, and by mixing with the light that comes through the teleſcope, would dilate the appearance of the lights and ſhades in the face of the moon. But to underſtand this mechaniſm more clearly we have added a deſcription of the figure.

The pole or maſt is *ab* Fig. 613.
The moveable board in the long channel, *cd*
The arm fixt perpendicular to it, *e*
The tranſverſe board that ſupports the object-glaſs, *ff*
The endleſs rope going round the pully, *gg*
The weight annext to it, *h*
The pully at the top of the pole, *a*
The tube that holds the object-glaſs, *i*
The ſtick faſtened to that tube, *kl*
The braſs ball fixt to the ſtick and lying in the ſocket, *m*
The lead at the end of the braſs wire, *n*
The ſhort wire at the end of the ſtick, *l*
The tube that holds the eye-glaſs, *o*
The ſtick fixt to the tube, *p*
The little moveable axis, *q*
The handle, *r*
The leaden bullet, *ſ*
The ſpool to wind the thread upon, *t*
The pins ſtuck croſs each other to make a hole for the line to paſs through, *u*

The

The slender silk-line, *lu*

The rest for the observer to lean upon, *x*

The Lantern, *y*

The triangular steps for climbing up the mast *a*, that would have embarrassed the figure, are omitted.

899. I now proceed to remove some objections that may arise in peoples minds, who have not tryed this mechanism. And first they may fear that the small deflection of the line, especially when so long as 100 or 200 feet, may possibly destroy the parallelism of the glasses to which it is connected: and indeed there would be good reason for this apprehension, if it were necessary to use a strong heavy line; for then it would require a very great force to reduce it to a tolerable degree of straightness. But the object-glass being so nicely poised in the manner described, is capable of being directed by the least touch of the lightest thread. Fifty foot of the silk-line we make use of, weighs but half a dram, and bears a tension by seven pound weight before it breaks. Therefore the small deflection of a line of this length or even much longer can do no harm, though but moderately stretched by a force equivalent to 2 or 3 pound weight. Especially as it is well known that a geometrical parallelism of the glasses is by no means necessary.

900. For it is certain that the forces requisite to stretch two lines of equal lengths to equal curvities, are as the weights of the lines. For example 50 foot of a line weighing one ounce, would require a force equivalent to 48 pounds, to reduce it to the same curve as 50 foot of our line is reduced to by 3 pounds weight. For it is the same thing whether 16 separate lines, each weighing half a dram, be stretched severally by 3 pounds apiece, or whether they compose a cord of an ounce weight, to be stretched by 16 times 3 pounds or 48 pounds.

901. But this flexure of the line may be farther examined by the joint assistance of geometry and experiment. For the curvity of the line, when so very small, may be considered as a portion of a parabola without sensible error; and when 150 feet of this line was stretched horizontally by a weight of 2 pounds and a half, we found by experiment that the depth of the lowest point of it, below the level of the extreams, was about a quarter of a foot. Let *abc* represent this parabolick thread, *db* its depth below the level *adc*. Let the lines *ae*, *cf* be tangents to the parabola, meeting the lines *ce*, *af* drawn parallel to *db*. Now by looking from the point *a* along the direction of the thread to the point *e*, the line of vision fell upon *e* about a foot below *c*. Whence it follows that *db* was a quarter of a foot. But *ce* and *af* are equal. Therefore the thread *cba* directs the axis of the object-glass placed at *c*, not to the point *a* but along the tangent *cf*; so that the eye at *a* is a foot too high; which does no harm at the distance of 150 feet. For the angle of deflection *cae* or *acf* is but two fifths of

Fig. 614.

of a degree, which though it may safely be neglected, yet I shall shew how it may be corrected once for all. If we take the distance *gb* double of *ac*, or 300 feet, so that *gabcb* may represent this bended line, the depth *kb* will be quadruple of *db*; but the angle of deflection only double of the former that is $\frac{4}{5}$ of a degree; as is easily understood by drawing the tangent *gl* meeting the perpendicular *bl*. For *bl* is quadruple of *kb* or *ce*; but *gb* was double of *ac*; therefore the angle of deflection *bgl* may be reckoned double of the former angle *cae*.

902. Now though this error of 48 minutes might be neglected without inconvenience, yet to take away all scruples I will shew how this deflection of the object-glass and all others may easily be corrected once for all. The object-glass being ballanced as before and drawn down to the level of the eye, stretch out the silk-line with one hand and apply it to your eye; and holding the lantern by the side of it with your other hand, while the line runs between your fingers in receding from the object-glass, take notice whether a double image of the candle appears in the middle of the lens; and if this shall happen when the whole line is run out (its length being equal to the focal distance of the lens) it is a certain sign that the lens has a true situation. But if only one of the images of the candle appears, the lens is ill situated, and worse if neither of them. By observing on which side of the string the reflected light is cast, the wire at the end of the stick, annext to the object-glass, must be bent a little towards the same side: and then the observer must examine the reflected light again, till he finds both the images of the candle to coincide at the end of the extended string; which must be drawn by a gentle force equivalent to two or three pounds weight; to which he must endeavour to accustom his hand. The position of the lens being thus adjusted will serve for any elevation of it. Too nice a judge may object indeed that when the line is obliquely situated to the horizon, as in making observations, the flexure of it will be less than when it was horizontal; but the difference will be exceeding small in a string so very light; and as we observed before, an exact parallelism of the glasses is not at all necessary.

903. If it be objected that the wind will occasion a great disturbance, by driving and curling the string, especially when so very long as we have mentioned; it must be remembered that very long tubes are much more exposed to the like commotions; insomuch that it is frequently impossible to make any observations while a moderate wind is stirring. But at these times it is well known that no observations can possibly be made for another reason; which is, that the transparency of the air, though it looks serene, is almost always vitiated by winds, and rendered unfit to observe in. Which happens also sometimes in a very calm and serene atmosphere, when the stars twinkle very much, by the interposition of some humid vapours; which occasion the edges of the moon and planets

to tremble and fluctuate in the telescope, and utterly destroy the distinctness of their appearance. Insomuch that the goodness of the glasses might often be suspected, if they had not been approved in a more favourable atmosphere. The same moisture does also frequently adhere to the surface of the object-glass, and makes objects appear dull. But this inconvenience may be prevented by warming the glass at a fire.

904. If the lantern does not project a sufficient light to a great distance, it may be improved both by using a larger wick to the candle, and also by a broader lens and of a less convexity in proportion as the object-glass is remoter.

905. There are several ways of preparing a pole of a proper height. A pole that is too weak may be strengthened by another of half the height, placed by the side of it and braced to it by transverse boards; or still much firmer by using two short poles placed at 2 or 3 feet distance from each other and from the long one in the form of a triangle and braced all together. By this means it is easy to elevate the object-glass to an hundred feet; and still much higher, either by making a stronger preparation of poles and boards at the ground, or by placing the pole upon the top of a turret or the corner of a high building, or even upon the middle of the building; where a person may stand, whose business it is to elevate or depress the object-glass according to direction from below.

906. Because unexperienced persons cannot easily find out and follow an object, with this sort of telescope, nor be shewn it by another without some method of fixing the eye-glass; we will now describe how this may be done, by a small machine placed upon a two-legged rest.
Fig. 615. Part of the top of this rest is represented at *aa*, and a variable rhombus made of brass plates at *bb*; two of whose sides are produced through their intersection *f* till they equal the sides of the rhombus. The length of each side is $5\frac{1}{2}$ inches and the breadth a little above an inch, and the thickness one tenth. This rhombus is fixt at *gg* to the side of the upper beam of the rest by an iron skrew-pin, passing through one angle of it, and through a thin circular springing plate, beaten concave a little; by which the motion of the rhombus about the pin may be equable and pleasant with a proper degree of stiffness. From the opposite and superior joint of the rhombus, a small axis or pillar projects about half an inch; and from the end of it there hangs a moveable plate 4 inches long and half an inch broad; which lyes out of sight, being covered by a wooden handle *d* of the same length and riveted to it. Into a dove-tailed channel made lengthways in the foreside of the wood, is inserted another plate *e*, which upon a very small axis supports the stick and tube that carries the eye-glass. And the whole is ballanced or counterpoised upon the axis *f* by proper weights *hh* fixt to the ends of the two sides of the rhombus that were produced.

907. Things being thus ordered to whatever place the obſerver ſhall move the eye-glaſs by the handle *d*, while it always hangs downwards, there it will remain at reſt; and thus may another experienced perſon ſucceed the obſerver to view any object already found for him. For the reſt being placed a little inclining, the tenſion of the line that connects the glaſſes will keep it from falling, though it has but two feet; and the gravity of the reſt thus leaning towards the ſpectator, will keep the ſtring to its due tenſion; ſo that a more convenient contrivance cannot be wiſhed for. The height of our reſt is 4 feet 9 inches, and its weight is 2 pounds and $\frac{3}{4}$ths. The weight of the eye-glaſs, tube and ſtick annexed is $\frac{1}{2}$ a pound. That of the rhombus and its counterpoiſe $2\frac{1}{4}$. Which I mention for the uſe of ſuch perſons as may be deſirous of having a like apparatus to ours, which by experience we have found moſt convenient.

908. To perfect this method of obſerving I will mention one contrivance more, which in ſome kinds of obſervations is very uſeful. In ſearching very diligently for thoſe ſatellits of ſaturn which *Caſſini* diſcovered, I perceived a great difficulty in finding them out, unleſs the night was very dark; and ſoon underſtood that a faintiſh light in the air was the cauſe of it; not that which comes through the object-glaſs, but what paſſes by, round about the ſides of it. To intercept this troubleſome light I put up the paper-umbrella round about the object-glaſs, which I uſed for the moon; and upon farther conſideration I added another expedient of greater efficacy; and this in effect was to contract the pupil of the eye, (which in the dark is very much dilated,) by looking through a very ſmall hole made in a thin plate held cloſe to it. Through this I could preſently perceive three ſatellits of ſaturn very plainly, but when I removed the plate I could only perceive the middlemoſt of them, which I formerly diſcovered. But becauſe the object is not ſo eaſily found when the pupil is thus contracted as when it is wider, therefore I fixt that little perforated plate to a ſmall joint at *k* ſhaped like a greek Λ, which turned upon a pin in the bottom of the tube that carries the eye-glaſs; in which bottom there is a larger hole to look through for finding the object, before the little plate with the ſmaller hole is ſlipped over it.

909. It may poſſibly be thought that the object ſhould appear darker through the ſmaller hole than through the wider. But this is certain, if the diameter of the ſmaller hole, be to the diameter of the aperture of the object-glaſs, as the focal diſtance of the eye-glaſs, is to that of the object-glaſs, that every thing will appear juſt as bright through the teleſcope as if that hole was removed. Neverthelefs it is better to double this breadth of the hole or rather more, to facilitate the finding of a ſtar and to keep it longer in view. The breadth of this hole applyed to our 34 foot teleſcope, is about the ſixteenth part of an inch, and its diſtance from the eye-glaſs is exactly $2\frac{1}{2}$ inches, which is alſo its preciſe focal diſtance.

ſtance. And this place of the hole muſt be accurately adjuſted; becauſe if it be ſituated any where elſe, you cannot ſee ſo much at one view through the teleſcope as uſual. And the place of it is eaſily adjuſted by bending the joint it is fixt to, either to you or from you; having placed it at firſt about half an inch from the end of the tube.

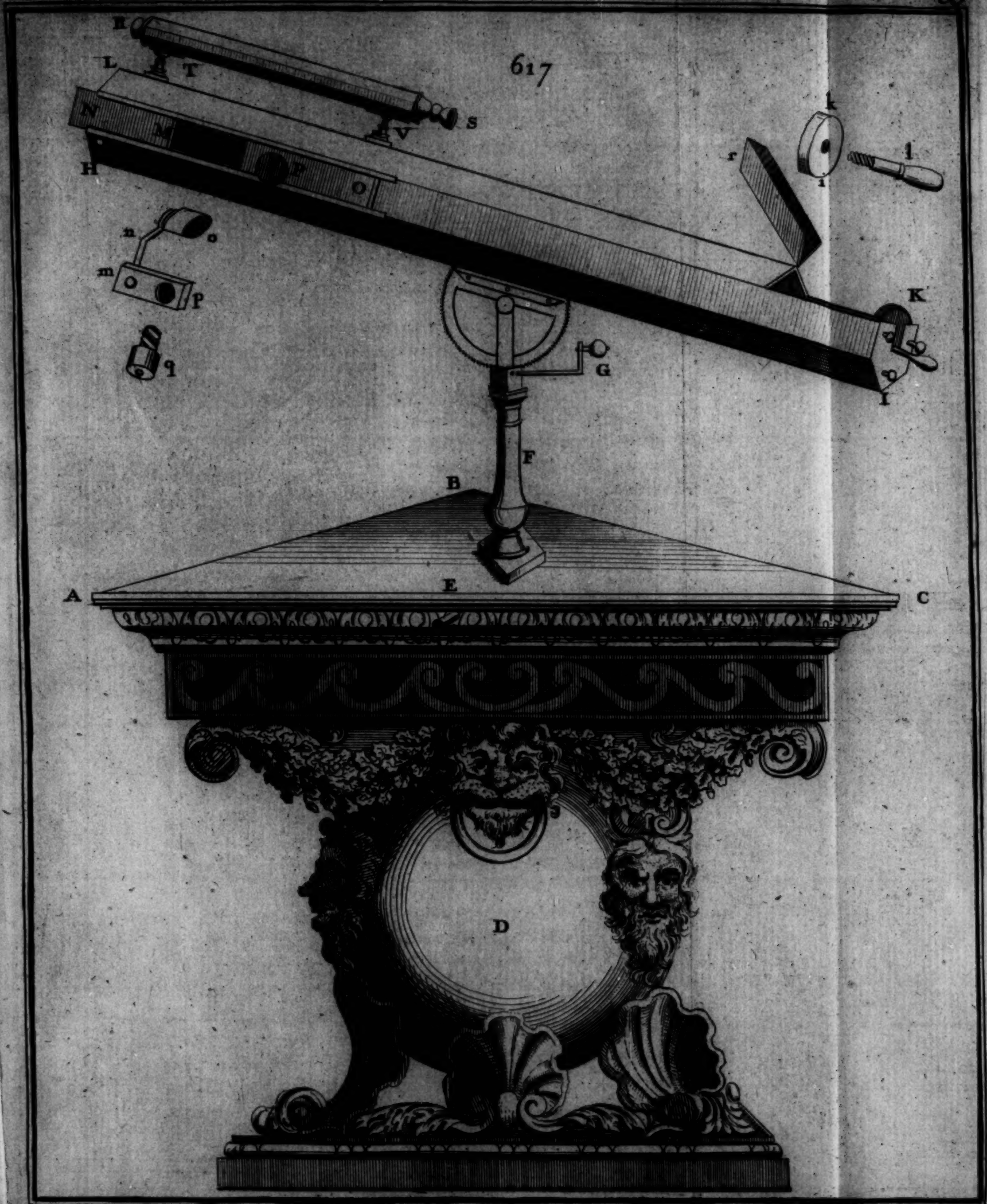
910. The diameter of the paper-umbrella ſhould be about the 45th part of the length of the teleſcope. And becauſe the interpoſition of it creates ſome difficulty in finding an object; to obviate this inconvenience, I erect a perpendicular ſight *m* upon the ſtick annexed to the eye-glaſs; the top of it being elevated as high above the axis of the teleſcope as the upper margin of the umbrella. Therefore by raiſing the eye till we can juſt ſee the ſtar over the upper edge of the umbrella, and then bringing the ſight or top of the ſtyle into the ſame line, when the eye is removed you will alſo find the ſtar in the teleſcope or very near it. By a little uſe and practice this method of obſerving becomes very eaſy. So far Mr. *Huygens*.

911. Signor *Blanchini* in his book upon the Rotation of Venus, pag. 59. refers to ſome farther improvements of this method by *Huygens* himſelf; which he ſays were printed in the Memoirs of the Royal Academy of Sciences at *Paris* about the year 1712, but I could not find them.

[a] Mem. de l' Acad. 1715. Fig. 616.

912. Mr. *de la Hire*'s little machine for managing the object-glaſs, is this [a], "I take, ſays he, an oblong piece of wood *EF* of a convenient ſhape and magnitude, and to the ends of the upper ſide of it I fix two cylindrical ſtaffs *GH*, *IK* to ſerve as an axis. Then I bore a hole through the middle of the piece *EF* at right angles to that axis: through this hole there paſſes another wooden axis *SLM*, two wooden nuts *L*, *M* being ſkrewed over each end of it, to keep it from dropping through the hole. To its upper end *S* there is fixt an upright ſquare board *AC* with a circular hole in it, to receive the object-glaſs. To the bottom corners of this board are fixt two equal wooden rulers, which meet at *N* and form an equicrural triangle, whoſe plane is perpendicular to the board *ABCD*. Under the corner *N* is ſkrewed in a wooden pin, to which the long ſtring for moving the object-glaſs is tied, as much below the line *CD* as the axis *GK*; which lyes upon two tenter-hooks *O*, *P*. To keep off the dew from the object-glaſs he preſcribes to include it in a paſteboard tube, made of ſpongy paper to ſuck up the humidity of the air. And to find an object more readily he preſcribes a broad annulus of white paſteboard to be put over the tube that carries the eye-glaſs, upon which the picture of the object being painted, an aſſiſtant that ſees it may direct the tube of the eye-glaſs into the place of it: or that the obſerver himſelf may ſee it, he preſcribes to receive it upon a ring of tranſparent oiled paper, paſted upon a circular frame inſtead of the paſteboard.

CHAP-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CHAPTER XI.

Sir Isaac Newton's *reflecting telescope made and described by the Honourable* Samuel Molyneux *Esquire, and presented by him to his Majesty* John V. *King of* Portugal: *with other kinds of mechanism for this and for Mr.* Gregory's *reflecting telescope.*

913. A*BC* represents a triangular board or table supported by the globe *D* and by the annexed carvings and masks; this serves for the basis or pedestal of this instrument. Upon occasion this board may be taken off by unskrewing three iron skrews, the heads of which lye near the three volutes at the three corners. At *E* is represented a small key or handle, which turns some wheel-work concealed under the board of the table, and which serve to give an horizontal circular motion to the pillar *F*, placed in the middle thereof, and to the superincumbent tube *HIKL*. If this should ever be out of order, by taking off the upper board as aforesaid it may be rectified. At *G* is represented another handle which gives the tube its perpendicular motion; so that while the observer sits with his right side applyed to the side of the table *AC* at the end *C*; by turning the two handles *E* and *G* he can give the tube any required elevation or azimuth, and thereby follow the motion of the heavenly bodies very commodiously. The pedestal and movements of the telescope. Fig. 617.

914. The reflecting telescope it self consists of two metalline specula and an eye-glass, which are to be duly placed in the tube *HIKL* left open at the end *HL*. The large concave spherical speculum *ik* is to be placed within the tube at *IK*, in which are fixt 3 stops or bits of wood, against which the polished surface of the speculum being applyed, the axis of reflection will fall exactly in the axis of the tube. In the brass-plate which closes this end of the tube, there are three skrews intended for holding the metal in this situation. As to this metal and the placing of it in the tube, many cautions are necessary to be observed. In the first place it is never to be touched but by skrewing into its backside a handle *l* which fits the hole therein. In the next place great care must be taken never to breath in the least upon it, and to expose it as little as possible to any damp air. If any thing of that kind happen, it should be wiped with a thorough dry linnen cloth before a fire; and it may be sometimes in like manner cleaned with a clean rag wetted in spirit of wine; provided the spirit be not left to evaporate; for that would leave an humid sediment thereupon, sufficient to hurt the fineness of its polish. Thirdly unless when in use it should be constantly kept with the face applyed downwards The reflecting metals.

wards upon the piece of plain glaſs, fixt for that purpoſe in the round cell in the red box.

Cautions in preſerving and placing them.

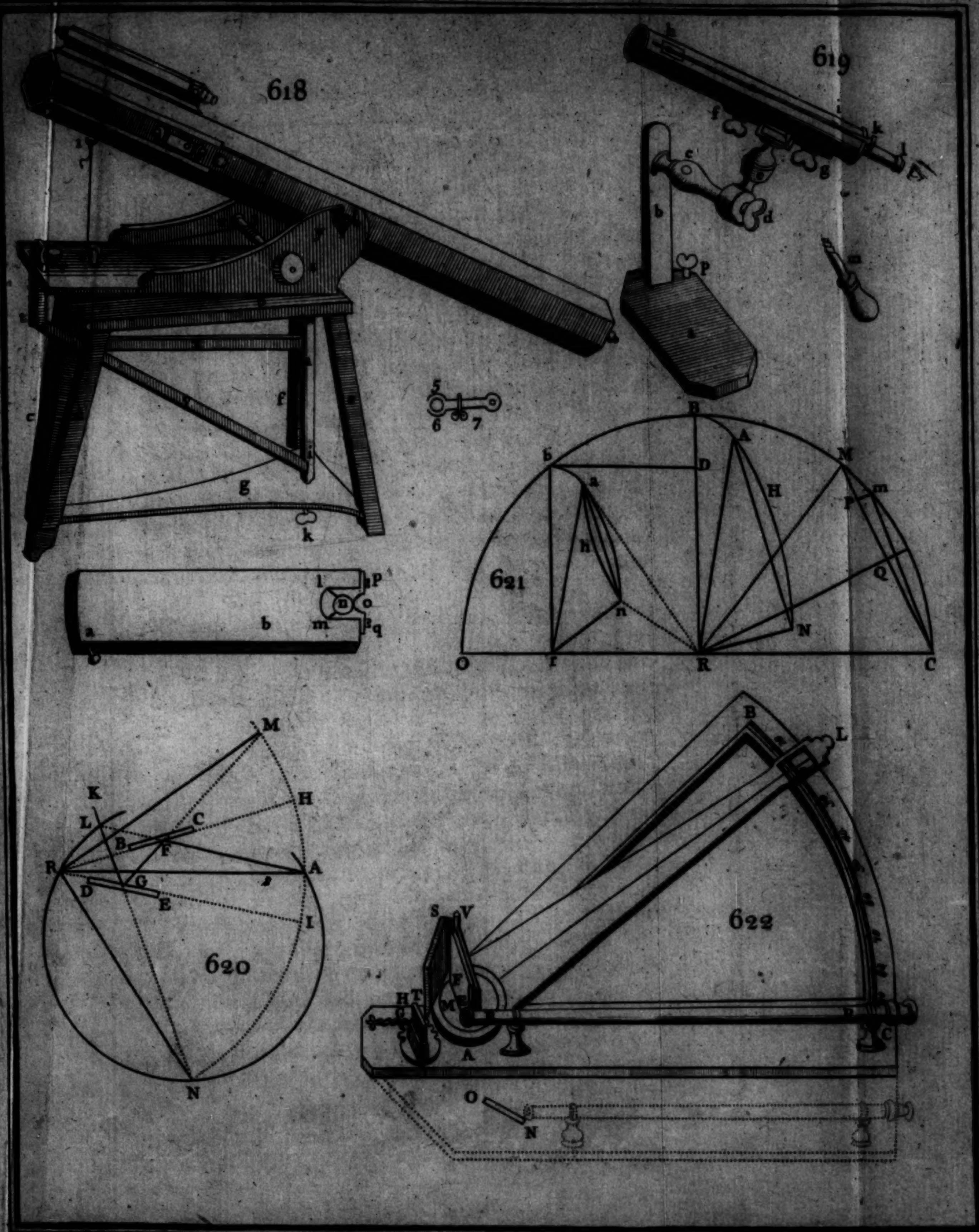
915. The ſpeculum is formed as a portion of a concave ſphere whoſe ſemiradius is about two foot 2 inches. The laws of reflection are ſuch that any error in its figure will produce about ſix times as great an irregularity in the picture formed in its focus, as the like irregularity would cauſe in a common refracting teleſcope. We have found by experience that an error of leſs than $\frac{1}{1000}$ inch is capable of vitiating its figure; ſo that great care muſt be taken in placing the metal in the tube for uſe, againſt the three ſtops aforeſaid; that the three ſkrews at *IK* be gently and tenderly ſkrewed, only juſt ſufficient to bear the metal truly againſt the ſtops; for the ſmalleſt exceſs of ſtreſs in the ſkrews againſt the back of the metal, may diſtort and very much damage its figure. There is alſo a piece of wood *m* having a round hole *p* in it, and carrying a ſmall braſs arm *n* which holds the other ſmaller ſpeculum *o*, which is a plane. This ſpeculum muſt be always preſerv'd from the air, when out of uſe. When the teleſcope is to be uſed the cover of the ſmall ſpeculum muſt firſt be taken off; then place it in the tube at the hole *M* which exactly fits the aboveſaid ſquare piece. Preſs it in pretty tight and true, which if duly performed, the center of this ſmall ſpeculum will be placed in the axis of the large concave one, and will reflect all the parallel rays (which enter at the open end *HL*) to the round hole *p* in the ſaid ſquare piece, in which hole one of the two eye-glaſſes in its cell *q* is to be placed; and then the inſtrument is prepared for uſe.

The poſture of the obſerver.

916. The obſerver is therefore to place himſelf at the ſide of the tube and to look in at *M* and he will then ſee the objects that lye at his left hand. This ſort of teleſcope in reſpect of a common glaſs one being to be uſed in the ſame manner as a German Flute is in reſpect of a common one. Some cautions are here to be given in reſpect of the little ſpeculum at *o*. In taking it out and putting it in again to its place in the box where it is laid by, when out of uſe, care muſt be taken not to ſhake or bend the arm; for the ſmalleſt accident of that kind will ſenſibly diſorder its ſituation. Three ſkrews may be ſeen at the back thereof, the middle one of theſe fixes it to the arm *mn*, the other two only preſs upon the back and ſerve only to adjuſt its ſituation to an exact angle of 45° with the axis of the great ſpeculum. This has been already done, but if any ſhock or other accident ſhould put it out of its place, it may be reſtored by theſe ſkrews. For whenever this inſtrument is uſed, the due performance of it abſolutely requires a true ſituation and direction in this ſpeculum.

The eye-glaſſes and magnifying power.

917. There are two eye-glaſſes whereof one that hath the largeſt aperture being made uſe of, the inſtrument will magnify as much as a common refracting teleſcope of about 20 or 22 feet long; and with the eye-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eye-glaſs that hath the ſmalleſt apertures it will magnify as much and as diſtinctly as one of 35 or 40 feet.

918. In comparing the effects of this inſtrument with thoſe of a teleſcope of that length, one very odd phenomenon is here to be mentioned; and that is, that ones imagination is always miſled in comparing their effects. For although by the minuteneſs of the parts viſible in the object obſerved, as alſo by the proportion between the focal diſtance of the eye-glaſs made uſe of and the focal diſtance of the great ſpeculum, it be demonſtrable that the diſtinctneſs and magnifying power thereof exceeds that of a refracting teleſcope of 35 feet; yet the ſpectator will always fancy that the refracter hath the advantage of this inſtrument. This may perhaps be attributed to the eye looking through ſo ſmall a hole as is neceſſary in this inſtrument; ſome other accounts may perhaps be given of this odd deception, but we need ſay no more here but that it will be certainly found upon a ſtrict examination to be but a deception, and probably an univerſal one to all perſons.

A deception in the magnifying power.

919. At *P* ſtands a round button of ivory, and at *Q* is repreſented a ſmall pin of ivory, which may be ſeen with a ſmall white thread fixt to it, at the end of the tube *H*. This thread at the other end is fixt in the inſide of the tube; and towards the middle of it, it is wound once round the inward end of the ivory button *P*; from this diſpoſition by turning round the ivory button *P*, the whole ſlider of black ebony wood *NO*, with the ſmall ſpeculum and the eye-glaſs applyed at *M*, may be made to approach to or recede from the large ſpeculum at the other end *IK*; and by this means its true diſtance and the diſtinct appearance of the object muſt be found; for various diſtances of the ſame object and for the various eyes of different obſervers; which variety in different perſons, from the great magnifying power of the eye-glaſs in this inſtrument, will be conſiderably more ſenſible than in a refracting teleſcope. But the true diſtance of the ſpecula will immediately be found in all caſes by turning this ivory pin *P* backwards and forwards very ſlowly and gently; and for celeſtial objects the true diſtance being once found for the obſerver's eye, a ſmall mark may be made acroſs the ſlider, and upon the edge of the tube to bring it ſpeedily and without any difficulty to its proper place at another time. By the little ivory pin at *Q* the ſtring may be tightened or relaxed to make the ſlider *NO* move moſt eaſily as occaſion ſhall require. Either of the eye-glaſſes, being applyed in the cylindrical hole *p* in the ſquare piece *mp*, may alſo be made to approach or recede from the focus, by turning round the ſmall tube *q* in which they are inſerted, the outſide whereof is wrought into a fine ſkrew for that purpoſe. And diſtinct viſion may alſo by that means be obtained for different eyes without moving the whole ſlider *NO*.

Diſtinct viſion how procured.

920.

The finder described.

920. *RS* represents a small refracting telescope whose axis is parallel to the axis of the reflecter. In its focus there are placed two cross hairs, and its only use is the more readily to find any object by the reflecter. The eye being applyed at *S*, turn the two handles at *E* and *G* till the point of the object to be viewed in the reflecter, falls exactly on the cross hairs; then the eye applyed at *M* to the reflecter will see the same object: with this caution that as the whole instrument with the whole basis can easily be moved, the most convenient situation for the observer will be to keep the tube *HI* nearly at right angles to the side *AC*, and to sit with the side *AC* flat against his right side near *C*, as hath been already mentioned. And in finding the object at first with the small refracting telescope, it is most convenient to stand at the corner of the table *C*. The handle *G* may be inserted at either side of the pillar *F* as convenience shall require. At the small pillars *TV* which support and hold the small telescope *RS*, there are some small skrews near *T*, which being relaxed the direction of the tube *RS* may be altered horizontally by pushing the tube with the hand sideways either way as occasion requires and then tightening the skrews again. And at *V* there are skrews and a springing piece of brass, which being relaxed or tightened will in like manner alter its elevation, so as to restore the parallelism of the tubes in case of any accident that may have disturbed them. In observing, it is convenient not to touch the table in any part, and only to move the handles as the motions of the star observed requires; for in an instrument that magnifies so much, the smallest motion or trembling is also magnified proportionably, and will be found inconvenient.

A pure air necessary for observations.

921. In considering the effects of this reflecting telescope, it must farther be noted, that for seeing clearly and well with it there is required a very clear equable and undisturbed air. For if there be any vapours moving and undulating in the atmosphere, which often happens though the night appears clear to the naked eye, these will entirely destroy the distinctness of the appearance: and it often happens that the air in this respect, at least here with us at *Kew*, will so suddenly and so totally alter, that the object will appear very distinct and very confused afterwards in 3 or 4 seconds of time; and the air is sometimes so very variable that objects will appear instantaneously to change, from being very clear to be confused, and then to be clear again. It will therefore be proper to accustome ones self to the fluctuating appearances of some land-objects, seen in the day time through the reflecter; lest the undulating appearances of the planets in the night may deceive one, and incline one to think this instrument doth not succeed so well as it is certain it will in a pure undisturbed air.

Mr. *Hadley's* machine for moving this telescope.

922. This other machine, for giving the horizontal and vertical motions to the telescope, is drawn according to Mr. *Hadley's* design a little altered

altered by experience [a]. Here *ab* is an oblong board firmly ſupported by four legs *c*, *d*, *e*, *f*; but the two foremoſt *c*, *d*, being inclined inwards and united in one foot, the machine reſts upon 3 feet connected by a ſtrong triangular board *g* lying parallel to the board *ab*. Between the hinder ends of theſe boards there paſſes a ſtrong wooden axis *hi*, having a ſteel point fixt in the lower end of it, which reſts in a hollow conical point, punched in the end of a thick braſs pin *k*, ſkrewed through a nut in the lower board *g*. The upper end of the ſaid axis *hi*, paſſes through a large notch *plmq* cut in the end of the upper board *ab*, repreſented ſeparately below the figure: and the ſaid axis is here ſurrounded by a braſs hoop *n* well poliſhed and firmly fixt to it; and this hoop is touched in two places by two pieces of poliſhed ſteel, drove into the ſides of the notch *lm* and in a third place *o*, by the back of a ſpringing arch *poq*, preſſed againſt it by the heads of two ſkrews that go into the end of the board. To the top of this axis *hi* another board *rs*, lying parallel to *ab*, is firmly fixt; and under its fore-end *r*, which projects a little beyond the under board *ab*, a ſhort piece *t* is fixt, and is joined to the axis *hi* by the braces *v*, *x*, ſo as to form a ſort of a crane; which by turning about the axis *hi*, gives an horizontal motion to the tube of the teleſcope, while it reſts upon two oppoſite pins fixt in the ſides of it and lodged in two braſs notches, ſkrewed to the tops of the parallel wooden cheeks *y*, *z*, that are fixt upright upon the oppoſite ſides of the board *rs*. And this motion is gradually communicated by turning the head of a pin *r*, which goes down through a hole in the board *rs*, juſt clear of the end of the board *ab*. For this end being formed into an arch concentrick to the axis *hi*, there is a ſtring fixt to one end of the arch, which being applyed againſt the convexity of the arch and lapped once round the pin *r*, is alſo fixt to a pin at *a* at the other end of it, and being lapped about the pin, is tightened thereby.

a Phil. Tranſ. N°. 376. Fig. 618.

923. The hinder end of the tube of the teleſcope is juſt made to preponderate by the weight of the metal lodged in it. And then by means of a ſilk line 1, 2, 3 hung by a loop upon a hook 1, fixt under the fore-end of the tube, and thence paſſing under a pully 2 fixt to the moveable board *rs*, and from thence winding round a cylinder 3, 4, that turns round in two holes in the cheeks *y*, *z*, the tube is gradually elevated or depreſſed, by turning the knob 4 at the end of the cylinder; whoſe motion is adjuſted to a proper degree of ſtiffneſs by compreſſing two braſs arches 5, 6, ſhaped like a pair of tongs, againſt the far end of the cylinder, by a ſkrew, paſſing through their legs, as repreſented at the ſide of the figure. The tube of the teleſcope ſhould have been drawn octangular; and under the board *ab* a drawer is omitted, where the metals and eye-glaſſes are laid by when out of uſe.

924. A little machine for ſupporting and managing Mr. *James Gregory*'s reflecting teleſcope, brought into uſe by Mr. *Hadley*, and generally made

Mr. *James Gregory*'s reflecting teleſcope.

Fig. 619. made about 16 inches long, (being then equivalent to a common dioptrick telescope of 15 or 16 feet,) is contrived in this manner. The base of the pedestal *ab* is a thick board *a*, resting upon 4 brass feet underneath it; one of which being a pin, *p*, that screws through the board, will make it steady upon any uneven plane: *b* is a small upright pillar about a foot long, fixt in the board *a*; and *cd* is a brass arm that screws into it; *de* is a short brass piece that turns round upon the end of the arm *cd* and is tightened and stayed by the screw *d*; *e* is a hollow socket having a round brass ball in it, moveable any way, and tightened and stayed by a screw or two not here expressed. The neck of this ball is fixt to the middle of a long brass piece *fg*, which is fixt along the side of the tube *hi*, by the screws *f*, *g*; and may be taken off at pleasure. Thus the tube is supported and made capable of being gradually moved and stayed in any position. The larger concave metal, having a hole in the middle of it, is lodged at the bottom of the thicker tube *hik*; and the smaller concave metal is held in the axis of the tube, near the mouth of it, by a small brass arm coming through a slit in the tube at *h*. The long iron wire *hik*, on the outside of the tube, screws through a hole in the arm at *h*; and is confined from moving lengthways by two shoulders on each side of a small hole in a fixt plate at *i*; and being turned round it self by the knob at *k*, it draws the arm and little concave backward or forward, in order to procure distinct vision of objects at various distances, and for eyes of different sorts; while the observer is looking in at the end *l*, of a short slender tube that is screwed into the end of the larger, and carries the eye-glasses. When this telescope is used at home, the pedestal *ab* may be placed upon a table near a window or upon the window-board; but when it is used abroad the pedestal may be left at home. For having tapped a hole in the side of a tree or any piece of wood with the hand-auger *m*, the wood-screw at the end *c* of the arm *cd* may be presently screwed into it; the auger *m* being put through the hole *c* to give power to the hand in turning the screw. The theory of this telescope has been shewn by the by in art. 302, and shall be farther explained in the Remarks upon art. 125.

CHAPTER XII.

The description of a new reflecting instrument for taking angles at Sea. By John Hadley *Esquire, Vice-President of the* Royal Society.

Phil. Transf. N°. 420.

925. THE instrument is designed to be of use, where the motion of the objects, or any circumstance occasioning an unsteadiness in the common instruments, renders the observations difficult or uncertain.

926.

926. The contrivance of it is founded on this obvious principle in catoptricks: that if the rays of light diverging from, or converging to any point, be reflected by a plane polished surface, they will, after the reflection, diverge from, or converge to another point on the opposite side of that surface, at the same distance from it as the first; and that a line perpendicular to the surface passing through one of those points, will pass through both. Hence it follows, that if the rays of light emitted from any point of an object be successively reflected from two such polished surfaces; that then a third plane, perpendicular to them both, passing through the emitting point, will also pass through each of its two successive images made by the reflections; and that these three points will be at equal distances from the common intersection of the three planes: and if two lines be drawn through that common intersection, one from the original point in the object, the other from that image of it which is made by the second reflection; they will comprehend an angle double to that of the inclination of the two polished surfaces.

927. Let *RFH* and *RGI* represent the sections of the plane of the figure by the polished surfaces of the two specula *BC* and *DE*, erected perpendicularly thereon, meeting in *R*; which will be the point where their common section, perpendicular likewise to the same plane, passes it; and *HRI* is the angle of their inclination. Let *AF* be a ray of light from any point of an object *A* falling on the point *F* of the first speculum *BC*, and thence reflected into the line *FG*, and at the point *G*, of the second speculum *DE*, reflected again into the line *GK*; produce *GF* and *KG* backwards to *M* and *N*, the two successive representations of the point *A*; and draw *RA*, *RM*, and *RN*. Fig. 620.

928. Since the point *A* is in the plane of the scheme, the point *M* will be so also by the known laws of catoptricks. The line *FM* is equal to *FA*, and the angle *MFA* double the angle *HFA* or *MFH*; consequently *RM* is equal to *RA*, and the angle *MRA* double the angle *HRA* or *MRH*. In the same manner the point *N* is also in the plane of the scheme, the line *RN* equal to *RM*, and the angle *MRN* double the angle *MRI* or *IRN*: substract the angle *MRA* from the angle *MRN*, and the angle *ARN* remains equal to double the difference of the angles *MRI* and *MRH*, or double the angle *HRI*, by which the surface of the speculum *DE* is reclined from that of *BC*; and the lines *RA*, *RM* and *RN* are equal.

929. *Corol.* 1. The image *N* will continue in the same point; although the two specula be turned together circularly on the axis *R*, so long as the point *A* remains elevated on the surface of *BC*: provided they retain the same inclination.

930. *Corol.* 2. If the eye be placed at *L*, (the point where the line *AF* continued cuts the line *GK*;) the points *A* and *N* will appear to it

at the angular distance ALN, which will be equal to ARN. For the angle ALN is the difference of the angles FGN and GFL; and FGN is double FGI, and GFL double GFR, and consequently their difference double FRG or HRI: Therefore L is in the circumference of a circle passing through A, N, and R.

931. *Corol.* 3. If the distance AR be infinite, those points A and N will appear at the same angular distance, in whatever points of the scheme the eye and specula are placed: provided the inclination of their surfaces remains unaltered, and their common section parallel to it self.

932. *Corol.* 4. All the parts of any objects will appear to an eye viewing them by the two successive reflections, as before described, in the same situation as if they had been turned together circularly round the axis at R, (keeping their respective distances from one another, and from the axis,) with the direction HI, *i. e.* the same way the second speculum DE reclines from the first BC.

933. *Corol.* 5. If the specula be supposed to be at the center of an infinite sphere; objects in the circumference of a great circle, to which their common section is perpendicular, will appear removed by the two reflections, through an arch of that circle, equal to twice the inclination of the specula, as is before said. But objects at a distance from that circle will appear removed through the similar arch of a parallel: therefore the change of their apparent place will be measured by an arch of a great circle, whose chord is to the chord of the arch equal to double the inclination of the specula, as the sines complements of their respective distances from that circle are to the radius: and if those distances are very small, the difference between the apparent translation of any one of these objects, and the translation of those which are in the circumference of the great circle aforesaid, will be to an arch equal to the versed sine of the distance of this object from that circle, nearly as double the sine of the angle of inclination of the specula, is to the sine complement of the same.

Fig. 621. 934. For let OBC in the annexed figure represent an infinite sphere, at whose center R are placed the two specula inclined to one another in any given angle, and let their common section coincide with the diameter ORC. Let BAN be the circumference of a great circle, to the plane of which the common section of the specula ORC is perpendicular, and BR its radius: let ban be the circumference of a circle parallel to BAN, and at the distance from it Bb: draw bD the sine, and br the sine complement of the arch Bb: BD is the versed sine of the same. Let A be a point of an object placed in the circumference of the great circle BAN, and N the point in which its image is formed by the two successive reflections,

flections, as before described; and let a be a point of another object placed any where in the circumference of the parallel ban, and n its image; and let abn be an arch of a great circle passing through the points a and n. The point a is at the same distance from the great circle BAN, as the point b, *i. e.* at the distance Bb. Draw AR, AN, RN, ar, an, rn, aR and nR.

935. By the fourth corollary the figures ARN and arn are similar, and consequently the line AN is to the line an as AR or BR is to ar or br, *i. e.* as the radius is to the sine complement of the distance Bb. But AN is the chord of the arch AHN of the great circle BAN equal to the translation of the point A, or double the inclination of the specula, and an is the chord of the arch abn of a great circle, measuring the angle aRn, by which the point a appears removed by the two reflections, to an eye placed in the center R. Therefore the translation, or apparent change of place of the point a is measured by an arch of a great circle, whose chord is to the chord of the arch AHN (equal to double the inclination of the specula) as the sine complement of its distance from the great circle BAN is to the radius.

936. From any point C of the circumference OBC draw the chords CN and Cm, to the same side of the point C, and equal to the chords AN and an respectively, draw the radius RM, and from R and m draw RQ and mP, both perpendicular to CM, and cutting it in Q and P. RQ is the sine complement, and CM double the sine of half the angle MRC, or ARN, or of the angle of inclination of the specula. The little arch Mm will represent the difference of the apparent translations of the objects in A and a; and if it be very small, may be looked on as a straight line, and the little mixed triangle MmP as a rectilinear one, which will be similar to RMQ, because RM is perpendicular to Mm and RQ to CM, and the angles at Q and P right angles. The line CP may be taken as equal to Cm, and MP as the difference of the lines CM and Cm. Therefore the little arch Mm is to the line MP nearly as RM to RQ: but CM (i. e. AN) was to Cm (i. e. an) as BR to br, and the difference MP of CM and Cm to the difference BD of BR and br, as CM to BR. Therefore Mm, the difference of the apparent translations, is to BD, the versed sine of the distance Bb, or to an arch equal to it, in the compound ratio of RM the radius to RQ the sine complement of the angle of inclination of the specula, and CM double the sine of the same to BR the radius, *i. e.* as CM to RQ.

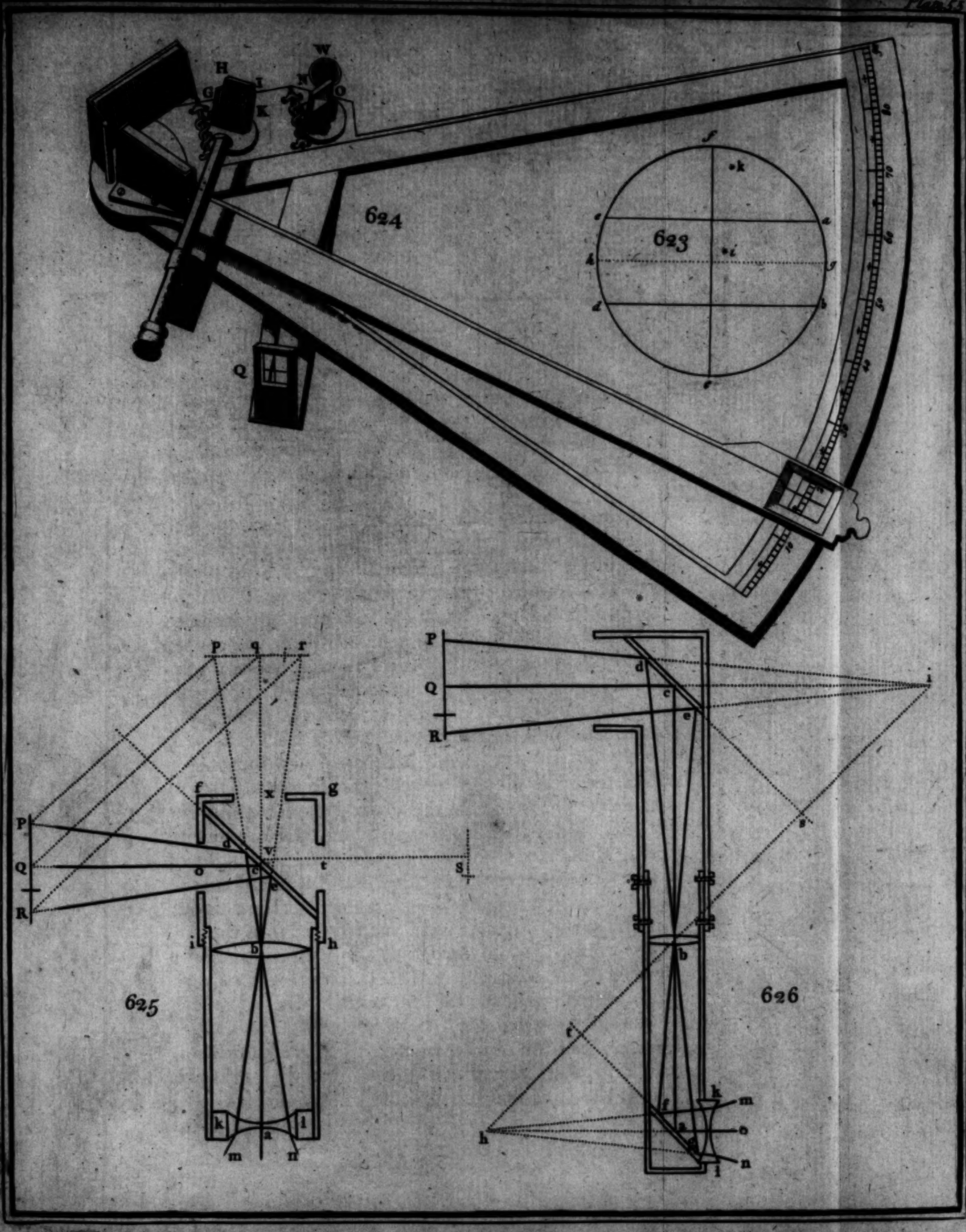
937. The observation may be corrected by one easy operation in trigonometry, as will appear from the first part of this corollary, *viz.* by taking the half of the angle observed, and then finding another angle, whose sine is to the sine of that half, as the sine complement of the distance Bb is to the radius: this angle doubled, will be the true distance

of the objects. But as this operation, though easy, will require the use of figures, I rather chose the method of approximation, because by that the observer, retaining in his memory the proportions of the sines of a few particular arches to the radius, may easily estimate the correction without figures, when the angle is not great, and by a line of artificial numbers and sines, may always determine it with greater exactness than will ever be necessary.

938. When the angle observed is very near 180 degrees, the correction may be omitted; for then it will be easy to keep the plane of the instrument so near that of the before-mentioned great circle, as not to want any, if the situation of that circle be known: if it be not, the observer, when he sees the two objects together, may turn the instrument on the axis of the telescope, till he finds that position of it by which he obtains the least angle; and this (if the specula are set truly perpendicular to the plane of the instrument) will always happen when the objects appear to coincide in the line gh, as expressed in the figure.

Fig. 622. 939. The instrument consists of an octant ABC, having on its limb BC an arch of 45 degrees, divided into 90 parts or half degrees; each of which answers to a whole degree in the observation. It has an index ML moveable round the center, to mark the divisions: and upon this, near the center, is fixed a plane speculum EF perpendicular to the plane of the instrument, and making such an angle with a line drawn along the middle of the index, as will be most convenient for the particular uses the instrument is designed for; (for an instrument made according to fig. 622 the angle LFM may be of about 65 degrees.) $IKGH$ is another smaller plane speculum, fixed on such part of the octant as will likewise be determined by its particular use, and having its surface in such direction, that when the index is brought to mark the beginning of the divisions (*i. e.* 0°) it may be exactly parallel to that of the other; this speculum being turned towards the observer, and the other from him. PR is a telescope fixed on one side of the octant, having its axis parallel to that side, and passing near the middle of one of the edges IK or IH of the speculum $IKGH$; so that half its object-glass may receive the rays reflected from that speculum, and the other half remain clear to receive them from a distant object. The two specula must also be disposed in such manner, that a ray of light coming from a point near the middle of the first speculum, may fall on the middle of the second in an angle of 70 degrees or thereabouts, and be thence reflected into a line parallel to the axis of the telescope, and that a clear passage be left for the rays coming from the object to the speculum EF by the side HG. ST is a dark glass fixed in a frame, which turns on the pin V; by which means it may be placed before the speculum EF, when the light of one of the objects is too strong: of these there may be several.

940.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940. In the diſtinct baſe of the teleſcope, repreſented by the circle *abcdef*, are placed three hairs, two of which, *ac* and *bd*, are at equal diſtances from, and parallel to the line *gh*, which paſſes through the axis, and is parallel to the plane of the octant: the third *fc* is perpendicular to *gh* through the axis. Fig. 623.

941. The inſtrument, as thus deſcribed, will ſerve to take any angle not greater than 90 degrees; but if it be deſigned for angles from 90 to 180 degrees, the poliſhed ſurface of the ſpeculum *EF* fig. 622 muſt be turned towards the obſerver; the ſecond *IKGH* muſt be brought forward to the poſition *NO* ſo as to receive on its middle the rays of light from the middle of the firſt in an angle of about 25 degrees, their ſurfaces being perpendicular to one another when the index is brought to the end of the divided arch next *C*; and this ſecond muſt ſtand five or ſix inches wide of the firſt, that the head of the obſerver may not intercept the rays in their paſſage towards it, when the angle to be obſerved is near 180°. The ſmaller ſpeculum is fixed perpendicularly on a round braſs plate, toothed on the edge; and may be adjuſted by an endleſs ſkrew.

942. In order to make an obſervation, the axis of the teleſcope is to be directed towards one of the objects, the plane of the inſtrument paſſing as near as may be through the other, which muſt lie to that hand of the obſerver, as the particular form of the inſtrument may require; *viz.* the ſame way that the ſpeculum *EF* does from *IKGH*, if it be compoſed according to this figure and deſcription. The general rule is, that when the index is brought to the beginning of the ſcale (*i. e.* to 0° when the inſtrument is deſigned for angles under 90°, or to 90° when it is deſigned for angles from 90° to 180°) if then a line be imagined to be drawn on it parallel to the axis of the teleſcope, or line of direction of the ſight, ſo as to point towards the object ſeen directly; which ever way this line is carried by the motion of the index along the arch from 0° towards 90° in the firſt caſe, or from 90° towards 180° in the ſecond, the ſame way the object ſeen by reflection ought to lie from that which is ſeen directly. The obſerver's eye being applied to the teleſcope, ſo as to keep ſight of the firſt object; the index muſt be moved backward and forward till the ſecond object is likewiſe brought to appear through the teleſcope, about the ſame diſtance from the hair *cf* (fig. 623) as the firſt: if then the objects appear wide of one another, as at *i* and *k*, the inſtrument muſt be turned a little on the axis of the teleſcope, till they come even, or very nearly ſo, and the index muſt be removed till they unite in one, or appear cloſe to one another in a line parallel to *cf*, both of them being kept as near the line *gh* as they can. If the inſtrument be then turned a little on any axis perpendicular to its plane, the two images will move along a line parallel to *gh*, but keep the ſame poſition in reſpect of one another; ſo that

that in whatever part of that line they may be obſerved, the accuracy of the obſervation will be no otherwiſe affected than by the indiſtinctneſs of the objects. If the two objects be not in the plane of the inſtrument, but equally elevated on, or depreſſed below it, they will appear together at a diſtance from the line *gh*, when the index makes an angle ſomething greater than their neareſt diſtance in a great circle: and the error of the obſervation will increaſe nearly in proportion to the ſquare of their diſtance from that line; but may be corrected by help of the fifth corollary. Suppoſe the hairs *ae* and *bd*, each at a diſtance from the line *gh*, equal to $\frac{100}{4146}$ of the focal length of the object-glaſs, ſo as to comprehend between them the image of an object, whoſe breadth to the naked eye is a little more than $2^\circ \frac{1}{4}$; and let the images of the objects appear united at either of thoſe hairs: then as the ſine complement of half the degrees and minutes marked by the index, is to the doubled ſine of the ſame; ſo is one minute to the error which is always to be ſubſtracted from the obſervation. Other hairs may alſo be placed in the area *abcdef*, parallel to *gh*, and at diſtances from it proportional to the ſquare roots of the numbers 1, 2, 3, 4, &c. and then the errors to be ſubſtracted from the ſame obſervation made at each of thoſe hairs reſpectively, will be in proportion to the numbers 1, 2, 3, 4, &c. This correction will always be exact enough if the obſerver takes care (eſpecially when the angle comes near 180°) to keep the plane of the inſtrument from varying too much from the great circle paſſing through the objects. When the angle is very near 180° the correction may be omitted, for then it will be eaſy to keep the plane of the inſtrument ſo near that of the beforementioned great circle, as not to want any, if the ſituation of that circle be known: if it be not, the obſerver when he ſees the two objects together, may turn the inſtrument on the axis of the teleſcope till he finds that poſition of it by which he obtains the leaſt angle; which (if the ſpecula are ſet truly perpendicular to the plane of the inſtrument) will always happen when the objects appear to coincide in the line *gh*, as expreſſed in fig. 623.

943. In regard to the workmanſhip, if an exactneſs be required in the obſervations, the arch ought to be divided with the greateſt care; becauſe all errors committed in the diviſion are doubled by the reflections. The index muſt have a ſteady motion on the center, ſo that the axis of it remain always perpendicular to the plane of the octant; for if that alter, it will be liable to vary the inclination of the ſpeculum it carries to the other: the motion muſt likewiſe be eaſy, leſt the index be ſubject to bend edge-ways: for the ſame reaſon it ſhould be as broad at that end next the center as conveniently can be. The ſpecula ſhould have their ſurfaces of a true flat; becauſe a curvature in either of them, beſide rendering the object indiſtinct, will vary its poſition, when ſeen by reflection from different parts of them: they muſt alſo be of a ſufficient length and breadth

for

for the telescope to take in a convenient angle without losing the use of any part of the aperture of its object-glass, and that in all the different positions of the index. They may be either of metal or glass plates foiled, having their two surfaces as nearly parallel as they can; yet a small deviation may be allowed; provided either their thickest or thinnest edges (and consequently the common section of their surfaces) be parallel to the plane of the octant: for in that case, though there are several representations of the object, they will be always very near one another in a line parallel to cf; and any of them may be used, except when the angle to be observed is very small. The chief inconvenience will be, that a small star will be more difficultly discerned, the light being divided among the several images. The telescope may be contrived to alter its situation, so as to receive the reflected rays on a greater or less part of its object-glass if the objects differ in brightness. The second speculum if it be made of a glass plate, may have a part unfoiled, that when one of them is sufficiently luminous, the less bright may be seen through it by the whole aperture. If the sun be one of the objects, or the moon be compared with a smaller fixt star; their reflected images must be still farther weakened by the interposition of one or more of the dark glasses ST. An exact position of the telescope is not necessary; and the instrument may be used without one, the disposition of the specula with regard to the sector and index, being such as may allow the eye to be brought as near the second speculum as may be, and make the instrument the most commodious for the observer.

944. It will be easy to judge, that scarce any greater degree of steadiness is requisite in the pedestal, or machine which carries this instrument, than what is sufficient for the telescope used with it: for although the vibrating motion of the instrument may occasion the images of the objects also to vibrate cross one another; their apparent relative motion will be very nearly in lines parallel to cf; and it will not be difficult to distinguish whether they coincide in crossing one another, or pass at a distance: and if the objects are near one another, and the telescope magnifies but about four or five times, it may be held in the hand without any standing support. In this manner the altitude of the sun, moon, or some of the brighter stars from the visible horizon may be taken at sea, when it is not too rough.

945. Fig. 624 shews an instrument designed for this purpose; differing from the foregoing description chiefly in the placing the specula and telescope, with regard to the sector and index; in this the line drawn along the middle of the index falls on the fore surface of the larger speculum, in an angle of about 4 or 5 degrees. The line of direction of the sight or the axis of the telescope (if that be used) falls on the surface of the speculum $IKGH$ in an angle of about 70° or 71°. It has also a third speculum NO disposed Fig. 624.

disposed according to the directions when the angle is greater than 90 degrees, whose use is to observe the sun's altitude by means of the opposite part of the horizon. On this the line of direction of the sight falls in an angle of about 32 or 33 degrees. In placing these two smaller specula, it will be farther necessary to take care that the speculum *IKGH* does not stand so as to intercept any of the rays coming from the greater one, fixed on the index, to the third *NO*; nor either of them hinder the index from coming home to the divided arch. *WQ* is a director for the sight; which is necessary when the telescope is not made use of. This consists of a long narrow piece, which slides on another fixed on the back of the octant, and carries at each end a sight erected perpendicularly on it: it may be removed at pleasure, and exchanged for the telescope, which slides on in the same manner, both serving indifferently with either of the two smaller specula. The eye is to be placed close behind the sight at *W*; and the thread stretched across the opening of the other sight at *Q* perpendicular to the instrument, is to assist the observer in holding it in a vertical posture; who is to keep this thread as near as he can parallel to the horizon, and the object near the upright one. How far an instrument of this kind may be of use at sea to take the distance of the moon's limb from the sun or a star, in order to find the ship's longitude, when the theory of that planet is perfected, I leave to trials to determine.

a Phil. Transf. N°. 425.

946. This instrument Mr. *Hadley* tells us[a] was made of wood, and was intended chiefly for taking altitudes of the sun, moon or stars above the visible horizon, either forwards or backwards: and that he has since procured another to be made of brass, by Mr. *Sisson*, for taking the distance of any kind of objects. It is supported by a single stem skrewed to it on the under side; the lower end of which may rest on the ground, to ease any observer of the weight of the instrument. This stem is also made to lengthen or shorten, by which means the instrument is brought to the proper height for any observer's eye, either standing or sitting. Instead of a ball and socket it has two circular arches fixed on its back, by which it is readily set to any position, which the situation of the objects may require.

The result of a trial of the instruments.

947. This most ingenious gentleman has also given us a particular account of a trial of these instruments, made at sea by order of the Right Honourable the Lords Commissioners of the Admiraly, in the presence of many skilful persons. The result of the trial was this, that after all due allowances were made, three observations of the interval between two stars, taken with the brass instrument, differed from Mr. *Flamsteed*'s observations at land, but about a minute at a medium; and that a dozen observations of the sun's altitudes, taken with the wooden instrument while the ship lay at anchor, were so consistent with one another as to differ from the truth but about half a minute at a medium; and lastly that another dozen

taken

taken under ſail while the wind blew hard, differed but a minute from the truth, and at another time they agreed together more exactly. Notwithſtanding ſo cloſe a conſiſtence among theſe obſervations, it might in all probability have been much exacter, had not ſeveral accidents concurred at that time to diſturb it; as that the horizon was not always clear of land, and conſequently not ſo readily diſtinguiſhable; that all the obſervers were perſons quite unaccuſtomed to the motion of a ſhip; which was here very great and quick; this veſſel being very light and ſmall, and conſequently more ſubject to be toſſed up and down by the waves. Now if the difference of the obſervers heights, thus occaſioned, was about 4 or 5 feet, as it was judged to be; it muſt neceſſarily raiſe and ſink the viſible horizon by turns near one minute, which is nearly the whole error in the altitudes abovementioned: by which it appears at laſt that a more accurate and commodious inſtrument for the buſineſs of navigation can hardly be wiſhed for.

CHAPTER XIII.

Of reflecting Perſpectives.

948. THERE is an inſtrument ſold in the ſhops which ſome call an Opera-glaſs, others a diagonal perſpective; it is properly a reflecting perſpective, ſo contrived for viewing a perſon in a publick place that no one can diſtinguiſh who it is you look at, even though they know the deſign of the inſtrument. The 625th figure repreſents a ſection of it made by a plane paſſing through its axis *abc* and cutting the plane of a little ſpeculum perpendicularly in the line *dce*. This reflecting plane is fixt obliquely in a round ſhort tube *fghi*, ſkrewed upon the end of the perſpective's tube *hikl*, ſo that its axis *abc* ſhall make about half a right angle *bcd* with the ſpeculum, whoſe poliſhed ſide faces the object-glaſs *hhi*. Therefore ſuppoſing rays as *abc* to go from the eye at *a*, and after reflection at *c* to paſs through a round hole at *o* in the ſide of the ſhort tube, and to fall upon a diſtant object at *Q*; this object will be ſeen by rays returning back in the ſame lines *Qcba*. The way then to find the object intended to be viewed in the perſpective, is to direct its axis at a right angle to the rays that come from the object. For the angle *acd* being made equal to half a right one, and being equal to *ecQ* *, the angle *Qca* muſt be a right angle to make up two right ones with the reſt. And if the object perceived through the perſpective, be higher or lower than the object ſought for, this will be brought into view by turning the perſpective to and fro about its axis.

The opera-glaſs. Fig. 625.

* Art. 9.

949. If the object be too near you to be ſeen diſtinctly with the perſpective, turn the other end of it towards your eye; and by looking

through the hole *x* you will ſee the object *s* by the rays *stvx* coming through another hole at *t*, oppoſite to the former hole *o*, and reflected at *v* from another plane ſpeculum parallel to the former and facing the eye at *x*. If the ſpectator be ſhort ſighted, a concave glaſs muſt be placed in the hole *x*, otherwiſe a plane one, to make the inſtrument more like a common perſpective. But though it be diſcovered to be a reflecting perſpective by the holes in its ſides; yet theſe holes being oppoſite, make it impoſſible to be known on which hand the object is placed that the ſpectator is viewing.

950. In both caſes the reflecting plane neither magnifies nor diminiſhes the appearance of the object. For in the perſpective, if its axis *ac* be produced till *cq* equals *cQ*, the reflected rays will diverge as from an image at *q* equal to the object at *Q*; and therefore will fall upon the object-glaſs *b* in the ſame manner as if they had come from the object it ſelf removed to *q*; with this difference only that the right ſide of the object will appear on the left hand, and the left ſide on the right. Becauſe the lines *Pp*, *Qq*, *Rr*, that join the correſponding points of the object and its image, are all perpendicular to the reflecting plane *de* produced[a], and conſequently parallel to one another. But the object appears upright, either by the ſingle reflection or through the perſpective provided its eye-glaſs be concave; becauſe higher points of the image *q* anſwer to higher points of the object *Q*, for the reaſon juſt mentioned. The whole inſtrument is generally about 4 inches long, and is very well made by Mr. *Scarlet* near St. *Anne*'s Church, *London*.

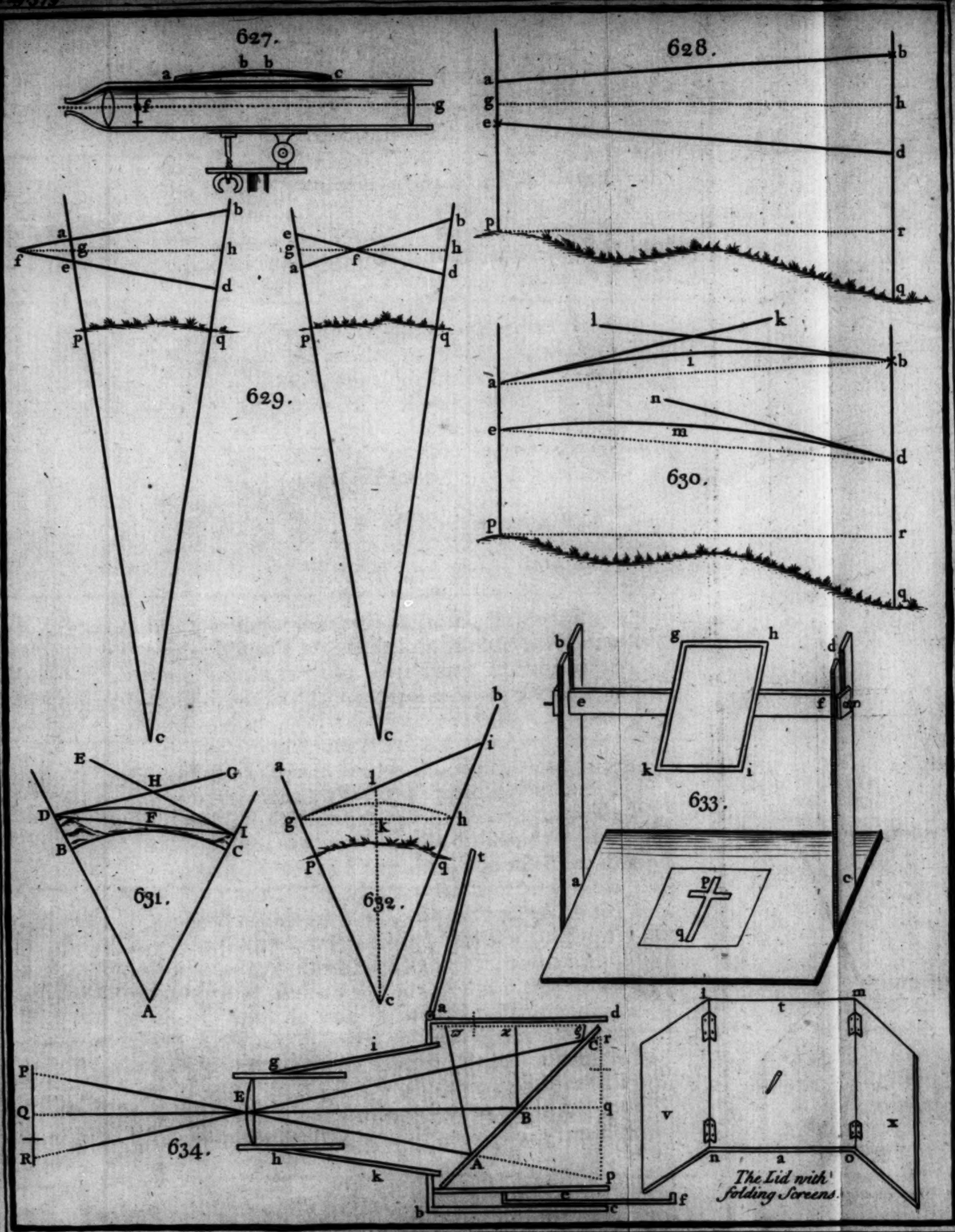
a Art. 23.

The Polimoſcope.

951. *Hevelius* in the preface to his *Selenography* deſcribes an inſtrument of this kind of his own invention, and commends it as uſeful in war eſpecially in ſieges, for diſcovering what the enemy is doing, while the ſpectator lies hid behind any obſtacle; and therefore he calls it a Polimoſcope. For this purpoſe he inlarges the interval *bc* between the object-glaſs and the ſpeculum by a tube of a length ſufficient to project the ſpeculum beyond the obſtacle that covers the ſpectator. And for a farther convenience of looking forward, as it were, he propoſes to place another plane ſpeculum *fg* at the other end of the tube, to reflect the rays through a hole *kl* in its ſide, in a direction *ao* parallel to, and according to the tendency of, the incident rays *Qc*; and to place the concave eye-glaſs in this hole. By this means the object will ſtill appear upright, and magnified juſt as much as if the two ſpeculums were removed, and the ſame eye-glaſs was placed in the axis of the tube. For in the rays *Qc*, *oa* produced through the ſpeculums *de*, *fg*, take *ci* equal to *cb* and *ab* equal to *ab*; and ſuppoſing rays to flow both ways through *b* the center of the object-glaſs, after reflection from the ſpeculums they will diverge from the points *b*, *i**. Let two of them fall upon the object in *P* and *R*, and ſince the angle *PiR* or *die* is equal to *dbe* or *fbg* or *fbg*; if the eye-glaſs was removed the

Fig. 626.

* Art. 23.


627.
628.
629.
630.
631.
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633.
634.
The Lid with folding Screens.

object would appear under the same angle *fhg* or *kbl* as it would appear under to the naked eye placed at *i*; but the reflected rays *fk*, *gl*, after refractions into *km*, *ln* through the eye-glass *kl*, are inclined in the same angle to each other as they would be, if the speculum *fg* being removed they had been refracted through the same eye-glass placed in the axis of the tube, at the same distance from *b* as it is now from *h*. And by tracing an oblique ray *Rebfkm*, it is manifest that the object appears upright; it also appears in the same posture with respect to right and left as to the naked eye.

952. The length of the perspective *ab* must not be very great; for then it will take in so little at one view as to make it difficult to find an object. And this was the Author's reason I suppose for ordering a longer tube to be joined to it. He says the length of his shorter tube was 8 inches and of the longer 22: by the shorter tube I suppose he means the perspective. Those that would be farther informed in the mechanism of the instrument, may consult this Author in his Selenography p. 24.

CHAPTER XIV.

A telescopick Level and its uses.

953. A TELESCOPE is of singular use in the art of levelling; that is in finding out such tracts of ground as have a continued and a sufficient declivity, for conducting water from place to place; either for making rivers navigable, or for draining low grounds, and the like useful purposes. Along the side of a telescope 2 or 3 feet long, having cross hairs in its focus, a tube of a spirit-level is fixt immutably in a position parallel to the telescope; and when the telescope is placed nearly horizontal upon a three legged supporter, there is usually a contrivance by a skrew for altering its elevation very gradually and steadily, in order to bring the extremities of the air bubble to rest exactly at two marks, made upon the glass tube with the point of a diamond. The line *fg* represents the line of sight drawn from *f*, the intersection of the cross hairs, through *g* the center of the object-glass; and the points *b*, *b* are the marks upon the glass tube *abc*. Now while these parts of the instrument are immutably fixt with respect to each other, it is manifest that as often as the air-bubble is exactly reduced to the marks *b*, *b*, the line of sight will always be reduced to the same position with respect to the horizon or to a plumb-line. It is not at all necessary in the business of levelling that the line of sight and plumb-line should be exactly at right angles; but only that the angles they make shall be always the same.

Description of a level.

Fig. 627.

954. The whole art of levelling consists in but little more than a repetition of the following problem: *p* and *q* being given points in two remote places,

The method of levelling. Fig. 628.

ces, it is required to find which is the lower and how much. Let *pa* and *qb* represent two straight staffs or poles fixt upright, by means of a plumb-line. Having placed the telescope by the side of the pole *pa*, and directed the line of sight to the pole *qb*, alter its elevation by the skrew above-mentioned, till the air-bubble rests exactly at the marks upon the tube. Then let an assistant mark the point *b*, which appears to be covered by the cross hairs; and also the point *a* exactly upon a level with the cross hairs; which is easily done by a common square applyed to the side of the pole *pa*. Then remove the telescope to the pole *bq*, and here let the same things be repeated; that is let *d* be the place upon a level with the cross hairs, and *e* the point upon the other pole *pa*, that appears to be covered by them while the air-bubble rests at the same mark as before. Bisect the interval *ae* in *g*, and the interval *bd* in *h*, and the points *g*, *h* will be upon a level: that is, if we suppose *gpqh* to represent a long canal full of stagnating water, the points *g*, *h* will both be in its surface; and consequently taking the lesser depth *pg* from the greater *qh*, their difference *qr* shews how much the point *q* is below the point *p* or *r*.

955. If the places *p*, *q* cannot be seen from each other, or if the difference of their heights be greater than the length of any common poles; then one or more intermediate stations must be chosen; and by repeating the same practice between every two successive stations, we shall find the level of the extreams.

956. When the points *g*, *h* are once found upon two poles not far asunder, it will be convenient by moving the cross hairs, to rectify the line of sight, so as to be nearly coincident with the line *gh* or with a line parallel to it; for then in future levellings, at greater distances, the marks *b*, *e* will be less subject to fall above or below the poles.

The reason of it.

957. The reason of this practice will appear from the following principles. 1st That a plumb-line is perpendicular to the surface of stagnating water directly under it, or to a plane conceived to touch the water at the point where the plumb-line produced would cut it. 2dly That the surface of a canal or lake of stagnating water, extended over the country to be levelled, may be taken for a portion of a spherical surface, notwithstanding that the figure of the earth is not exactly spherical. 3dly That any two points are upon a level which lie in the surface of the lake; and consequently are equidistant from the center of it, or from the concourse of two plumb-lines produced.

Fig. 629.

958. Now let *acb* be the angle made by the plumb-lines *ae*, *bd* produced downwards, till they meet at *c*. At any point *a* of the line *ae* make an angle *cab* of any given magnitude; and at any point *d* of the other line *bc*, make the angle *cde* equal to *cab*; and let *ab* and *de* (produced) meet in *f*; and let the line *fgh*, which bisects the angle *bfd*, cut the plumb-line *ae* in *g*, and *bd* in *h*. I say the points *g*, *h* will be upon a level

level, or that cg and cb will be equal. For the angles cab, cde being made equal, and the angle at c being common to the triangles cab, cde; the remaining angles cba, ced are alſo equal [a]. Hence the exterior angle $cbf = (bbf + bfb = ced + bfb = gef + gfe =$ the exterior angle $agf =) cgb$; and by conſequence the legs cg, cb of the triangle cgb are equal, which was to be proved. a Euc. I. 32.

959. Now in the practice of levelling the intervals ae, bd are very ſmall in compariſon to the diſtance gb, and conſequently the angle afe or bfd is very ſmall; and therefore its legs af, fe and bf, fd are very nearly in a ratio of equality; and conſequently the line gb, which biſects the angle at f, will alſo biſect the intervals ae, bd very nearly; becauſe the ſegment $ga : ge :: af : fe$*, and likewiſe the ſegment $hb : hd :: bf : fd$, that is in a ratio of equality without ſenſible error in the practice of levelling. * Euc. VI. 3.

960. This reciprocal way of levelling ſeems to be the moſt exact of any I have met with, eſpecially if it be performed by two inſtruments made to agree together beforehand; which may be done by placing them together, and by altering the croſs hairs in either of them, till the ſame mark upon a remote object, is covered by both the croſſes, while both the bubbles reſt at their marks upon the tubes. Then may two obſervers find the marks upon the oppoſite poles at the ſame time; and conſequently the refractions of the rays in the air, whatever be their quantities, will be equal as near as poſſible; and then the reſult of the practice will be as accurate as if there had been no refractions at all. For let the curve bia repreſent the courſe of the viſual ray from b to a; and let the lines ak, bl touch it at a and b. Then becauſe the points a, b are very nearly upon a level, the denſity and conſtitution of the air and vapours at the ſame inſtant, will be nearly the ſame in each place; and by conſequence the curve aib and its tangents at a and b, will be equally inclined to the chord ab. For the ſame reaſons the curve emd will be ſimilar and equal to the curve aib, being ſituated ſo very near to it. Therefore the angle edn, under the chord ed and tangent dn, will be equal to the angle abl or bak; and conſequently ſince the angles qdn, pak are made equal in the two obſervations, by taking away the equal angles edn, bak, cauſed by the equal refractions, the remaining angles qde, pab will be equal to each other, as if there were no refractions at all.

The refraction of the rays by the air, conſidered.

Fig. 630.

961. If the reciprocal obſervations be made about the middle of the ſame day when the air is the pureſt, there will ſcarce be any occaſion for two inſtruments; but if they be made near the morning or evening, even on the ſame day, an equality of refractions cannot be depended upon unleſs they are made at the ſame inſtant. The members of the Royal Academy of Sciences at *Paris* tell us, in their account of the Meaſure of the

Earth,

Earth*, they often found, " that an object which at break of the day appeared in the level and sometimes a little above it, did afterwards, when the sun was up, appear below it. And on the contrary, after the setting of the sun, objects far distant appeared to be raised so sensibly, that in less than half an hour their apparent height was augmented more than three minutes. As to the cause of these appearances they add, that the coolness of the night condenses the vapours, which descend to a lower place, leaving the air in the higher stations more pure than in the day time. And on the contrary when the heat of the sun has made a part of the vapours to mount to more elevated stations, there must be less difference of the mediums, and consequently a less refraction. They also teach us the following method whereby the quantity of any particular refraction may be determined by observations.

Fig. 631. 962. " Let *A* be the center of the earth, *BC* its ordinary surface, and *D*, *I* the tops of two mountains. We are to consider that the earth being inveloped with an atmosphere or vaporous air composed of different regions which are gradually more subtil the farther they are removed from the earth; it comes to pass that the visual ray, which comes from a higher place to a lower, as from *D* to *I*, and passes obliquely from a more subtil to a more gross air, is continually bent in its way in proportion as it changes the medium; which gives it the position of a curve line much like that of *DFI*. But the eye placed at *I* receives the curve ray as if it were the tangent *IE*, in which direction it sees the object *D*. For the same reason if we suppose another eye at *D*, it sees the object *I* in the straight line *DG*, tangent to the same bended ray *DFI*. And supposing the two tangents *IE* and *DG*, which are instead of the visual rays, to cut each other in *H*; one may imagine, that there happens the same thing as if the two objects *D* and *I* were respectively seen with only one refraction, which should be made in *H*, and which should be equivalent to all those of the true ray *DFI*.

963. For discovering these refractions, and also for knowing the total value of them, which we suppose reduced to the angle *DHE* or *IHG*, the two angles *AIE* and *ADG* must be observed; and moreover the angle at *A* must be determined by measuring the distance *BC* or *ID*, and by changing it into minutes and seconds of a great circle of the earth. Because the excess of these three angles *AIE*, *ADG* and *DAI* above 180 degrees, is the total refraction equal to the angle *IHG* or *DHE*.

964. They also add the following experiment to make it appear (contrary to the opinion of some authors) that even at noon day there remains somewhat of refraction; when the distance is so great that the visual ray cannot pass from one place to another without approaching the

* Translated by Mr. *Waller* and joined to the Memoirs for a Natural History of Animals. *London* 1688.

earth

earth. "The last summer (say they) being on the top of the towers of *Nostre Dame* at *Paris*, we pointed the quadrant towards the tower of *Montlehery*, and we found that the foot of this tower was precisely in the apparent level *. This was about noon in a very serene time. Some days after at the same hour, the top of the tower of *Nostre Dame* observed from the foot of *Montlehery* appeared below the apparent level 11′. 30″. whereas conformable to the distance of 12796 toyses, which there are between these two places, this angle ought to have been 13′. 30″. Whence it appears that the ray had two minutes of refraction in the whole; that is, the angle DHE contained two minutes. In this calculation they took the diameter of the earth to be 6538594 toyses.

965. Setting aside the curvity of a ray, which Mr. *Picard* tells us is scarce sensible about noon, when the distance of the object does not exceed 1000 toyses; the line of sight through the telescope may be set perpendicular to a plumb-line or parallel to the horizon in this manner. Having found two points g, h upon a level as before; let gi be perpendicular to gc and cut ch in i, and having computed the line hi (as follows) and made a mark at i, place the level at g and alter the place of the cross hairs in the focus, till they appear to cover the point i when the air-bubble is at its marks, and the business is done. Now the line hi is equal to the square of gh applyed to $2gc$; and consequently may be found by measuring the distance gh and dividing its square by the diameter of the earth; which may be supposed equal to $2gc$, though it is not exactly so, the earth being not exactly spherical. For bisecting gh in k, draw ck cutting gi in l; and since the triangles kgl, kcg are similar, we have $kl : kg :: kg : kc$, and by doubling them all we have $ki : gh :: gh : 2kc$. Mr. *Picard* computes that when the distance gh is 300 toyses or 1800 *Paris* feet, the line hi is one inch; and hence any other hi may be found for any other known distance; it being as the square of the distance gh.

Other methods of levelling.

Fig. 632.

966. Hence when the instrument is thus rectified, the point h upon the level with g, may be found by one observation; that is by marking the point i covered by the cross hairs, and by computing ih by the rule above. As the intervals between the stations must be but small in this method, because of refractions as was said above, the readiest way is to make them all equal; which may be known exact enough for this purpose, by observing whether the pole be removed to such a distance, that its image (or the image of any given part of it) in the focus of the telescope shall be always of the same length, being measured by the distance between two parallel hairs in the focus: and then the same allowance must always be made for the depth of the point h below i.

* A line parallel to the horizon of the station they call the apparent level.

967. Laſtly by means of theſe parallel hairs, it is eaſy to find when the teleſcope is placed in the middle between two ſtations; and then the points upon a level at each pole are preſently found, by directing the teleſcope firſt to one pole and then to the other, and by marking the points covered by the croſs hairs. And theſe points will be upon a level notwithſtanding any refractions of the viſual rays, becauſe the refraction of each ray will be equal. Thoſe that are deſirous of fuller inſtructions in the practice of levelling may conſult Mr. *Picard*'s Traite du Nivellement 8°. who deſcribes ſeveral forms of inſtruments for levelling, and gives ſeveral hiſtories of levels actually taken.

CHAPTER XV.

Optical machines for making pictures of objects; and their uſes in drawing.

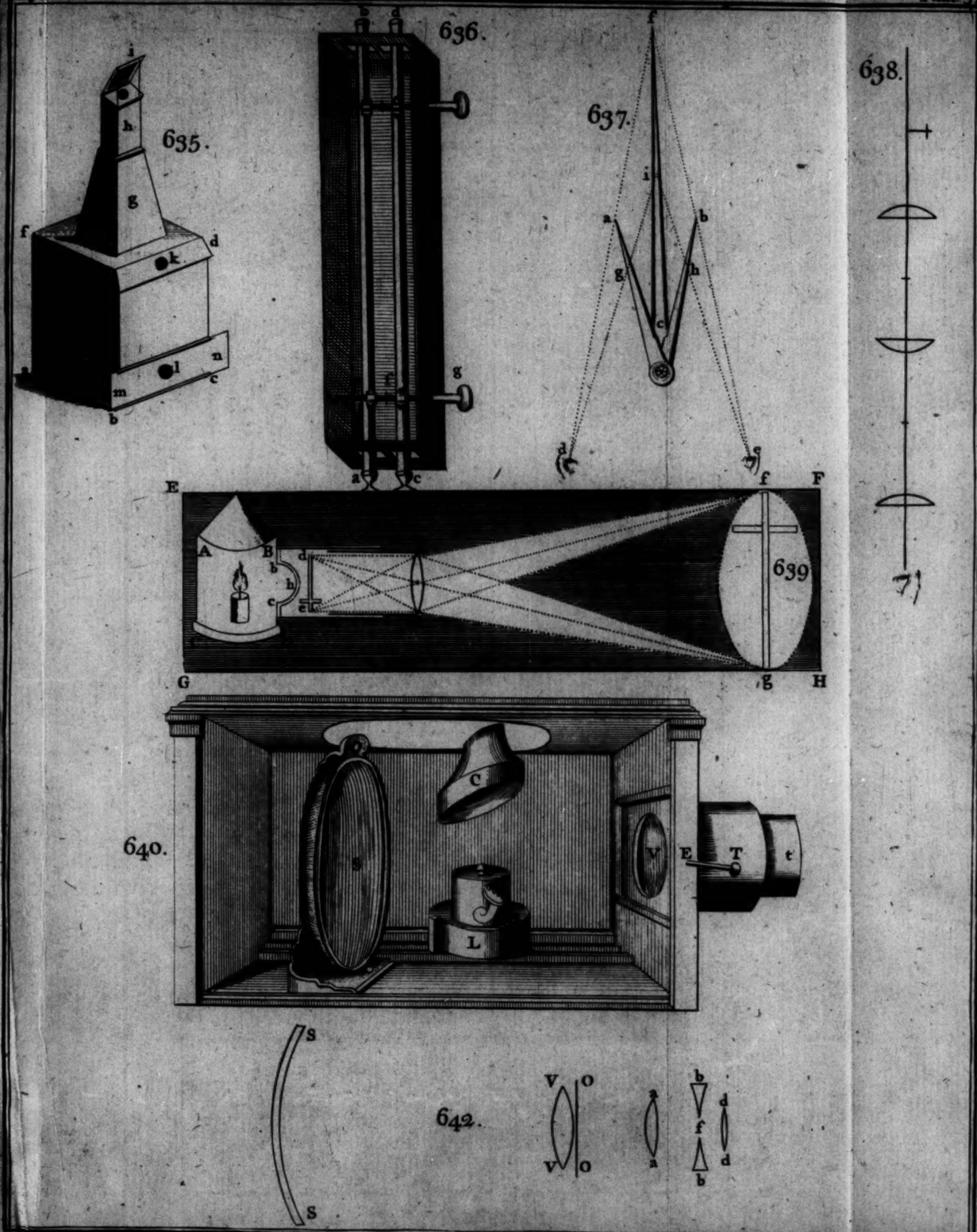
Of pictures in a dark room.

968. IN the 67th article I explained the reaſon of that famous and admirable experiment of forming pictures of external objects upon a white paper, by a lens placed in the window-ſhutter of a dark room; and ſhewed how the inverted pictures might be viewed upright by reflecting them downwards upon a table. For the convenience of directing the axis of the lens towards any object, it is uſually placed in a large cylindrical hole, bored through the middle of a wooden ball, commonly called a ſky-optick-ball; which is eaſily moveable about its center within a hollow zone made of wood and faſtened to the window-ſhutter: this zone conſiſts of two half zones ſkrewed together in the middle after the ball was let in; and the concavity of the zone hinders the light from paſſing between it and the ball. The pictures of objects will be ſo much the larger as the focal diſtance of the lens is longer, and ſo much the brighter as its aperture is larger. The focal diſtance of the lens being 8 or 10 feet, it is convenient to receive the pictures upon a large ſkreen, covered with linnen or white paper, and to have it moveable upon ſmall wheels for the purpoſe of placing it readily at the due diſtance from the lens.

A remarkable appearance in theſe pictures.

969. The broadeſt lens of this kind that ever I ſaw, is at Mr. *Scarlet*'s ſhop by St. *Anne*'s Church *London*: where I remember I was not only diverted with ſeeing the experiment in ſo great perfection, but was ſurpriſed alſo with an unexpected appearance in it: *viz.* that the pictures of perſons walking along the ſtreet, beſides their progreſſive motion, had alſo a perpetual dancing or undulating motion up and down, like the appearance of a perſon carried in a chair, but quicker and more ſenſible. Poſſibly this may ariſe from nothing elſe but a real undulation in the progreſſive motion of a perſon in walking; for I queſtion whether he could walk freely between two parallel boards, if the upper board

be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be but little higher than his head when he ſtood ſtill; and if not, whether the undulation of his head compared with the fixt board would not be ſenſible to a by-ſtander. And if ſo, it will be more ſenſible in the picture upon the ſkreen, in proportion as the picture is nearer to the eye than to the lens. For if the eye be placed by the ſide of the lens, looking firſt at the picture and then at the man himſelf, their apparent motions would be equal, becauſe the rays croſs in the center of the lens. Now upon the ſkreen the picture of a board over the man's head is needleſs; ſince the fixt parts of the ſkreen are ſeen at the ſame view with the picture upon them, and are ſufficient for diſtinguiſhing its motion. If theſe pictures upon the ſkreen be viewed by reflection from a looking-glaſs held in ones hand almoſt horizontal, ſo that the rays coming from the picture may be reflected upwards to the eye, the people in the ſtreet will appear upright and without any undulating motion of their heads; which confirms the cauſe I have been given of it. For now they do not appear like flat paintings upon the ſkreen, but to walk upright upon level ground; the ſkreen being ſcarce taken notice of now, as it is when we view it directly or without a looking-glaſs.

970. For drawing copies of prints or paintings, or the perſpective of ſolids, by tracing the out-lines of their pictures formed by the lens; let the original be placed without doors at a proper diſtance, and let its picture be received upon a ſheet of paper or upon a large plane glaſs, left unpoliſhed on one ſide. This glaſs being fixt upright with its rough ſide turned from the window, it is eaſy with a black lead pencil to trace upon it the out-lines of the pictures. Then having ſtrained a ſheet of fine paper over the glaſs, the ſtrokes of the pencil will appear through it, when held againſt the ſky-light; and thus the picture may be drawn upon the paper. The eaſieſt way of making the image diſtinct upon the glaſs when fixt, is to place the lens in a tube that ſhall ſlide within another ſhort tube fixt in the window-ſhutter. Uſes in drawing.

971. But to ſave the trouble of making two draughts, we may proceen in this manner. Having ſtrained the drawing paper upon a ſmooth board, let it be laid upon a ſmall firm table, to be placed under the lens in the window-ſhutter; and let an inclined looking-glaſs, to reflect the picture upon the paper, be fixt over the table in ſome ſuch manner as this: *ab* and *cd* are two boards fixt upright on the table on each hand of the drawing paper; and *ef* is a third board equal in length to the interval between the upright boards, and having a round pin at each end of it. When this board is laid over the backſide of a looking-glaſs and ſkrewed to the frame of it, the pins muſt be lodged in two ſlits at the tops of the upright boards; and by two nuts that ſkrew upon the pins the glaſs may be ſtayed at a proper inclination, for throwing the picture directly down Fig. 633.

upon the paper below; and then it may be made diſtinct by moving the tube inwards or outwards that holds the lens.

The portable *camera obſcura.* Fig. 634.

972. The portable *camera obſcura*, or dark chamber, commonly ſold in the ſhops, will require but a ſhort deſcription. The theory is this; the rays that come from the object PQR after paſſing the lens E are tending to form an image pqr; but being reflected upwards by the looking-glaſs ABC, they form an horizontal image $\varpi\chi\varsigma$ upon a glaſs plane, whoſe unpoliſhed ſide lyes uppermoſt; upon which a copy of the picture may be ſketched out with a black lead pencil; and to the ſpectator facing the object, the picture appears upright. This figure repreſents a ſection of the machine through the axis of the tube that holds the lens, and through the middle of the ſquare box and the looking-glaſs within it. The ſection of the ſide oppoſite to the tube is not here repreſented, it being a door that opens ſideways; the edges of the rough glaſs at the top are placed in two grooves upon the ſides of the box; and being taken off, it is placed in a drawer ef at the bottom of the box; the looking-glaſs ABC may alſo be drawn out of the grooves in the ſides of the box and lodged in the ſame drawer. The ſquare wooden tube conſiſts of 3 parts; the innermoſt that carries the lens, draws outwards or inwards to make the pictures diſtinct. The parts gh and ik, being fixt together and to the box with ſmall bolts, may be taken aſunder and put into the box; then the lid at at the top, and the door at the end, being both ſhut and fixt, the machine becomes more commodious for carriage. The inſide of the lid whoſe ſection is at, has two wings, that open to right angles on each ſide of it, and reſt upon the ſides of the box, to ſhade the image upon the rough glaſs.

Fig. 635.

973. Another portable *camera* for the buſineſs of drawing is made by Mr. *Scarlet* in this manner. Into the middle of the top of a ſquare box there is inſerted an upright tube, or rather a piece of a ſquare pyramid; in the top of which there ſlides a ſhort ſquare tube, having a broad object-glaſs fixt in a hole at the top of it; the focal diſtance of the glaſs being ſomewhat leſs than its utmoſt height above the bottom of the box, where the pictures of objects are to be formed by it. Upon one edge of the top of this tube there turns a lid, having a plane looking-glaſs fixt flat upon the inſide of it; and thereby capable of being ſtayed at any inclination proper for reflecting the rays, that come from an object, directly downwards through the object-glaſs, to the drawing paper fixt upon the bottom of the box; where the picture of the object will be diſtinctly painted when the object-glaſs is adjuſted to a proper height. The box being quite cloſed and dark within, this picture is viewed through a ſmall hole in the upper edge of the box ſloped off, from the ſide oppoſite to the object, where the draughts-man ſtands and puts his hand and pencil through a hole made in this ſide, or rather in a long piece that ſlides in it horizontally, accord-

according as he has occasion to move his hand. The square box is *abcdef*; the fixt tube, *g*; the sliding tube, *h*; the object-glass, *o*; the inclined looking-glass, *i*; the hole for the eye, *k*; the hole for the hand, *l*, in the slider *mn*; the picture *p* of the object *P*.

CHAPTER XVI.

A Binocular Telescope described.

974. It is a common observation that objects seen with both eyes appear more vivid and strong than to a single eye; especially when both are equally good. A person not short-sighted may soon be convinced of this fact by looking attentively at objects pretty romote, first with one eye and then with both. *P. Cherubin* in his *Vision Parfaite*, says the experiment succeeds also in looking at near objects; to me it does not answer so well. But a short-sighted person may be perfectly satisfied of it by applying a concave glass first to one eye, and then another concave to his other eye. This observation gave rise to the contrivance of a binocular telescope, in which the said appearance is still more manifest.

975. The binocle consists of two distinct telescopes severally directed from each eye to the same object, and combined together as follows. In the 636th figure, *ab* and *cd* are two equal telescopes laid in a long box nearly parallel to each other; the interval between the eye-stops *a*, *c* being equal to the interval of the pupils, and that of the centers of the object-glasses somewhat less than the other. Both ends of the telescopes pass through oblong slits in both ends of the box; and the interval between them may be widened or contracted at either end by a long skrew-pin, laid over each end of both the telescopes; the threads of each half of the skrews being wrought contrary ways and called a right and left handed skrew. For these halves being put through two nuts *e*, *f* fixt to the upper sides of the telescopes, it comes to pass that by turning the skrew-pin one way, the two telescopes will accede to, and the other way, they will recede from, each other; till by one of these skrew-pins the interval between the eye-stops *a*, *c* becomes equal to the interval of the pupils of the observer, and by the other, the axes of the telescopes become directed to the same object: which will be known exactly if there be cross hairs in the focus of each telescope; or even without them. For before this position is obtained the objects will appear double, and afterwards single; and then also besides a much stronger and brighter appearance of the object through both telescopes, than through either alone, there is another phenomenon very remarkable. Fig. 636.

976. In the focuses of the two telescopes there are two equal rings as usual, which terminate the pictures of the objects there formed; and of A remarkable appearance in the binocle.

 con-

consequence the visible area of the objects themselves. These equal rings by reason of the equal eye-glasses, appear equal and equally remote when seen separately by each eye, the other being shut; but when seen united by both eyes, they appear much larger and remoter too; and the objects seen through them do also appear larger, though circumscribed by the united rings in the same places as when seen separately.

The like appearance to the naked eye.

977. Endeavouring to solve this odd appearance, I found out another still more surprising. Having opened the points of a pair of compasses somewhat wider than the interval of your eyes; with your arm extended hold the head or joint in the ball of your hand, with the points outwards and equidistant from your eyes, and somewhat higher than the joint. Then fixing your eyes upon any remote object lying in the line that bisects the interval of the points, you will first perceive two pair of compasses (each leg being doubled [a]) with their inner legs crossing each other, not unlike the old shape of the letter w. But by compressing the legs with your hand, the two inner points will come nearer to each other; and when they unite, (having stopt the compression,) the two inner legs will also entirely coincide and bisect the angle under the outward ones; and will appear more vivid, thicker and longer than they do, so as to reach from your hand to the remotest object in view, even in the horizon it self, if the points be exactly coincident. This appearance will continue the same wheresoever you direct your eyes to any other collateral objects; nor will it vanish by variously inclining the plane of the legs to the horizon; or by any other means than by looking directly at them. The like appearance will happen when two equal round slices of a cork, or any two equal surfaces, are stuck upon the points of the compasses; and likewise when two straws, or two tobacco-pipes, or any two things of equal thickness are used in the shape and instead of the compasses; with this difference only, that the ends of the pipes, or the corks, will not shoot out so far as the naked points of the compasses. If the pipes be held parallel to each other and perpendicular to the visual rays, their apparent union will also seem parallel to them, and somewhat remoter; and the same will happen when two tapering legs of two pair of compasses, are also held parallel to each other; but if they converge as towards a joint, the points will shoot out farther than the other parts of the legs, as in the first experiment, when the plane of them was not perpendicular to the visual rays. In the 637th figure, the points of the compasses are *a* and *b*; the head or joint *c*; the eyes of the observer *d* and *e*; the imaginary leg *cf*, bisecting the angle *acb* and extended to the object *f*.

[a] Art. 137.

Fig. 637.

The reason of it.

978. I apprehend the reason of the phenomenon to be this: by shutting the eyes *d*, *e* alternately, it will be evident that the points *d*, *a*, *f*, are in one straight line, and the points *e*, *b*, *f* in another; and so the reason of the apparent union of the points *a*, *b* at the object *f*, is because their

pictures

pictures are upon the same points of the retina's as those of the object it self. For the same reason, while the legs of the campasses are gradually compressed, if your eye be kept fixt upon the imaginary point of the united legs, this point will appear to approach gradually: which shews why any two points of the legs, as *g*, *h*, equidistant from the extreams, appear nearer and nearer to the eyes as they lye nearer and nearer to the head of the compasses. The appearance of the round corks upon the points of the compasses, is so exactly analogous to that of the two holes in the focuses of the two telescopes, that what has been said of the corks is sufficient to explain the phenomenon of the holes.

979. For want of a two handed skrew, the ends of the telescope may be made to accede to, or recede from, each other, by several other contrivances, to be seen in *P. Cherubin*'s *Vision parfaite*; who has wrote immensely upon this instrument. To exclude all useless and hurtful light from the eyes, the eye-stops are made hollow and very broad, to cover some part of the temples; and their inner parts are cut away to admit the upper part of the nose between them. Mr. *Scarlet*, who makes these binocles at *London*, says that 3 plano-convex eye-glasses for each telescope, with their sides situated as in the 638th figure, are better than 3 double convex ones. But a binocle made with two of Mr. *James Gregory*'s reflecting telescopes hereafter described, would be less cumbersome and magnify more than these. The instrument may be supported upon a stand, whose top has an horizontal circular motion, and also a vertical one upon an horizontal axis; which may be governed by a string, tied to the lighter end of the telescope and wound about a peg, that turns easily in a hole made in the pillar of the stand. Fig. 638.

CHAPTER XVII.

An explication of the Magick Lantern, sometimes called Lanterna Megalographica.

980. THE contrivance is briefly this; *ABCD* is a tin lantern, from whose side there proceeds a square or round arm or tube *bnkclm*, consisting of two parts; the outermost whereof *nklm* slides over the other, so as that the whole tube may be lengthened or shortened thereby. In the end of the arm *nklm* is fixt a convex glass *kl*: about *de* there is a contrivance for admitting and placing an object *de* painted in dilute and transparent colours on a plane thin glass; which object is there to be placed inverted. This is usually some ludicrous or frightful representation, the more to divert the spectators: *bbc* is a deep convex glass, so placed in the other end of the prominent tube, that it may strongly cast the Description of it by Mr. *Molyneux*. Fig. 639.

the light of the flame *a* on the picture *de* painted on the plane thin glaſs. And here it is to be noted, that the glaſs *bbc* is only deſigned for the ſtrong illumination of the picture *de*, and has nothing to do in the repreſentation; and therefore in ſome of theſe lanterns, inſtead of the glaſs *bbc*, we ſhall find a concave ſpeculum ſo placed, that it may ſtrongly caſt the light of the flame *a* on the picture at *de*; and ſometimes both are uſed.

981. Wherefore let us now conſider the picture *de* as a very lightſome object of diſtinct colour and parts. And let us conceive *de* more remote from the glaſs *kl* then its focus. It is then manifeſt that the diſtinct image of the object *de*, ſhall be projected by the glaſs *kl* on the oppoſite white wall *FH* at *fg*; and here it ſhall be repreſented erect. For now the whole chamber *EFGH* is dark, the lantern *ABCD* incloſing all the light; ſo that in effect this appearance of the magick lantern is no more than what is already declared concerning the repreſentation of outward objects in a dark room by a convex glaſs; and here we may obſerve, that if the tube be contracted, and thereby the glaſs *kl* brought nigher the object *de*, the repreſentation *fg* ſhall be projected ſo much the larger; and ſo much the more diſtant from the glaſs *kl*; according to the rules before laid down [a]. So that the ſmalleſt picture at *de* may be projected at *fg* in any greater proportion required, within due limits. From whence the name of Lanterna Megalographica. And conſequently, protracting the tube and drawing the glaſs *kl* more diſtant from the object *de*, will diminiſh the repreſentation *fg*, and project it nigher the glaſs *kl*. As to the mechanick contrivance of this lantern, the moſt convenient proportion of the glaſſes, &c. this is ſo ordinary amongſt the common glaſs-grinders, that it is needleſs to inſiſt farther thereon in this place. It is ſufficient to me that I have explained the theory thereof.

a Art. 48.

Agother magick lantern deſcribed by Dr. 's *Graveſande*.

982. "There are ſeveral machines made by the combination of mirrors and lenſes, which afford uſeful and pleaſant appearances; whoſe explanation may be eaſily deduced from what has been ſaid. Among many other, I ſhall only chuſe to explain one; by which, figures that are painted upon ſmall pieces of glaſs, are repreſented monſtrouſly large upon a white plane. This is a phenomenon wonderful enough to deſerve a particular explanation. The inſtrument that performs this, is called a magick lantern, which optick writers have not altogether paſſed by, but yet have not ſufficiently improved.

Fig. 640.

983. Let there be a wooden box about a foot and an half long, 14 inches high, and as wide; there muſt be a concave mirror *S*, of 8 inches diameter, and a portion of a ſphere of 18 inches: this mirror is fixt to a foot which moves upon rulers, along the length of the box. There is alſo in this box a lamp *L*, ſuſtained by a wooden foot which is moveable lengthways between two rulers, in the ſide of the box. The pipe of the lamp ſtands forward in ſuch manner, that the center of the flame is over

against

againſt the center of the ſurface of the mirror; this flame is made up of four little flames, which by touching one another, make one ſquare flame, two inches wide. In the top, or upper plane of the box, there is an oblong hole, which has a cover that ſlides in two grooves, or between two rulers or ledges: through this cover paſſes the chimney *C*, which (as you ſee in the 641ſt figure) ſtands up about one half foot above the box. Fig. 641.
The chimney is moveable with the cover, whilſt the opening remains ſhut; that the chimney may be always over the lamp. In one of the little ſides of the box, which is over againſt the mirror, there muſt be a round hole about 5 inches wide; which muſt have in it a convex lens of glaſs of the ſame bigneſs *V*, convex on both ſides, which are portions of a ſphere of one foot diameter: the axis of this glaſs, being continued to the ſurface of the mirror, will be perpendicular to it, and fall upon its center, as likewiſe to the plane of the flame, through whoſe middle point it alſo paſſes. This hole is ſhut and opened by a plane moveable in a groove, which is moved by a cylinder that ſtands out of the box at *E*. To this hole without the box anſwers the tube *T*, whoſe length and diameter is of about 6 inches, at the end of which there is a ring, in which the ſecond tube *t* moves, of about 4 inches diameter, and 5 or 6 inches long. In the leſſer tube there are two lenſes; the firſt in that end which is thruſt into the tube *T*, and it is of the ſame convexity as the glaſs *V*, and three inches and a half diameter. The ſecond lens is three inches from the firſt, and flatter, being terminated by portions of a ſphere of four feet diameter. Between theſe lenſes, at the diſtance of about one inch from the ſecond, there is placed a wooden circular ſtop, or aperture, which ſhuts up all the tube, except a hole of an inch and a quarter diameter in the middle of the wood. The objects that are to be repreſented, are to be painted upon a flat thin piece of glaſs, which muſt be moved without the box over againſt the glaſs *V*, between it and the tube *T*, the picture being in an inverted poſition. If theſe pictures are round, they may be of 5 inches diameter: that they may be moved eaſily, they are put into flat boards, three in one board. The picture alſo may be painted upon long glaſſes, which may be ſucceſſively made to ſlide before the glaſs *V*.

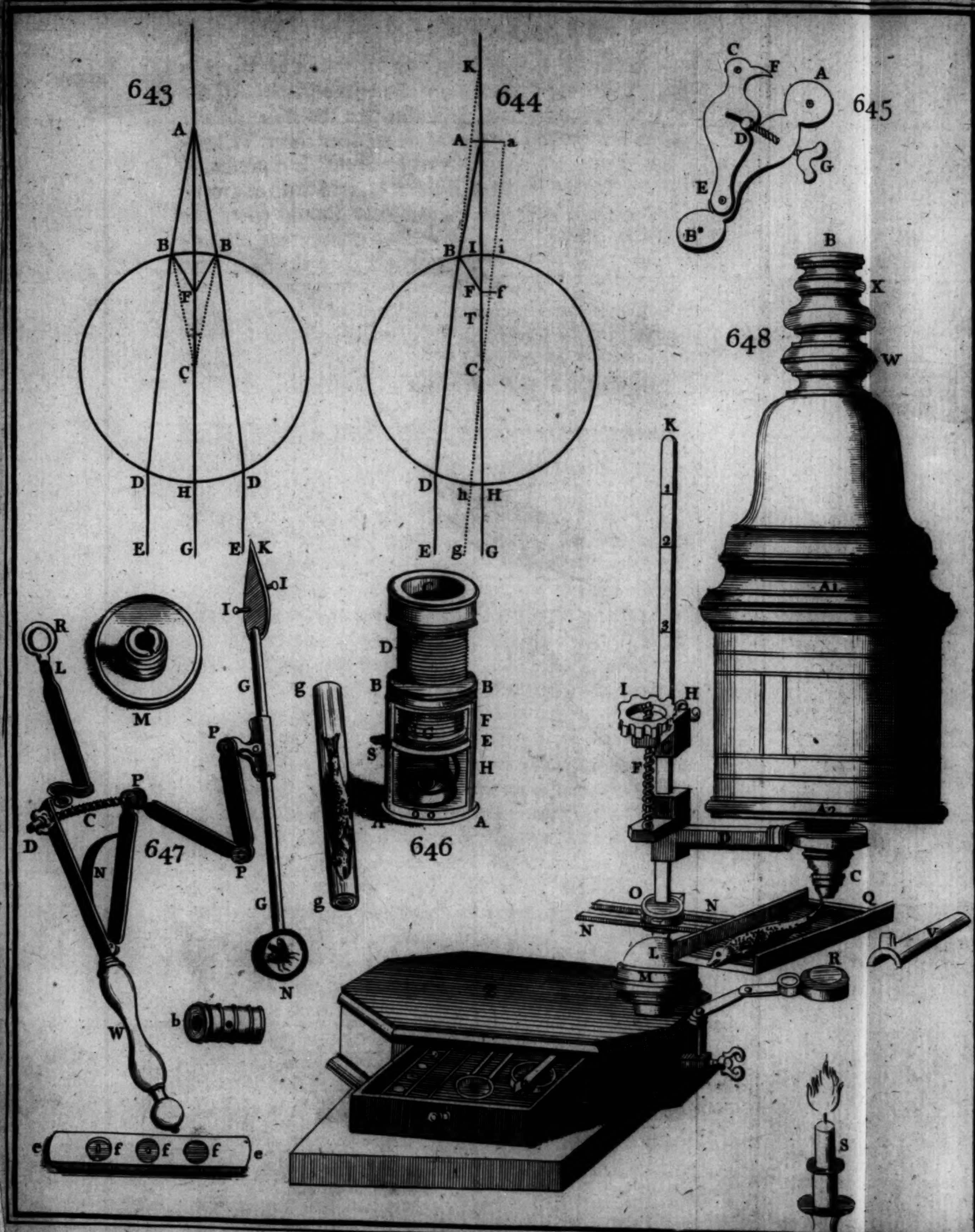
984. This whole box ſtands upon a frame or foot, made ſo that it may be fixed at different heights. There are flat pieces of wood fixed to the box, at bottom, which ſlide in grooves in the frame; each of them has a ſlit in it; ſo that the box may be made faſt at any deſired height, by the help of ſkrews joined to the frame, and moveable in the ſlits. The whole machine is placed at the diſtance of 15, 20, or 30 feet from a white plane, which diſtance muſt be different according to the bigneſs of the plane; for this diſtance may be equal to the length of the plane: the box muſt be juſt at ſuch an height that the tubes, in the ſide of the box, may be exactly oppoſite to the middle of the plane. The lamp being lighted, the box

muſt

must be shut, and the figures which are painted upon the glass will be represented upon a white plane. By moving backwards and forwards the tube that has the two lenses in it, you will find the proper position of the glasses required to give a distinct representation. As for the disposition of the several parts of the machine, which immediately serve for exhibiting this appearance, we shall here more particularly explain it.

Fig. 642. 985. In this figure the parts are shewn separately; *SS* is the mirror, *ll* the flame which consists of four flames in the line *ll*; *VV* is the glass *V* of the 640th figure; *OO* is a picture painted upon a flat thin piece of glass; *aa* the bigger lens; *dd* the flattest lens; *bb* the wooden stop between the lenses; *f* the aperture or hole in the middle of the wooden circle. These things being disposed as has been already explained, and as may be seen in the figure, the rays which proceed from a point of the picture *OO*, by going through the lens *aa*, become less diverging, and fall upon the lens *dd*, as if they came from a point more remote; from this lens they go out converging, and are collected upon the surface of the white plane, where they exhibit the point of the picture that is painted on the glass. This picture is illuminated both by the rays that proceed from the flame *ll*, and by the rays reflected by the mirror *SS*. For the perfection of this machine it is required, 1st, that the picture *OO* be enlightened as much as possible; 2dly, that it be equally enlightened in all its points; 3dly, that all the light, by which every point of the picture is enlightened, go through the lenses *aa* and *dd* to the white plane, and serve to make the representation; 4thly, that no other light but that go out of the box, lest the representation should be less lively, by reason of extraneous light. The first requisite depends upon the bigness of the flame and of the mirror, and of its concavity; the more concave it is, the nearer it is to be brought to the flame, and then the more rays will be intercepted and reflected; yet care must be taken that the mirror (which may be made of very good glass) be not too much heated. When the flame and mirror are so contrived, that the picture is the most enlightened that it can possibly be, and every where equally enlightened, the flame and mirror must be so placed, that the inverted image of the flame shall fall just upon the picture. Now as the representation of the flame can be increased and diminished, the mirror and flame must be so disposed, that the representation of the flame shall cover the whole picture upon the glass, but so as not to exceed it: for then the picture is as much enlightened by the reflected light as it can be, and all its points are equally illuminated; the direct light also does pretty near equally fall upon all the points of the picture; this light would indeed be increased by bringing the flame nearer; but the reflected light would be diminished, and the diminution of this last would be greater than the increase of the other. The glass *VV* serves to inflect the light, by which the picture *OO* is illuminated, be-

fore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fore it comes to the picture; by which inflection all the light comes to the lens *aa*, and serves for the representation on the white plane. All the light that is of use for this representation, goes through the hole *f*; and the rays coming from different points, intersect one another there. Wherefore the picture upon the glass, which is placed inverted, is represented erect upon the white plane. By the ring *bb* all the rays, which do not serve to form the representation, are intercepted, lest they should enter the room, and make the picture less distinct. This ring or aperture also intercepts those rays by which one point is more enlightened than another, whereby the light, which (from what has been said) is equally enough diffused, is yet made more equal. But unless the stop or aperture *bb* be just where the rays intersect, it does a great deal of mischief.

CHAPTER XVIII.

The mechanism of various Microscopes, with some microscopical observations.

986. HAVING considered the theory of single and double microscopes in the 118, 129, 127, 690, 705, 715, 720, &c. articles, and in the remarks upon them, I proceed here to a description of several ingenious contrivances for placing the objects at a due distance from the glasses, and for casting a proper light upon them. But first I will shew how the little glass globules are made for single microscopes; for the manner of making little lenses, being much the same as of making object-glasses for telescopes, need not here be repeated.

987. Mr. *Butterfield* tells us[a] he had tryed several ways of making glass globules of the bigness of great pins heads and less, as in the flame of a candle made of tallow or of wax; but that the best sort of flame for making them clear and without specks was that of a lamp, made with rectified spirit of wine. Where instead of a cotton wick, he made use of fine silver wire doubled up and down like a skein of thread. Then having prepared some fine glass, beaten to powder and washed very clean, he took a little of it upon the sharp point of a silver needle wetted with spittle, and held it in the flame, turning it about till it melted and became quite round, but no longer for fear of burning it. The art lyes in giving the globule an exact roundness, which can only be learned by experience. When a great many globules are thus formed, he rubs them clean with a soft leather. Then having several small pieces of thin brass plates, twice as long as they are broad, he doubles them up into the form of a square, and punches a fine hole through the middle of them: and having rubbed off the bur about the holes with a whetstone, and blacked the insides of the plates with the smoke of a candle, he places a globule

To make glass globules for microscopes.

[a] Phil. Transf. N°. 141.

 between

between the two holes, and tacks the plates together with two or three rivets. Then he tries how they magnify ſmall objects, and keeps the beſt of them for uſe.

988. Dr. *Hook* uſed to take a very clear piece of glaſs and to draw it out into long threads in a lamp; then he held theſe threads in the flame, till they ran into round globules hanging to the end of the threads. Then having fixt the globules with ſealing wax to the end of a ſtick, ſo that the threads ſtood upwards, he ground off the ends of the threads upon a whetſtone and poliſhed them upon a ſmooth metal plate with a little putty.

a Phil. Tranſ. No. 221, 223.

989. Mr. *Stephen Gray* tells us[a] that for want of a ſpirit lamp, he laid a ſmall particle of glaſs, about the bigneſs of the intended globule, upon the end of a charcoal; and by the help of a blaſt-pipe, with the flame of a candle he ſoon melted it into a globule. By this means he made them indifferently clear, and the ſmalleſt very round; but the larger by reſting upon the coal were a little flatted, and received a roughneſs on that ſide. Therefore he was was wont to grind and poliſh them upon a braſs plate, till he reduced them to hemiſpheres. But he found that the ſmall round globules, beſides that they magnified more, ſhewed objects more diſtinct than the hemiſpheres.

Particles within a tranſparent globule how magnified.

990. Theſe experiments led him to another which is very curious. Having obſerved, ſays he, ſome irregular particles within the glaſs globules, and finding they appeared diſtinct and prodigiouſly magnified, when held cloſe to my eye; I concluded that if I conveyed a ſmall globule of water cloſe to my eye, in which there were any opacous or leſs tranſparent particles than the water, I might ſee them diſtinctly. I therefore took on a pin a ſmall portion of water, which I knew to have in it ſome minute animals, and laid it on the end of a ſmall piece of braſs wire, that then lay by me, about $\frac{1}{10}$ of an inch in diameter, till there was formed ſomewhat more than an hemiſphere of water. Then keeping the wire erect, I applyed it to my eye, and ſtanding at a proper diſtance from the light, I ſaw them and ſome irregular particles as I had predicted; but moſt enormouſly magnified. For whereas they were ſcarce diſcernable by my glaſs microſcope, they appeared within the globule not much different in form nor leſs in magnitude than ordinary peaſe. They cannot well be ſeen by day-light, unleſs the room be darkened, after the manner of the famous dioptrical experiment, but moſt diſtinctly by candlelight. They may alſo be ſeen very well by the full moon-light.

Fig. 643.

991. Mr. *Gray* explains the reaſon of this appearance as follows. Let the circle *DBBD* repreſent a ſphere of water, *A* an object placed in its focus, ſending forth a cone of rays, two of which are *AB*, *AB*; which coming into the water at *B* and *B*, will be refracted from their direct courſe and become *BD*, *BD*; and at *D*, *D* they will, at their paſſing into

to air, again be refracted into *DE*, *DE*, and so run parallel to one another and to the axis of the sphere *AFCG*. Now it is a known principle in opticks, that the angle of reflection is equal to the angle of incidence: wherefore let the rays *BD*, *BD*, be imagined to have come from some point *F* of an object placed within the sphere of water, by being reflected from the interior surface of the sphere at *B*, *B*. But *CBD* is the angle of reflection, to which making *CBF* equal, the point *F* will be the place from whence if an object sends forth a cone of rays, two of them as *FB*, *FB*, will be reflected into the lines *BD*, *BD*; and then coming to the other side of the globule at *D*, *D*, they will be refracted into *DE*, *DE*, as before: and consequently will be fit for distinct vision, whether the object be placed in *F* within, or in *A* without, the sphere, if its interior surface be considered as a concave speculum.

992. This explanation of Mr. *Gray*'s may be carried a little farther, to shew the reason why the animals appeared so enormously great, as he describes them. The conclusion is this; the animals within the water globule are magnified $3\frac{1}{3}$ times more in diameter, than they would be if placed without the globule at its principal focus: and this increase of the magnifying power of the smallest globules is very considerable. To demonstrate this conclusion, let the perpendiculars *Aa*, *Ff* to the diameter *HFI* be cut by any other diameter *hfi* in *a* and *f*; and then the unequal objects *Aa*, *Ff*, whereof the former is seen by refraction, and the latter by reflection, will appear under the same angle *HCh*, and consequently of the same magnitude. Therefore one and the same object placed at *F* will appear bigger than at *A* in the ratio of *CA* to *CF**. Imagine the rays *ED*, *GH* to go backwards or from the eye, and after the first refraction let them meet at *K*, and after the second refraction at *A*; which will bisect the line *IK**. Therefore bisecting *CI* in *T*, we have $CA:CT$ :: ($2CA$ or $HK:HC$, that is as) the sine of incidence (m) to the difference of the sines ($m-n$)*; and disjointly we have $TA:TC :: n:m-n$. Hence $CA = \frac{m}{m-n}CT$ and $TA = \frac{n}{m-n}CT$. Now because *K* is the focus of incident rays as *DB* upon the surface *BI*, their focus *F* after reflection will be had by taking $TF:TI$ or $TC :: TC:TK$*, and conjointly $CF:CT$ :: ($CK:TK$ that is :: $2TA:2TA-TC :: \frac{2n}{m-n}TC : \frac{2n}{m-n}TC - TC$ ::) $2n : 3n-m$. Therefore $CF = \frac{2n}{3n-m}CT$, and $CA:CF :: \frac{m}{m-n} : \frac{2n}{3n-m}$. Hence putting $m:n :: 4:3$ as in water, we have $CA:CF :: 10:3 :: 3\frac{1}{3}:1$, which was to be proved.

And in what proportion.

Fig. 644.

* Art. 222.

* Art. 227.

* Art. 224.

* Art. 207.

993. In a glass globule putting $m:n :: 3:2$, we have $CA:CF :: 2\frac{1}{4} : 1$, which shews how much any small particles within the glass, are more magnified than if they were placed beyond it at its principal focus.

994. Hence alſo the animals within the water globule are more magnified than if they were ſeen through a glaſs globule of the ſame bigneſs, in the ratio of $2\frac{1}{2}$ to 1. For in glaſs $CA = 3\,CT$, and in water $CF = \frac{6}{5}CT$. Therefore $CA : CF :: 2\frac{1}{2} : 1$.

995. Hence alſo the animals within the water globule, appear of the ſame magnitude as they would do through a glaſs globule, whoſe diameter is $\frac{2}{5}$ths of the diameter of the water globule. For ſuppoſing CA or $3\,CT$ in glaſs, equal to CF or $\frac{6}{5}CT$ in a larger water globule; we have CT in the glaſs globule, to CT in the water globule, as $\frac{6}{5}$ to 3, or as $\frac{2}{5}$ to 1.

996. But Mr. *Gray* tells us theſe little animals will appear more diſtinctly, if drops of water be conveyed by a pin's point into a round hole made in a braſs plate, whoſe thickneſs is about one tenth of an inch, and the diameter of the cylindrical hole a little leſs than half a tenth, obſerving to fill it till near an hemiſphere of water be extant on each ſide of it. Now ſuppoſing the axis of this cylinder of water to be terminated by equal ſpherical ſurfaces, and to be exactly equal to three diameters of the ſphere of thoſe ſurfaces, I find the little animals, ſeen by reflection from the farther ſurface, will appear juſt twice as big in diameter, as if they were placed in the focus of one of thoſe ſpheres of water, and were ſeen through it as in common microſcopes.

Deſcription of little animals in water &c.

997. The author's account of the little animals thus obſerved, being very curious, deſerves to be tranſcribed. " They are of a globular form and but little leſs tranſparent than the water they ſwim in. They have ſome times two dark ſpots diametrically oppoſite; but theſe are rarely ſeen. There are ſometimes two of theſe globular inſects ſticking together; where they are joined it is opacous: poſſibly they may be in the act of generation. They have a twofold motion; a ſwift progreſſive regular one, and at the ſame time a rotation about their axes, at right angles to the diameter that joins the dark ſpots: but this is ſeen only when they move ſlowly. They are almoſt of an incredible minuteneſs. Mr. *Leeuwenhoek* is moderate enough in his computation, when he tells us (Phil. Tranſ. N°. 213. p. 198.) he ſaw inſects in water ſo ſmall that 30000 could hardly equal a courſe ſand. But I believe it will ſeem a paradox to him, when he is told he may ſee them by only applying his bare eye to a portion of water wherein they are contained. I have examined many tranſparent fluids, as water, wine, brandy, vinegar, beer, ſpittle, urine, &c. and do not remember to have found any of theſe liquors without more or leſs of theſe inſects. But I have not ſeen many in motion, except in common water that has ſtood, for ſometimes a longer at others a ſhorter time, as has been obſerved by Mr. *Leeuwenhoek*; though I do not remember he has obſerved that they are exiſtent in the water before they revive. In the river, after the water has been thickened by rain, there are ſuch infinite numbers of them, that the water ſeems in great part to owe its opacity and whiteneſs

neſs to theſe globules. Rain water ſo ſoon as it falls has many, and ſnow water has more of them. The dew that ſtands on glaſs windows has many of them: and foraſmuch as rains and dews are continually aſcending or deſcending, I believe we may ſay the air is full of them. They ſeem to be of the ſame ſpecifick gravity with the water they ſwim in, the dead remaining in all parts of the water. Of many thouſands that I have ſeen I could diſcern no ſenſible difference in their diameters; they appear of equal bigneſſes in water that has been boiled; they retain their ſhapes and will ſometimes revive.

998. There is another ſort of inſects I have ſeen this way, but theſe are not ſo frequently to be found, at leaſt this winter ſeaſon. They are much longer than the former, and can transform themſelves into many ſhapes. They are elliptical for the moſt part; but ſometimes they contract themſelves ſo as to be almoſt globular; and ſometimes they extend themſelves ſo as to be two or three times longer than broad. Theſe ſometimes turn themſelves round, on their axes and diameters, as they go, and conſiſt of tranſparent and opacous parts.

999. The ſame ingenious author deſcribes another water-microſcope of his own invention, as follows. "*AB* I call the frame of the microſcope; it may be about $\frac{1}{16}$ inch in thickneſs. At *A* there is a ſmall hole, near $\frac{1}{30}$ inch in diameter, in the middle of a ſpherical cavity about $\frac{1}{8}$ inch in diameter, and in depth ſomewhat more than half the thickneſs of the braſs. Oppoſite to this at the other ſide of the braſs, there is another ſpherical cavity, half as broad as the former; and ſo deep as to reduce the circumference of the ſmall hole abovementioned, to almoſt a ſharp edge. In theſe cavities the water is to be placed, being taken upon a pin or a large needle, and conveyed into them till there be formed a double convex lens of water; which, by the concavities being of different diameters, will be equivalent to a double convex lens of unequal convexities. By this means I find the object is rendered more diſtinct, than by a plano-convex of water or by a double convex, formed on the plane ſurfaces of a piece of metal. *CDE* is the ſupporter whereon to place the object, if it be water in the hole *C*, if a ſolid on the point *F*. This is fixt to the frame of the microſcope by the ſkrew *E*, where it is bent upwards, that its upper part *CF* may ſtand at a diſtance from the frame *AB*. It is moveable about the ſkrew *E* as a center to the end that either the hole *C* or the point *F* may be expoſed before, the microſcope *A*, and that the object may be brought to, and fixt in, its focus. There is another ſkrew about half an inch in length, which goes through a round plate in the frame of the microſcope *AE*, the ſkrew and plate taking hold of the ſupporter about *D*; where there is a ſlit ſomewhat larger than the diameter of the ſkrew; which is requiſite for the admiſſion of the hole *C* or point *F* (according to the nature of the object) into the focus of the watery lens

Another water microſcope by Mr. *Gray*. Fig. 645.

at

at *A*. For by turning the ſkrew *G*, the ſupporter is carried to or from the ſame; which may be ſooner done, if while one turns the ſkrew with one hand, the other holds the microſcope by the end *B*; and one be looking through the water till the object be ſeen moſt diſtinctly. The ſupporter muſt be made of a thin piece of braſs well hammered, that by its ſpring it may better follow the motion of the ſkrew. I chuſe rather to fix the ſupporter by the ſkrew *E* than by a rivet; becauſe it may now, by the help of a knife be unſkrewed, and by the other ſkrew *G* be brought cloſe to the frame of the microſcope, without weakening its ſpring, and ſo become more conveniently portable. If the hole *C* in the ſupporter be filled with water, but not ſo as to be ſpherical, all objects that will bear it are ſeen therein more diſtinctly.

1000. The hole at *B* is made for ſeeing animals in water by reflection from its farther ſurface, as above deſcribed. So far Mr. *Gray*.

A deſcription of Mr. *Wilſon*'s pocket microſcope, taken from Phil. Tranſ. N°. 281. Fig. 646, 647.

1001. This ſet of microſcopes has nine different magnifying glaſſes; eight of which may be uſed with two different inſtruments, for the better applying them to various objects. One of theſe inſtrumens is repreſented at *AABB* and is made of ivory or braſs; it has 3 thin braſs plates at *E* and a ſpiral ſpring of ſteel-wire *H* within it; to one of the thin plates of braſs is fixed a piece of leather *F*, with a ſmall furrow *G* both in the leather and braſs to which it is fixed: in one end of this inſtrument there is a long ſkrew *D* with a convex glaſs *C*, placed in the end of it: in the other end of the inſtrument there is a hollow ſkrew *oo*; wherein any of the magnifying glaſſes, *M*, are ſkrewed when they are to be made uſe of. The 9 different magnifying glaſſes are all ſet in ivory, 8 of which are ſet in the manner expreſſed at *M*. The greateſt magnifier is marked upon the ivory, wherein it is ſet, with N°. 1, the next N°. 2, and ſo on to N°. 8; the ninth glaſs is not marked, but is ſet in the manner of a little barrel box of ivory, as at *b*. At *ee* is a flat piece of ivory, whereof there are 8 belonging to this ſet of microſcopes; (though any one who has a mind to keep a regiſter of objects may have as many of them as he pleaſes;) in each of them there are 3 holes *fff*, wherein 3 or more objects are placed between two thin glaſſes, or talks, when they are to be uſed with the greater magnifiers.

1002. The uſe of this inſtrument *AABB* is this: Having taken the handle *W* from the inſtrument in fig. 647 and ſkrewed it upon the button *S* in fig. 646, take one of your flat pieces of ivory *ee*, or ſliders, (if you pleaſe to call them ſo,) and ſlide it betwixt the two thin plates of braſs at *E*, through the body of the microſcope, ſo that the object you intend to look upon be juſt in the middle; remarking that you put that ſide of the plate *ee*, where the braſs rings are, fartheſt from the end *AA*: then you are to ſkrew into *oo* (the hollow ſkrew in the end of the body of your microſcope) the 3d, 4th, 5th, 6th or 7th magnifying glaſs *M*; which being

done put the end *AA* cloſe to your eye, and while you are looking through your magnifying glaſs upon the object, you are to ſkrew in or out the long ſkrew *D*, which moving round upon the leather *F*, held tight to it by the ſpiral wire *H*, will bring your object to the true diſtance; which you will know by ſeeing it clearly and diſtinctly: but ſeeing that in the greater magnifiers you can ſee but a ſmall part of the object, *viz.* the legs or claws of a flea; while you are looking upon any part of the object, if you take hold of the end of the plate or ſlider, *ee*, whereon the object lyes, and move it gently, you may ſee the whole object ſucceſſively, or any part of the object you pleaſe; and if that part of the object you deſign to look upon be out of the true diſtance, remember your end ſkrew *D*, can always bring it in, by ſkrewing it one way or the other.

1003. After this manner may be ſeen all tranſparent objects, duſts, liquids, chryſtals of ſalts, ſmall inſects, ſuch as fleas, mites, &c. If they be inſects that will creep away, or ſuch objects as one intends to keep, they may be placed between the two regiſter glaſſes *ff*. For by taking out (with the point of a pen-knife or ſmall plyers) the ring that keeps in the glaſſes *ff* where the object lies, they will fall out of themſelves; ſo you may lay the object between the two hollow ſides of them, and put the ring in as it was before: but if the objects be duſts or liquids, a ſmall drop of the liquid, or a little of the duſt laid on the outſide of the glaſs *ff* and applyed as before, will be ſeen very eaſily.

1004. As to the firſt ſecond and third magnifying glaſſes, being marked with a + upon the ivory wherein they are ſet, they are only to be uſed with thoſe plates or ſliders that are alſo marked with a +, wherein the objects are placed between two thin talks; becauſe the thickneſs of the glaſſes in the other plates or ſliders, hinder the object from approaching to the true diſtance from theſe greater magnifiers. But the manner of uſing them is the ſame with the former. Only remember to be careful when you put in or pull out the plate or ſlider *ee*, whereon the object lyes, or move it from one object to another, not to let it rub your magnifying glaſs; which is done by unſkrewing a little the end ſkrew *D*, when you put in or pull out your plate, or move it from one object to another.

1005. For ſeeing the circulation of the blood at the extremities of the arteries and veins, in the tranſparent parts of fiſhes tails, &c. there are two glaſs tubes, the one bigger and the other leſſer, as expreſſed at *gg*, wherein the fiſh is to be put; when theſe tubes are to be uſed, you are to unſkrew the end ſkrew *D* in the body of the microſcope, until the tube *gg*, can be received eaſily into that little cavity *G* of the braſs plate faſtened to the leather *F*, under the other two thin plates of braſs at *E*: when the tail of your fiſh lies flat to the glaſs tube, ſet it oppoſite to your magnifying-glaſs, and by ſkrewing in or out your end ſkrew *D*, as is ſaid before,

fore, you may eafily bring it to the true diftance, and fee the blood circulate with great pleafure.

1006. If you would fee the blood circulate in a frog's foot, choofe fuch a frog as will juft go into your tube; then with a little ftick expand the hinder foot of the frog, and apply it clofe to the fide of the tube, obferving that no part of the frog hinders the light from coming on its foot; and when you have it at the juft diftance, by means of the fkrew *D*, as abovefaid, you will fee the rapid motion of the blood in its veffels, which are very numerous, in the tranfparent thin membrane that is between the frog's toes. For this object the 4th and 5th magnifiers will do very well; but you may fee the circulation in the tails of water-newts with the 6th and 7th glaffes, by reafon the globules of the blood of thofe newts are as big again as the globules of the blood of frogs or fmall fifh, as has been taken notice of in N°. 280. of the Philofophical Tranfactions, pag. 1184.

1007. The circulation cannot fo well be feen by the firft fecond and third magnifyers, becaufe the thicknefs of the glafs tube wherein the fifh lyes, hinders the approach of the object to the focus of the magnifying glafs.

1008. The other inftrument fig. 647 is made of brafs or prince's metal, with joints *PPP* to turn eafily any way, and with a fmall pair of tongs *GG*, which open at the points *K*, by preffing together the two heads of the pins *II*, for taking up of objects. At the other end of thefe tongs *GG*, is fkrewed on a round piece of black wood *H*, with a piece of ivory let into it, for placing opake objects on, according to their difference of colour. Upon the end L there is a fkrew, upon which the glafs *b* fet in the barrel-box may be fkrewed. When the other glaffes are to be ufed, there is a ring *R* of brafs to be fkrewed on the end *L*, into which ring all the other glaffes *M*, may be fkrewed. So when any object is taken up in the points of the tongs *K*, or laid upon the other end *H*, it may very eafily (as one who fees the inftrument will perceive) be applyed to the true focal diftance of any of the glaffes *M*, by the help of the joints *PPP*, and by means of the fkrew *C*, with the wheel *D*, which being regulated by a fpring *N*, will bring the object to the exact diftance for diftinct vifion.

1009. The glafs placed in the manner of a barrel-box at *b*, is only to be ufed with the brafs inftrument (or in your hand) being the leaft magnifier, for greater objects, fuch as flies and common infects, &c. remembring to put the hole *b* next to your eye.

1010. In the viewing objects, one ought to be careful not to hinder the light from falling upon them, by the hat, peruke, or any other thing, efpecially in looking at opake objects: for nothing can be feen with the beft of glaffes, unlefs the object be at a due diftance, with a fufficient light. The beft lights for the plates or fliders, where the object lyes between

between the two glasses, is a clear sky-light, or where the sun shines on any white thing, or the reflection of the light from a looking-glass. The light of a candle is likewise good for the viewing of very small objects, though it be a little uneasy to those who are not practised in microscopes to find it out. The only use of the convex lens at *C* is to collect the light into a narrower compass where it falls upon the object, after it has passed through a moderate hole in the leather *F*.

1011. For the convenience of those who would draw, or make any sketches or designs of microscopical objects, they may also have a pedestal to fix the two instruments above described, and make them stationary to any convenient light. This pedestal may be placed on a table, and after the object and light are fixed, as many persons as please may view the object, without any trouble or difficulty in finding the light.

A description of Mr. *Marshal*'s double microscope by Dr. *Harris* in his Lexicon Technicum. Fig. 648.

1012. The eye-glass is at *W*, the object-glass at *C*, the middle glass at *A* 1; *B* is the cover or lid, to keep out the dust from the eye-glass *W*; *X* is the place of the eye, *W* a skrew where the eye-glass lyes; *A* 1 a skrew where the middle glass lyes; *A* 2 the draw, where the outermost tube *A* 1 *A* 2 is disjoined from the inner one, of the same length; *Z* the frame or basis on which the microscope stands firm; *T* a small drawer in the frame or basis, with a ledge or till in it, having six partitions to hold so many several object-glasses, one magnifying more than another, and fixed in brass cells ready to skrew on at *C*, and marked 1, 2, 3, 4, 5, 6; these partitions are also marked, 1, 2, 3, 4, 5, 6; the other part of the drawer serves to hold the object-plate *a*; a pair of small nippers *b*, to take up, or handle any object conveniently; another object-plate *d*, having one side white and the other black, to fix your objects upon, as black upon white, and white objects on black. *L*, *M* is a brass ball and socket, on which the whole body of the microscope is moveable, so as to lye in any position for the light. *K*, *O* a square brass pillar on which the microscope is moveable up and down, by means of the collar *E*, into which the arm *D*, (holding the microscope) is continued. *G* another brass collar sliding up and down on the pillar *KO*, having a small skrew *H* by which it is, as occasion serves, fixed fast to the said pillar, at any height. *I* a large brass nut, in whose center is a female skrew, fitted to the male skrew *F*, which is fixed in the collar *E*: by the turning of which nut *I*, (the collar *G* being first fixed to the pillar by the skrew *H*) the microscope is raised up or down on the pillar, and made to come nearer or go farther from the object *Pc*: and, which is also a very great advantage, the axis of the microscope is always kept perpendicular to that point of the object, over which it was first placed; so that here is not the inconvenience which occurs in other glasses, of often losing the sight of the object, by skrewing the glass *C* higher or lower. *PQ* is a glass object-plate fixed in a brass frame, whose arm *NN*, is fixed to the pillar by means of the nut *O*. The arm *NN* hath in

it a ſlit, by which it is eaſily put on, or taken off the pillar, and by which it may be fixed upon it at any diſtance. *P* a ſmall fiſh lying on the glaſs-plate, that the circulation of the blood may be ſeen in the end of the tail-fin, at *c*. *R* a convex-glaſs, by whoſe help a bright ſpot of light is brought from a candle at *S*, ſtanding on the ground, while the microſcope ſtands on the edge of a table or ſtool, which ſpot of light *c* ſerves to render the circulation more conſpicuous. *V* a lead coffin to be put on the fiſh, to hinder it from ſpringing away, and moving his tail out of the light. 1, 2, 3, 4, 5, 6, are marks on the pillar *KO*, to ſhew the reſpective diſtances of the object-glaſs from the object you look upon, according as the object-glaſſes you make uſe of magnify more or leſs. Thus, for inſtance, if you uſe the object-glaſs 5 or 6 (either of which will ſhew the circulation of the blood) you muſt fix the upper edge of the collar *E*, at the mark 5 or 6 on the pillar. And then the microſcope will be very near its exact diſtance from the object; ſo that by a ſmall turn or two of the nut *I* one way or the other, to be found by tryal, you may ſoon fit it exactly to your own eye.

1013. By this microſcope liquors alſo may be very commodiouſly examined; for if you place a ſmall drop of any liquor on the glaſs-plate juſt in the middle of the ſpot of light *c*, the parts of it will become very viſible, and its animalcula, if it have any, will be diſcovered. And thus may the eels in vinegar, the ſmall creatures in black-pepper water, or in waters where wheat, barly, &c. has been infuſed, the eels and other ſmall living creatures in puddle water, be as plainly ſeen as by almoſt any other microſcope.

1014. There is one thing I ought not to omit to mention on this occaſion; which is, that I have often with this glaſs, ſeen the circulation of the blood in the fins of the tails of tadpoles; and indeed more conſpicuouſly here than in any other creature: for the fins growing all round the tail, and coming but a little way out beyond the body of it, both the ejaculation of the blood out by the arteries, and its return again by the veins, is much quicker than in the tails of fiſhes; and abundance more ſtreams, turns, and windings of the moving blood are here viſible, than I could ever ſee in any other animal. To which I may add, that the creature will live a good while out of the water, and will lie very ſtill. So far Dr. *Harris*.

1015. Dr. *Hook* tells us in the preface to his Micrography that in moſt of his obſervations he uſed a double microſcope; retaining the broad middle glaſs, when he wanted to ſee very much of an object at one view, and taking it out when he would examine the ſmall parts of the objects more accurately. For always the fewer the refractions, the more bright and clear the object appears. Accordingly he tells us that objects appear more diſtinct and more magnified through a ſingle glaſs globule, prepared as

as above[a], than through any double microſcope. Nevertheleſs he ſeldom made uſe of a ſingle microſcope upon account of the difficulties in placing the objects ſo near the globule as they ought to be.

a Art. 987.

Illumination of microſcopical objects.

1016. Since the aperture of the object-glaſs of a double microſcope muſt be very ſmall to ſhew the object diſtinct, and conſequently can receive but little of its light, the way to remedy this inconvenience is to illuminate the object as much as poſſible. Dr. *Hook* made choice of a room having but one window towards the ſouth; where at the diſtance of three or four feet from the window, he placed his double microſcope upon a table, and by a glaſs globe full of water and a thick plano-convex lens he collected the light upon the object. Or when the ſun ſhone he placed a piece of oiled paper very near the object, and with a very large burning-glaſs he threw the ſun's rays upon the paper, ſo that a great quantity of them might be tranſmitted through it to the object. But the paper being ſubject to take fire, if the focus of the rays fell upon it, inſtead thereof he ſometimes ſubſtituted a piece of plane glaſs, not poliſhed but only rough ground with fine ſand; which when warmed gradually would endure much more heat than oiled paper, and conſequently would ſuffer more light to paſs through it to the object. By this means he ſays the light of the ſun was more equally diffuſed upon the parts of the object than the ſun's direct light; which coming but in one direction, would be ſo ſtrongly reflected from ſome parts of the object, as to drown the appearance of all the reſt that lay in the ſhade. In the night time he illuminated his object with the light of a lamp, firſt refracted through his globe full of water, (or clear brine which refracts more than water) and then collected into a ſmaller ſpot upon the object by a plano-convex lens. On the ſide of the lamp oppoſite to the globe, he alſo placed a poliſhed concave metal, to reflect part of the rays upon the globe. And thus he ſays he could illuminate an object with a ſmall lamp as much as it would well bear; and that he drew moſt of his repreſentations of objects by the light of his lamp. Why the Dr. always preſcribes the uſe of a plano-convex lens, I do not underſtand. For it is evident that a double convex lens will contract a given pencil of rays into a much ſmaller ſpace than a plano-convex one of the ſame ſphere, becauſe its principal focus is as near again to the lens. But after all though greater degrees of illumination are great helps to greater degrees of magnifying with ſmaller object-glaſſes, yet after a certain degree of magnifying, he obſerves, they leave us again in the dark.

Mr. *Folkes*'s account of *Leeuwenhoek*'s microſcopes. Phil. Tranſ. N°. 380.

1017. Mr. *Leeuwenhoek* on the contrary in all his obſervations which he continued to communicate to our Royal Society for above 50 years together, made little or no uſe of any other microſcopes but ſingle ones, as we are informed by our worthy Vice-Preſident *Martin Folkes* Eſquire, in the account he has given us of Mr. *Leeuwenhoek*'s legacy to the Society of

of 26 of these microscopes. Those that are conversant in microscopical observations may be usefully entertained with this account, part of which I will here transcribe. "These microscopes are all single consisting each of a small double convex-glass, let into a socket, between two silver plates riveted together, and pierced with a small hole. The object is placed on a silver point or needle, which by means of skrews of the same metal, provided for that purpose, may be turned about, raised or depressed, and brought nearer or put farther from the glass, as the eye of the observer, the nature of the object, and the convenient examination of its several parts may require. Mr. *Leeuwenhoek* fixed his objects, if they were solid, to this silver point with glew; and when they were fluid or of such a nature as not to be commodiously viewed unless spread upon glass, he first fitted them on a little plate of talk, or excessively thin-blown glass; which he afterwards glewed to the needle in the same manner as his other objects. The observation indeed of the circulation of the blood and some others require a somewhat different apparatus; and such a one he had, to which he occasionally fixed these same microscopes. But as it makes no part of this legacy, I shall omit giving any farther account of it; only taking notice it may be seen in a letter to the Royal Society, on the 12th of *January* 1689, and printed in his *Arcana Naturæ detecta* N°. 96. But I was willing to mention just so much to shew the universal use of these single microscopes; and as it induces me, among other things to believe, these were the kind of microscopes generally, if not solely, used by this curious gentleman in all his observations, and to which we are obliged for his most surprising discoveries.

1018. The glasses are all exceedingly clear and shew the object very bright and distinct; which must be owing to the great care this gentleman took, in the choice of his glass, his exactness in giving it the true figure, and afterwards among many, reserving such only for his use, as upon tryal he found to be most excellent. Their power of magnifying are different as different sorts of objects may require; and as on the one hand, being all ground glasses none of them are so small and consequently magnify to so great a degree as some of those drops frequently used in other microscopes, yet on the other hand, the distinctness of these very much exceeds what I have met with in glasses of that sort; and this was what Mr. *Leeuwenhoek* ever principally proposed to himself, rejecting all those degrees of magnifying in which he could not so well obtain that end. For he informs us in one of his letters, where he is speaking of the excessive praise some give to their glasses on this account, that though he had above 40 years had glasses by him of an extraordinary smalness, he had made but very little use of them; as having found in a long course of experience, that the most considerable discoveries were to be made with such glasses,

glasses, as magnifying but moderately, exhibited the object with the most perfect brightness and distinction.

1019. Nor ought we to forget a piece of skill in which he very particularly excelled, which was that of preparing his objects in the best manner to be viewed by the microscope: and of this I am perswaded any one will be satisfied who shall apply himself to the examination of some of the same objects as do yet remain before these glasses. At least I have my self found so much difficulty in this particular, as to observe a very sensible difference between the appearances of the same object, when applyed by my self and when prepared by Mr. *Leeuwenhoek*, though viewed with glasses of the very same goodness. I have the rather insisted upon this, as it may be a caution to us, that we do not rashly condemn any of this gentleman's observations, though even with his own glasses we should not be able to verify them ourselves. We are under great disadvantages for want of the experience he had, and he has himself put us in mind, that those who are the best skilled in the use of magnifying glasses, may be misled, if they give too sudden a judgement upon what they see, or till they have been assured by repeated experiments. But we have seen so many and those of his most surprising discoveries, so perfectly confirmed by great numbers of the most curious and judicious observers; that there can surely be no reason to distrust his accuracy in those others which have not yet been so frequently or so carefully examined. So far Mr. *Folkes*.

Micrometers applicable to double microscopes.

2020. Those that are curious in making exact draughts of the appearances of objects seen in double microscopes, may be very much assisted by a lattice made with fine silver wires, or with the strokes of a diamond upon a plane glass, put into the place of the image formed by the object-glass; and by transferring the parts of the object, seen in the squares of the lattice, into corresponding squares of a similar lattice drawn upon paper. It may also be of singular use in philosophical enquiries to know the exact measures of the several vessels and other parts of animal and vegetable substances; and this, as Mr. *Balthasaris* has observed in his little treatise upon Micrometers, may be done very exactly by a micrometer of the same form as is used in a telescope. For by opening the hairs of the micrometer till they exactly comprehend an object of a known length, suppose $\frac{1}{10}$ inch, and by observing the number of revolutions in this opening; the diameter of any other object, answering to a known number of revolutions, may be found by the golden rule.

Dr. *Jurin's* way of measuring microscopical objects.

1021. But the most ingenious Dr. *James Jurin*, in his excellent Dissertations upon Physico-mathematical subjects pag. 45, has taught us a very accurate and ready way of doing the same thing even without a micrometer. First he twists a very fine silver wire a great many times about a pin or a slender cylinder, so closely as to leave no interval between the

wreaths

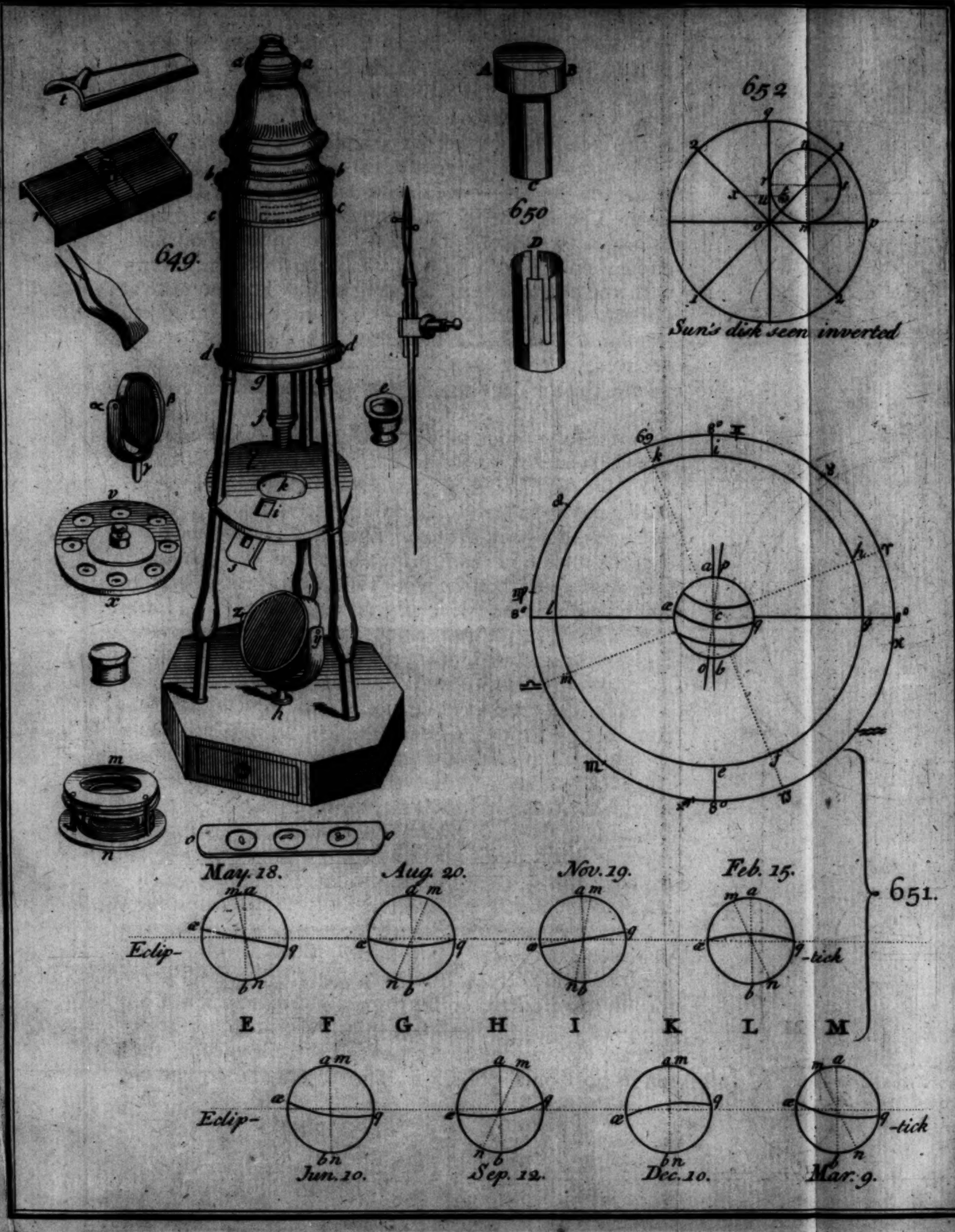
wreaths, which he carefully examines by a magnifying glaſs; then he takes the interval of the outermoſt wreaths between the points of a pair of fine compaſſes, and applies this extent to a diagonal ſcale of inches; and by dividing this meaſure of that extent, by the number of wreaths therein contained, he obtains the thickneſs of the wire it ſelf. Then he cus it into very ſmall bits, and ſcattering them upon the object-plate, he places the object upon them, if it be tranſparent, or the wires upon the object, if it be opake; and by the eye he compares the parts of the object with the thickneſs of thoſe wires that happen to lye contiguous to them.

Applyed to blood-globules.

1022. Thus he obſerved that 4 globules of human blood would generally cover the breadth of a wire which he had found to be $\frac{1}{485}$ part of an inch, and by conſequence that the diameter of a ſingle globule was $\frac{1}{1940}$ part of an inch. Which was alſo confirmed by Mr. *Leeuwenhoek's* obſervations upon human blood, made with a piece of the ſame wire tranſmitted to him from Dr. *Jurin*. Phil. Tranſ. N°. 377.

Other obſervations on blood-globules by Dr. *Jurin*.

1023. We are alſo obliged to this learned and judicious gentleman, my highly eſteemed friend, for two other microſcopical obſervations, to rectify two vulgar errors. The firſt is, that the globules of blood were thought lighter than the ſerum; which gave riſe to a ſecond miſtake, that they contained either air or ſome elaſtical fluid within them. I have often placed, ſays the Dr, a drop of ſerum upon a clean glaſs before a microſcope, in which I had diſſolved a very ſmall quantity of blood; and obſerved, that when the glaſs was held in a perpendicular poſture, the blood-globules ſubſided to the bottom of the drop: and inverting the glaſs, the globules again deſcended through the ſerum to the bottom. I had alſo the ſame ſucceſs with a ſmall quantity of ſerum and blood in a capillary tube. And the ſame has alſo been obſerved by the famous Mr. *Leeuwenhoek*. Hence it is by no means probable that the blood globules are veſicles filled with air, or any other fluid lighter than the ſerum; and that they are not filled with any ſort of elaſtick fluid, will appear by the following experiment. In a ſmall quantity of ſerum I diſſolved ſo much human blood as that the globules might not lye too thick together, to hinder their being ſeen diſtinctly. Then having lodged a ſmall drop of this liquor on the ſide of a thin glaſs tube, I fitted the tube to the air-pump and placed a microſcope by it, ſo that I could ſee the blood-globules through the tube. This being done I cauſed the tube to be exhauſted, keeping my eye upon the globules all the time, in order to obſerve whether they dilated themſelves, as the air was withdrawn; but could not perceive the leaſt alteration; they appearing exactly of the ſame bigneſs in the Vacuum as they had done before. Whereas if they had been filled with an elaſtick fluid, they would either have burſt, or have been dilated to at leaſt 70 or 80 times their former magnitude. The ſtop-cock being afterwards turned and the air ſuffered


649.
650
652
Sun's disk seen inverted
651.
May. 18.
Aug. 20.
Nov. 19.
Feb. 15.
Eclip-
-tick
E F G H I K L M
Eclip-
-tick
Jun. 10.
Sep. 12.
Dec. 10.
Mar. 9.

fered to re-enter the tube, the blood-globules ftill retained the fame bigneſs as in vacuo. So far Dr. *Jurin.* Phil. Tranf. N°. 361.

1024. In this microſcope, made by Mr. *Culpeper* and Mr. *Scarlet*, the inner tube *ab*, which ſlides in the outer *cd*, holds all the glaſſes. The eye-glaſs is at *aa*, the broad middle glaſs at *bb*, and the object-glaſs, being ſet in a button at *e*, is ſkrewed upon the end of a narrower tube *fg*; which being fixed in the baſe of the inner tube, paſſes freely through a hole in the baſe of the outer. The buttons that contain the ſeveral object-glaſſes are marked 1, 2, 3, &c. and the convexity of the inner tube is alſo marked with dotted circles, numbered 1, 2, 3, &c. in order to bring that circle to coincide with the mouth *cc* of the outer tube, whoſe number is the ſame as that of the object-glaſs then made uſe of. But if the object does not yet appear quite diſtinct, the button *e* upon the ſnout *fg* muſt be gradually unſkrewed, to bring the glaſs nearer to the object placed below it. Of theſe glaſſes the greater magnifiers are known by their having ſmaller apertures.

A double microſcope, in which the objects are illuminated by reflection. Fig. 649.

1025. The baſe *dd* of the outer tube is ſupported by three braſs pillars, fixt into a wooden pedeſtal *h*; and a little below the object-glaſs *f*, a circular plate *ikl* is lodged like a ſtage between the pillars, three ſemicircular notches being cut in its circumference to receive the pillars, and to reſt upon three rings that ſurround them. Three ſmall braſs circles *mn* with holes through the middle of them, are to be placed over the hole *k* in the middle of the ſtage; and then the ivory plate *o* may be put between the two uppermoſt of theſe circles, which are preſſed together by a ſpiral ſpringing wire lodged between the two undermoſt; the two outermoſt being held together by three ſmall pillars paſſing through three holes in the circumference of the middle circle. For viewing the circulation of the blood, the button *p*, on the under ſide of the frame of a broad plane glaſs *qr*, being put through a ſlit *i*, made in the ſtage, a ſmall braſs plate *s* under the ſtage, muſt be ſhoved inwards, till a ſmaller ſlit in it embraces the neck of the ſaid button; and then the fiſh being laid upon this glaſs and covered with the coffin *t*, its tail may be brought exactly under the object-glaſs by turning the glaſs *pq* about the button, or by ſhoving it inwards or outwards along the ſlit *i* in the ſtage. The circular objectplate *vx* has a like button in its center, to be put into the ſame ſlit as before; and then the different objects, placed between two talks in the holes made round the circumference of the plate, may be viewed ſucceſſively by turning the plate about its center.

1026. All theſe tranſparent objects are illuminated mighty well in this microſcope, either by candle-light or ſky-light reflected upwards from a concave looking-glaſs *yz* placed in a frame upon the center *h* of the pedeſtal. While you are viewing the object through the microſcope, turn this concave upon its horizontal poles *y*, *z*, and you will ſoon find out

that

that position of it wherein it reflects the most light through the hole *k* upon the object; and this happens when it reflects the rays very obliquely. Opake objects, when laid upon a black ebony or a white ivory plate, put into the hole *k* upon the stage, may be illuminated by candle-light transmitted through a double convex lens $\alpha\beta$; the foot γ, of the frame $\alpha\beta\gamma$ in which it turns, being put into the hole *l* in the stage. The candle must be placed in a line drawn from the object through the middle of this lens, at such a distance from it as shall cause the spot of light upon the object-plate to be the narrowest. By day-light this lens gives little or no advantage to the direct sky-light.

An extempore apparatus for a reflecting microscope. Fig. 649, 650.

1027. For want of a proper apparatus for a reflecting microscope, I made the tryals I mentioned in art. 738 with this before us. Having removed the concave looking-glass and its frame *xyz*, and having cemented the back of a concave metal *AB* to the top of a small wooden cylinder *C* about an inch long, I placed this cylinder within a hollow brass tube *D*; whose sides were slit downwards to make it spring towards the cylinder *C*, and thereby support it at any given height. Then I skrewed the bottom of this tube to the center of the pedestal *h* by a skrew whose head lay under it. I placed my object over the hole *k* upon the glass-plane *qr*, for laying fish upon; and enlightened it, in the night time, by candle-light refracted through the lens $\alpha\beta$: then having took out the object-glass and the broad middle glass *bb*, I adjusted the distance of the eye-glass *aa* from the concave, by raising or depressing the inner tube that contained it; or by inserting a longer tube. I believe the experiments would have succeeded better, had I encompassed the reflecting metal with a wide loose tube, blacked within and extended from the pedestal up to the hole *k* under the object-plate; to exclude all other light from falling upon the metal, but that which descended from the object it self.

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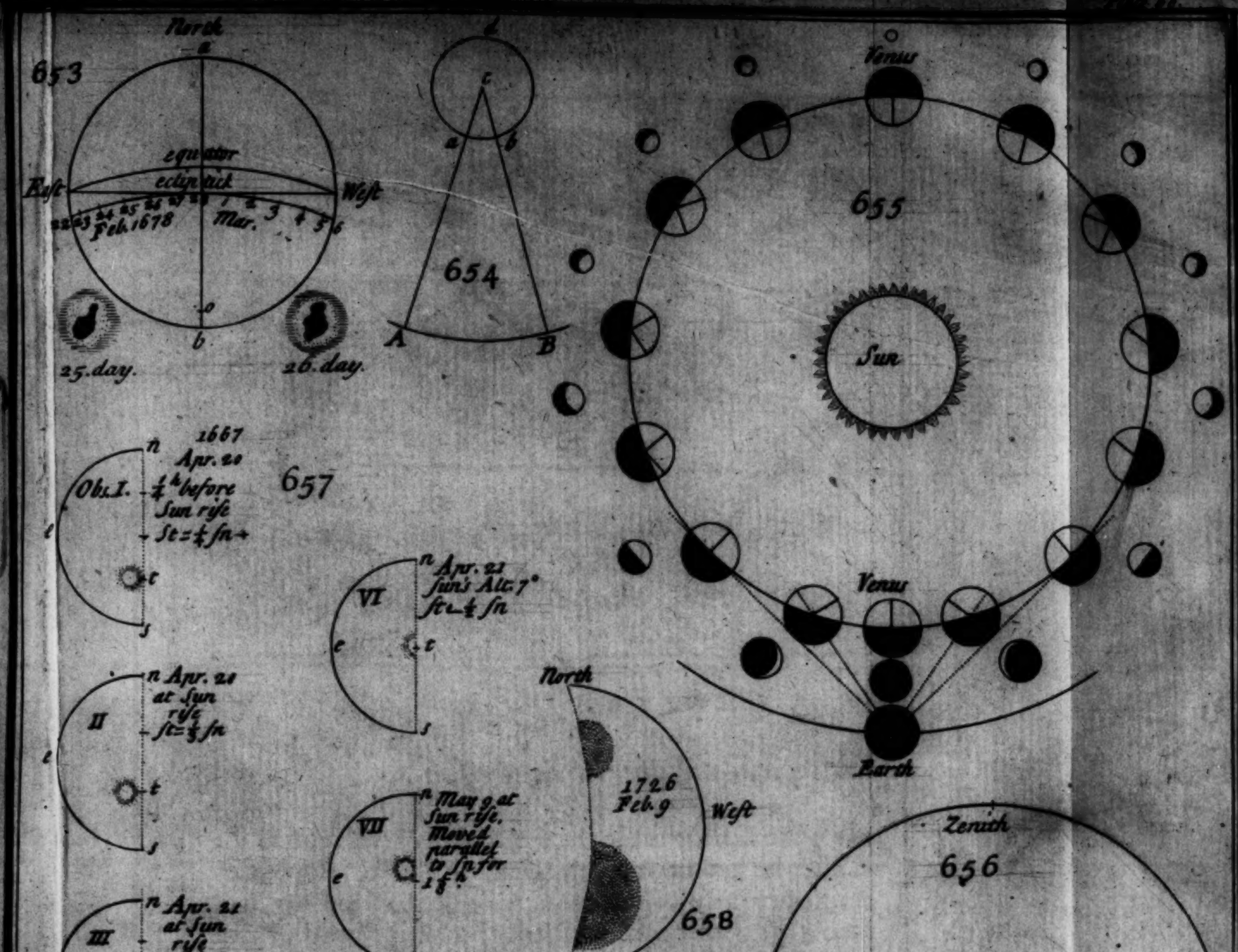
653
North
East
West
equator
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Feb. 1678
Mar.
25. day.
26. day.
654
A
B
655
Venus
Sun
Venus
Earth
656
Zenith
South
The Sun's Disk
North
Venus
Nadir
657
1667
Apr. 20
Obs. I.
¼ h before
Sun rise
st = ¼ sn +
II
Apr. 21
at Sun
rise
st = ⅓ sn
III
Apr. 21
at Sun
rise
st = ¼ sn
IV
Apr. 21
Sun's Alt.
4°
st = ⅖ sn
V
Apr. 21
Sun's Alt.
6°. 10'.
st = ½ sn +
VI
Apr. 21
Sun's Alt. 7°
st = ⅕ sn
VII
May 9 at
Sun rise.
Moved
parallel
to sn for
1 ⅝ h.
VIII
May 10 & 13
before
Sun rise
XI
June 5 & 6
before
Sun rise.
658
North
1726
Feb. 9
West
South
659
North
G
1726
Feb. 26
F
West
E
South
660
661

A
COMPLEAT SYSTEM
OF
OPTICKS.
BOOK IV.
A
Philosophical Treatise,
Containing the history of telescopical discoveries in the heavens.

CHAPTER I.

Telescopical discoveries in the sun.

1028. IN the year 1610 *Galileo* made the first discovery of spots in the sun, soon after he had finished his telescope; and in opposition to the received opinion of the unalterable state of the heavens, maintained, that these spots consisted of a kind of matter that suffered very great and sudden alterations; that they adhered to the sun, which carried them round an axis of his globe in about a month; that he took this axis at first to be perpendicular to the plane of the earth's annual orbit, because the courses of the spots appeared on the sun's disk as straight lines parallel to that plane; which then happened through a particular direction of the floating motions of the spots upon the sun's surface, as he soon collected from the visible alterations in their shapes and situations to one another when several of them appeared together; and thence he concluded, that the general course of the spots was frequently varied a little, in like manner as, to a person viewing the earth from a great distance, the clouds would appear to move parallel to one another and to the equator, with the uniform velocity of the earth's diurnal revolution, unless disturbed a little from that regular course by their floating in the atmosphere. *Galileo's discovery of spots in the sun.*

1029. At first, I say, he took the apparent motion of the sun's spots to be rectilinear and parallel to the ecliptick; but a year or two after, upon observ-

ing

ing an entire tranſit of a large ſpot, and by marking its place at noon, day after day, upon a circular paper anſwering to the ſun's diſk, he found its courſe was a little incurvated; and thence he concluded that the ſun's axis was a little inclined to the plane of the ecliptick.

1030. The fame of this diſcovery ſoon excited others to make the like obſervations, and occaſioned ſome diſputes upon the nature of theſe ſpots. Thoſe who contended for *Ariſtotle*'s doctrine of the unalterable ſtate of the heavens, maintained, they were either illuſions of the teleſcope, or, if real bodies, were a large ſyſtem of ſatellites, which like Mercury and Venus moved round the ſun, in ſuch narrow circles as even in their greateſt digreſſions from the ſun to be totally obſcured by the ſplendor of his rays; and therefore never viſible but as ſpots on his body at the times of their inferior conjunctions with it.

1031. In oppoſition to that hypotheſis *Galileo* alledged two phænomena; firſt, that many ſpots are obſerved to break out near the middle of the ſun's diſk, others to decay and vaniſh there or at ſome diſtance from the limb; which plainly ſhews them to be generated and diſſolved. For if without generation and diſſolution, they were brought into view by progreſſive motion only, their ingreſs and egreſs at the limb of the ſun would always be viſible.

1032. Secondly, from the changes of their apparent velocities and ſhapes, conſtantly regular, and exactly agreeable to the rules of opticks, he inferred their contiguity to the ſun's ſurface; ſeeing that immediately after their ingreſs, and towards their egreſs, their apparent motions upon the diſk were exceeding ſlow, and quickeſt at the middle of it; and that their ſhapes were oblong and ſlender near the limb of the diſk, but grew broader and rounder towards the middle of it; and that theſe variations of velocity and ſhape, when duly obſerved and computed, agreed exactly to the motions and poſitions of ſpots adhering to the ſun, but differed enormouſly from their motions in concentrick circles, even at very ſmall diſtances from the ſun's ſurface. Beſides, ſays he, were they ſpherical bodies like other planets and ſatellites, they would appear always round, even at the limb of the diſk, as well as the middle; it being peculiar to a ſpherical body, to appear always ſpherical. Therefore the apparent contraction of the ſpots towards the margin of the diſk, and dilatation towards the middle, plainly ſhews them to be a ſort of ſubſtance whoſe thickneſs is but ſmall in compariſon to its length and breadth, and conſequently ſomething that adheres to, or floats upon, the ſurface of the ſun.

1033. From what has been ſaid it appears that *Galileo* made a compleat diſcovery of all that we know at preſent of the nature and properties of theſe ſpots; what others have ſince added, is only an exacter determination of the poſition of the ſun's axis, and of the periodical time of his mo-

tion

tion round it, which without ſome viſible ſpots could never have been known to us.

1034. Conceiving a perpendicular *ab*, to the plane of the earth's annual orbit *efghiklm*, to paſs through the ſun's center *c*, it is collected from the apparent curvilinear courſes of the ſun's ſpots, that his axis *pco* is inclined to the perpendicular *acb* in an angle of $7\frac{1}{2}$ degrees; and that a plane paſſing through the axis *po* and perpendicular *ab*, cuts the ecliptick in the 8th degree of Piſces, on the ſide next the ſun's north pole *p*, and conſequently in the 8th of Virgo on the ſide next his ſouth pole *o*. *Scheiner* determined the angle *acp* to be 7 degrees, and *Caſſini* 8, but chuſes to take a medium of $7\frac{1}{2}$; in the reſt they both agree. The inclination of the sun's axis and equator to the ecliptick. Fig. 651.

1035. Conſequently the plane of the ſun's equator *æq* is inclined to the plane of the earth's annual orbit in an angle of $7\frac{1}{2}$ degrees, and being produced will cut the orbit in two oppoſite points *e*, *i*, at which when the earth arrives, the ſun's equator will appear to an eye in the plane of it, as a ſtraight line upon his diſk. Conſequently if the earth ſtood ſtill at *e* or *i* for about a fortnight, a ſpot upon the ſun's equator would appear to deſcribe a diameter of his diſk, inclined to the ecliptick in an angle of $7\frac{1}{2}$ degrees.

1036. But while the earth is moving in her orbit from *e* towards *g*, or from *i* towards *l*, the perpendicular diſtance of the ſpectator's eye from the plane of the ſun's equator produced, will increaſe continually; and conſequently the viſible projection of the ſun's equator upon his diſk (or any plane parallel to the diſk) will become elliptical and grow wider continually; and when the eye arrives at *g* or *l*, will be an ellipſis whoſe long axis is to its ſhort one, as the radius to the ſine of $7\frac{1}{2}$ degrees, or as 100 to 13; becauſe two lines drawn from *g* or *l* to *æ* and *q* will be nearly perpendicular to *ab*.

1037 The 8 figures below at *E*, *F*, *G*, *H*, *I*, *K*, *L*, *M*, repreſent the apparent ſhapes and poſitions of the ſun's equator ſeen upon his diſk *aæbq* from the places *e*, *f*, *g*, *h*, *i*, *k*, *l*, *m* of the earth's annual orbit, on the days noted at the ſides of theſe figures, and deduced from the ſun's apparent places in the 8th degree of ♊, ♍, ♐, ♓ abovementioned, and likewiſe on the intermediate days of the ſolſtices and equinoxes for the preſent age. Fig. 651.

1038. On theſe 8 days the meridian plane of any obſerver's place cuts the ſun's diſk at noon in the line *mn*, whoſe poſition and inclination to the ecliptick, and its axis *ab*, will be found on thoſe days, as here repreſented, by ſeeking the ſun's place in the ecliptick of a globe, by bringing it to the brazen meridian, and by obſerving the angle and poſition in which the meridian croſſes that part of the ecliptick. Apparent courſe of the ſun's ſpots at any given time.

1039. Therefore by turning towards the ſouth and holding the diameter *mn*, belonging to any one of thoſe eight days, in an upright poſture, that your eye may view it in your meridian, the line *æq* ſhews you the

apparent ſhape and poſition of the ſun's equator to his vertical diameter *m n* at the noon of that day; and by holding the diameter *m n* in the plane of any other hour circle, paſſing through the ſun's center, you have the poſition of his equator and parallels at any hour of thoſe 8 days: which is ſufficient to give an idea of the courſe of the ſun's ſpots, not only upon thoſe particular days but any intermediate day in the year.

How to obſerve the courſe of a ſpot. Fig 652.

1040. The agreement of this theory with the courſe of any ſpot may be practically examined, by obſerving it day after day with a teleſcope furniſhed with four fine wires in its focus, croſſing one another in equal angles of 45 degrees, and fixt in ſuch a poſition to the courſe of the ſun's diurnal motion, that his upper or under limb may touch any one of the wires as *op* and move along it *. At the inſtant of time when the ſun's antecedent limb arrives at the wire *oq*, perpendicular to *op*, and touches it in *r*, imagine his motion to be ſtopt till you draw his diameter *r s*, and from the ſpot *t* the line *t u x*, perpendicular to *o q*, cutting it in *u* and either of the oblique wires in *x*. Then by obſerving the number of ſeconds in which the ſpot *t* will be carried to *u* and to *x*, and the point *s* to *r* by the ſun's uniform motion, you have three numbers proportional to the lines *r s*, *t u*, *u x* or *u o*, whereby the poſition of the ſpot is determined with reſpect to the diameter *r s*, to which the diameter *m n* in this and the foregoing figures is perpendicular, as being a portion of an hour circle. And by repeating the like obſervation day after day, you may find as many points of the ſpot's courſe as you pleaſe; but be the obſervations ever ſo exact, thoſe points can never form an exact portion of an ellipſis or a ſtraight line; becauſe while the earth is moving in her orbit, the poſition of the obſerver's eye, to the circle deſcribed by the ſpot in turning about the ſun's axis, is continually changing. But if a correſponding number of points of the ſpot's courſe be determined upon the principles of the theory, they will agree very well with the poſitions of the obſerved points; of which many inſtances may be ſeen in the Memoirs of the Royal Academy of Sciences: I will tranſcribe only one.

* Art. 874.

An obſervation to confirm the foregoing theory. Recueil de Mem. de l'Acad. Tom. X. p. 601. Fig. 653.

1041. In the year 1678. Feb. 25. N. S. 8^{h}. a. m. *Caſſini* ſaw a ſpot in the ſun ſurrounded by the uſual nebuloſity, and advanced ſo far upon the diſk, that he judged it might have been ſeen three days before had the ſky been clear. It lay a little below the ecliptick upon the diſk, and at 8^{h}. 40^{m}. its diſtance from the ſun's eaſtern limb was $\frac{1}{6}$ of the diameter of the diſk. This very day the ſun's ſouthern pole arrived at its greateſt elevation *b o* of $7\frac{1}{2}$ degrees above the ſouthern limb of the diſk, which always happens when the ſun is in the 8th degree of Piſces. Conſequently the ſun's equator was alſo elevated $7\frac{1}{2}$ degrees above the ecliptick of the diſk, and the ſpot was then about as much below the equator of the diſk; by which he concluded it would paſs very near the center of the diſk in the morning of Feb. 28, as repreſented in the figure; which he drew beforehand.

hand. According to this prediction, on Feb. $28^d. 9^h. 30^m$. a. m. he saw the north side of the spot upon the center of the disk, and at the same hour of the subsequent mornings he saw the spot in the very places he had marked in his figure; which entirely confirmed the elements of his theory; namely, that the sun's axis was then inclined $7\frac{1}{2}$ degrees to the plane of his disk, that the spot adhered to the sun's surface, and made an apparent revolution to the center of the disk in $27\frac{1}{2}$ days.

1042. In the year 1639 *Scheiner* published his *Rosa Ursina* containing, he says, near 2000 observations of solar spots for 20 years together, in which time he frequently saw above 50 at once, but for 20 years after, betwixt 1650 and 1670, scarce any appeared *. He found that many of his spots scarce exceeded 25 days in making an apparent revolution from a given place on the disk to the same again, that others took up 27 days, and some others near 28. An apparent revolution results from a combination of three different motions; of the sun about his axis, which no doubt is uniform; of the earth about the sun, whose variations are regular and known; and of the spot upon the sun's surface, whose irregularities are unknown. Since the sun's motion about his axis, and the earth's about the sun, have the same direction, it follows that, about midsummer, when the earth moves slowest, the periodical time of a spot's revolution to a line connecting the centers of the sun and earth, is the least of all, *cæteris paribus*, and the greatest about midwinter when the earth moves quickest. But the difference in time thus arising, cannot exceed four hours, and therefore the greater part of the difference in the periods abovementioned must arise from the floating motion of the spots upon the sun's surface. *Scheiner* and *Cassini* the son* have also observed that the periodical times of the nearer spots to the equator are shorter than those of the remoter; and *Cassini* the father observed the like property in Jupiter's spots *.

Periodical time of a spot's apparent revolution.

* Phil. Trans. N°. 75.

* Mem. de l' Acad. 1703 p. 142. 8vo.

* Mem. de l' Acad. 1692

1043. These irregularities seem to create great difficulties in determining the time of the sun's exact revolution about his axis; nevertheless *Cassini* the father, observing that several large spots had often appeared and disappeared in the very same parallel to the sun's equator, imagined that some particular place or places in it might be disposed, at different times, to supply the matter of these spots; and if so, that they seldom went far from the place of their origin: just as the fumes of mount *Ætna*, if seen from the sun, at the times of their greatest eruption, would appear as a spot on the earth's disk, returning constantly to the same situation after every 24 hours very nearly; sometimes a little sooner, at other times a little later, according as the wind might happen to drive the fumes eastward or westward from the mountain. He then considered that if this comparison were applicable to the sun, the apparent periodical time

of

of a ſpot, ſo nearly fixt to a certain place, would be nearly the ſame as that of the ſun it ſelf, and be a common meaſure of ſeveral large intervals of time betwixt the ſpot's appearances in the ſame apparent place of its parallel. Accordingly by comparing many diſtant obſervations together, he found a common meaſure of many ſuch intervals of time to be $27^{d}. 12^{h}. 20^{m}$. very nearly *.

* Rec. de l' Acad. Tom. X. p. 727.

1044. For example, betwixt an obſervation of his own on May 14. N. S. 1684., and another on Apr. 29. 1686, there paſſed 715 days, that is 26 times $27\frac{1}{2}$ days; again by dividing the days betwixt the former of thoſe obſervations and a later on May 13. 1688, by 54 revolutions, the quotient gave him $27^{d}. 12^{h}. 40^{m}$. for a mean revolution; and a mean betwixt this and the former is $27^{d}. 12^{h}. 20^{m}$; although he had obſerved that a ſingle revolution of ſome of thoſe ſpots was performed in $27\frac{2}{3}$ days, of ſome others in $27\frac{1}{3}$, and of others in $27\frac{1}{2}$.

1045. Laſtly by turning over the earlieſt obſervations made by *Scheiner* and *Hevelius*, for ſpots ſeen alſo in the month of May and in the ſame parallel with thoſe he had conſidered, he found one in 1625 and another in 1644, which he compared with the three abovementioned, and ſettled the limits of ſix grand intervals of time, whoſe common meaſure was about $27^{d}. 12^{h}. 20^{m}$, within 4 or 5 minutes at moſt. This period may therefore be attributed to the ſun it ſelf rather than any other, as being of a middling length among many ſingle periods of various ſpots, and deduced from obſervations all made in the ſame month of May or near it, and conſequently leſs affected by the inequalities of the earth's annual motion.

The time of ſun's revolution about his axis. Fig. 654.

1046. Hence the periodical time of the ſun's revolution to a fixt ſtar is $25^{d}. 15^{h}. 16^{m}$. For in $27^{d}. 12^{h}. 20^{m}$. of the month of May, the earth deſcribes an angle *AcB* or *acb*, about the ſun's center *c*, of $26^{o}. 22'$, and as $360^{o} + 26^{o}. 22'$, to 360^{o}, ſo is $27^{d}. 12^{h}. 20^{m}$, the time of the ſun's apparent revolution to the moveable line *cbB*, to the time of his abſolute revolution to the fixt line *caA*, which therefore is $25^{d}. 15^{h}. 16^{m}$.

Fig. 653.

1047. All ſpots conſiſt of a black part in the middle, of ſome irregular figure, encompaſſed with a nebulous border of a colour leſs dark; and it often happens, after a gradual decay and diſappearing of the black part, that its place ſeems brighter than the reſt of the ſun, and continues ſo for two or three days. Theſe brighter ſpots are called *faculæ*.

Abſolute magnitude of ſpots.

1048. The magnitude of the ſurface of a ſpot may be eſtimated by the time of its tranſit over a hair in a fixt teleſcope *Galileo* reckons ſome ſpots to be larger than all *Aſia* and *Africa* put together. But had he known the ſun's parallax and diſtance as exactly as we do, he would have found them much larger than the whole ſurface of the earth. For in 1612 he obſerved a ſpot ſo large as to be plainly viſible to the naked eye; it therefore ſubtended an angle at the eye of about a minute *. The diameter of the earth, if removed to the ſun, would ſubtend an angle of but 20 ſeconds.

* Art. 97.

conds. Therefore the diameter of the ſpot, was to the diameter of the earth as 60 to 20 or 3 to 1, and conſequently the ſurface of the ſpot, if circular, to a great circle of the earth, as 9 to 1, and to the whole ſurface of the earth as 9 to 4 or $2\frac{1}{4}$ to 1. *Gaſſendus* obſerved a ſpot whoſe diameter was $\frac{1}{20}$ of the ſun's, and therefore ſubtended an angle at the eye of above a minute and a half. Its ſurface was therefore above five times larger than the whole ſurface of the earth. He tells us he ſaw above 40 ſpots at once, but did not perceive that the light of the ſun was ſenſibly diminiſhed; nevertheleſs the paleneſs of the ſun mentioned by hiſtorians, as after the death of *Julius Cæſar*, might poſſibly be cauſed in this manner, provided we admit the fact, and do not regard it as an invention of flattery.

1049. From the perpetual emiſſion of the ſun's light, how minute and delicate ſoever, it is reaſonable to conclude that his body muſt waſte, and therefore could not have exiſted from eternity, as a luminous body, without a ſupply of new matter. Some ſupplies may come now and then, as Sir *Iſaac Newton* imagines, from the falling of comets into the ſun. He computes that the comet in 1680 approached towards the ſun's ſurface within leſs than a ſixth part of the ſun's diameter; and, by moving with an immenſe velocity in that nearneſs, he concludes that it muſt have been retarded by the reſiſtence of the ſun's atmoſphere, and conſequently muſt approach nearer and nearer after every revolution, till at laſt it falls into the ſun. It muſt be confeſſed that theſe ſupplies can happen but very rarely, and that the bodies of comets are very ſmall in compariſon to the vaſtneſs of the ſun; nevertheleſs, conſidering the inconceiveable minuteneſs of the particles of light, though continually flying from the ſun, a few ſuch ſupplies may abundantly ſuffice for keeping up his bulk for innumerable ages, without ſenſible diminution to the inhabitants of the ſyſtem. The moſt ancient obſervations of the ſun's diameter compared with the preſent, are quite inſufficient for determining whether it be diminiſhed or not.

Sun's decreaſe yet imperceptible.

CHAPTER II.

Teleſcopical diſcoveries in Mercury and Venus.

1050. WHEN *Copernicus* revived the ancient Pythagorick Syſtem, aſſerting that the earth and planets moved round the ſun in the center of their orbits, the Ptolemaicks objected, if this were true, that the phaſes of venus ſhould reſemble thoſe of the moon. *Copernicus* replied, that ſome time or other that reſemblance would be found out. This prediction was fulfilled by *Galileo*, who ſent this firſt diſcovery of the phaſes of venus in a letter, written from *Florence* in 1611, to *William*

Phaſes of Venus.

liam de' Medici, the Duke of *Tuſcany*'s Ambaſſador then reſiding at *Prague*, deſiring him to communicate it to *Kepler*, firſt mathematician to the Emperor *Rudolphus* II. The letter is extant in the preface to *Kepler*'s Dioptricks, and in engliſh is this. "Being undoubtedly certain of the truth of the diſcovery concealed in the cypher ſent you ſome weeks ago, it is now time to explain it to your Excellency, and by you to Sig^r *Kepler*. Know then, that about three weeks ago, when venus became viſible in the evenings, I began to obſerve her very diligently with my teleſcope, hoping to ſee with my eyes, what my underſtanding did not at all doubt
Fig. 655. of. At firſt then I ſaw venus perfectly round, neat and diſtinctly terminated, but very ſmall; which figure ſhe retained till ſhe approached nearer to her greateſt digreſſion from the ſun, increaſing continually in apparent bulk. From that time her figure began to fail of its roundneſs on its eaſtern ſide which lay from the ſun, and in a few days was reduced to a perfect ſemicircle; and continued ſo without the leaſt alteration, till ſhe left the tangent to her orbit, and began to return towards the ſun. At preſent the ſemicircle becomes more and more hollow every day, its angles being changed into horns, which will grow ſharper and ſharper, till they become ſo thin as to vaniſh at her occultation in the beams of the ſun. Hence paſſing by to her morning appearance, we ſhall ſee her fine horns again, pointing from the ſun, and growing fuller and fuller up to the angles of a ſemicircle, at the other greateſt digreſſion from the ſun, which ſhape will continue many days without ſenſible change. Hence forward the ſemicircle will ſwell out gradually till it be almoſt compleatly round, and will keep ſo for ſome months. At preſent the apparent diameter of venus is about five times greater than it was at her firſt evening appearance. By theſe obſervations of this admirable appearance, we have the moſt certain, ſenſible deciſion and demonſtration of two grand queſtions, which to this very day have been doubtful and diſputed among the greateſt maſters of reaſon in the world. One is, that the planets in their own nature are opake bodies, attributing to mercury what we have ſeen in venus. The other is, that venus neceſſarily moves round the ſun, as alſo mercury and the other planets; a thing well believed indeed by *Pythagoras*, *Copernicus*, *Kepler* and my ſelf, but never yet proved, as now, by ocular inſpection upon venus. *Kepler* and the Copernicans may therefore glory with good reaſon, in having philoſophiſed ſo well as not to ſeem vainly credulous, though hitherto looked upon as fools and bigots by all thoſe that ſeek philoſophy in books only. As to the cypher, ſent you in letters tranſpoſed into theſe words, *Hæc immatura a me fruſtra leguntur*, *o*, *y*, the ſame put in order run thus, *Cynthiæ figuras æmulatur mater amorum*, that is, venus imitates the phaſes of the moon. I am &c.

Galileo Galilei.

Florence 1. *Jan.* 1611.

1051. *Galileo*'s telescope scarce magnified enough to discover the like phases of Mercury, whose excessive brightness was another impediment. But from this and his shorter digressions from the sun, than those of Venus, he justly concluded, that the orbit of Mercury lay within that of Venus. Mars he plainly perceived was sometimes gibbous, or deficient from perfect roundness. Jupiter appeared to him always round, and Saturn too, which he took to consist of three contiguous globes, as explained hereafter. From these discoveries of the phases of the planets and their degrees of brightness in proportion to their nearness to the sun, he soon composed the very first, and still the best, as being the shortest and plainest demonstration of the true system of the universe; which till then had been always dubious and disputed; followed indeed by the best judges, but for no other reason than that it afforded more simple and easy solutions of phænomena, than could be deduced from *Ptolemy*'s hypothesis; which tho' sufficient to determine their choice, was still too far from a satisfactory proof of the real order of the heavenly bodies. Phases of the other planets.

1052. M. *de la Hire* was the first person that discovered and observed the transits of fixt stars and planets over the meridian at noon day*. This he did through the moveable telescope belonging to the mural quadrant at the Royal Observatory at *Paris*, and says, that he never failed of seeing the transit of venus, though within two degrees of the sun, to the right or left, upwards or downwards; that in 1700 Aug. $27^d. 0^h. 24^m$, he observed her transit with a 16 foot tube, which magnified her diameter (of one minute) about 90 times, and consequently shewed it three times larger than that of the moon viewed by the naked eye; that she then appeared in the form of a fine slender crescent, with her horns in an horizontal line and her back upwards; that in the interior arch of the crescent he saw some inequalities more considerable than those of the new moon; that he had observed the like at other times, by which he judged she might have spots upon her, visible at other times, like those of the other planets; that the splendor of the atmosphere, so near the neighbouring sun, took off so much from the excessive lustre of venus, as to render her appearance much neater, better defined and freer from colours than at any other time and place; and that the eye-glass of the telescope need not then be smoaked. Lastly, that in Nov. 1691 he saw her at noon very near her superior conjunction with the sun, appearing round and very small*. What has been said of venus is applicable to mercury; whose phases were first discovered by *Martinus Hortensius*. Venus and mercury visible at noon day.

* Mem. de l' Acad. 1700. p. 288. 4to.

* Mem de Math. & Phy. Tom. X. p. 20.

1053. Nothing is more commodious for settling the motions of mercury and venus than their appearances upon the sun's disk, a delightful sight to astronomers, and long wished for in vain before the invention of telescopes. The writer of the life of *Charles* the great relates, that mercury was seen in the sun for 8 days together, 17. Mar. A.D. 807*. but this Venus and mercury visible in the sun's disk.

* Du Ham. Hist. R. Ac. Sci. p. 469.

this no doubt was some large spot in the sun; since mercury is far too small to be seen there with the naked eye. *Kepler*, before he finished his tables, had placed the nodes of mercury's orbit at the beginning of the signs of gemini and sagittary, and thence predicted his appearance in the sun, 20 May 1607; on which day he observed the sun very diligently, viewing his image cast upon a paper in a dark room through a small hole in the window-shut. Then also there happened to be a large spot in the sun, which *Kepler* took for mercury. It was lucky for him that telescopes were invented soon after, which presently convinced him, that what he took for mercury was only a solar spot. Upon this he rectified his tables, and by placing the nodes of mercury about the middle of taurus and scorpio, he predicted, that his first appearance in the sun would be on the 7th Novem. 1631.

1054. *Gassendus* was then at *Paris* and made due preparation to observe it, in the manner he had used to observe spots upon the sun, by receiving its rays through a telescope a little lengthened, in order to collect them into an image cast upon a white paper *; and in watching for mercury he fell into the contrary mistake to that of *Kepler*, taking mercury himself for a solar spot, just sprung from the sun, having found it quite free from them the day before. However he took the position of the spot once or twice, intending to compare it with that of mercury, which he thought would appear much larger. But in taking its third position, and observing it moved quicker than spots usually did, he began to suspect it might be mercury himself; and being farther assured of it by his fourth observation, he prepared to observe its egress from the sun's disk, which happened Novemb. 7^d. 10^h. 20^m. *mane Sty. N.* at the distance of $33\frac{1}{2}$ degrees from the sun's vertical point. This was the first time that mercury was ever seen in the sun, and the only observation then made of that transit. Several subsequent observations of the like transits are collected in *du Hamel*'s Hist. of the Royal Acad. of Sci. p. 470. edit. 2d, from which I extracted that of *Gassendus*.

* Rem. 128.

1055. In the year 1691 Dr. *Halley* published* a most accurate theory for finding the visible conjunctions of mercury and venus with the sun, together with a specimen of it, in tables of all the times of their appearance in the sun's disk for two centuries, beginning from the time of the invention of telescopes; which tables agree with the latest and best observations to a wonderful degree of exactness. For the observed time of the conjunction of mercury on the 29th of Octob. 1723, was but $4\frac{1}{2}$ minutes sooner than the prediction, and the latitude of mercury but six seconds more southerly; the error in longitude being little more than two diameters of this exceeding small planet, and in latitude scarce a single diameter: so that for the future, astronomers may trust that table of transits to a few minutes, as its author has justly observed *, and not wait with

* Phil. Trans. N. 193.

* Phil. Trans. N. 386.

with the uncertainty of hours, nay days, as had been lately done. The diameter of mercury was then obſerved with an excellent micrometer applied to the Hugenian teleſcope of 123 feet, and found to be 10″¼.

1056. A ſight of venus in the ſun was wiſhed for by aſtronomers with more impatience than that of mercury, as being the propereſt means for adjuſting the ſun's diſtance from us, and ſome other important points in aſtronomy. But *Kepler*, having declared in his Optical Aſtronomy, publiſhed in 1604, and alſo in his epitome of the Copernican Aſtronomy, in 1621, that no ſuch thing could happen in that century, nor the next till the year 1761, all thoughts of it were quite laid aſide. Neverthele ſs upon correcting his tables in the year 1630, he altered his opinion and concluded that venus would be ſeen in the ſun in the year following, and accordingly gave publick notice of it in the ſame advertiſement, wherein he predicted that tranſit of mercury which *Gaſſendus* obſerved. But as to venus, which was impatiently watched by *Gaſſendus* and others, the event did not anſwer.

1057. Notwithſtanding theſe diſappointments, which occaſioned a general deſpair of obtaining a ſight of this rare phænomenon, yet it was accidentally predicted and ſoon after obſerved by our countryman *Jeremias Horrockes*, a youth of an admirable genius, at Hool a ſmall village in Lancaſhire about 15 miles northwards from Liverpool. In the treatiſe he wrote upon this ſubject, (which was publiſhed by *Hevelius* at the end of his *Mercurius in ſole viſus)* he tells us, that meeting with *Lanſbergius*'s tables and finding them much extolled by their author, he thence computed ſeveral diaries of the planetary aſpects, in order to examine their agreement with the heavens, and correct them by his own obſervations; which he did accordingly. Among other aſpects he computed the conjunction of venus with the ſun, by theſe and alſo by *Kepler*'s tables, and found they agreed well enough in the ſame concluſion. This was contrary to *Kepler*'s computation from his own tables, as I ſaid above, but *Horrockes* had corrected them by ſome of his own obſervations. At the approach of the deſired time for obſerving this conjunction, through a teleſcope he projected the ſun's image upon a white paper in a dark room, and after waiting many hours with great impatience, at laſt on the 24th of Novemb. 1639 at 3 hours and ¼ paſt 12, O. S. juſt after he came from church, he had the pleaſure to find a large round ſpot near the limb of the ſun's image, as repreſented at number 1. He made three obſervations of its poſitions 1, 2, 3, and as many of its magnitude, and found its diameter was $\frac{1}{25}$ of the ſun's at moſt; the ſpot was perfectly round and blacker than ſolar ſpots, from which it differed alſo in the quickneſs of its motion, along the ſpace 1, 3, which it deſcribed in half an hour. He was deprived of a longer ſight of it by the ſetting of the ſun. According to his draught of this appearance, the interior contact of venus with the Fig. 656.

 ſun's

ſun's limb is about 61 degrees from its loweſt point. He had given notice of this conjunction to his friend *Will. Crabtree* at Mancheſter, who juſt got a ſight of it before ſun-ſet, and obſerved the diameter of venus to be $\frac{7}{200}$ of the ſun's. Theſe two were the firſt and only perſons that ever ſaw venus in the ſun. Whoever peruſes the little treatiſe abovementioned, from which I have taken this account, muſt needs admire the ſpirit and genius of this young aſtronomer, who died at the age of 22 in 1641. His theory of the moon and other poſthumous works were publiſhed in 4to by Dr. *Wallis* in 1673.

1058. By Dr. *Halley*'s table of the viſible conjunctions of venus in the ſun, it appears that *Horrockes* ſaw the laſt that can happen till the year 1761, when on May the 26th, near ſix in the morning at *London* venus will appear in the ſun's diſk not above four minutes ſouth of its center. This tranſit will continue almoſt 8 houres, from 2 in the morning till almoſt 10, and therefore the ingreſs into the ſun's diſk cannot be ſeen in any part of England. Dr. *Halley* farther aſſures us, from his own obſervation of mercury in the iſland of St. *Helena*, that the beginning and end of the tranſits of theſe planets over the ſun's diſk, may be obſerved to the exactneſs of one ſecond of time. I had the good luck, ſays he, to ſee mercury as he was entering the ſun's limb and made a black notch in it, and was certain of the inſtant of his total ingreſs, by a fine thread of the ſun's light, which immediately broke out from the interior contact of the two limbs and ſtruck my eye inſtantaneouſly; and at the beginning of the egreſs of mercury, the like thread of light was broken and vaniſhed inſtantaneouſly at the other interior contact. If the like obſervations be duly made upon the enſuing tranſit of venus, with good clocks and teleſcopes, in ſeveral diſtant countries, he ſhews in what manner we may thence determine the ſun's parallax and diſtance from the earth, to the exactneſs of a 500th part of the whole *; whereas by the beſt obſervations hitherto made, we are not abſolutely certain of thoſe quantities to leſs than about a ſeventh part of them.

* Phil. Tranſ. N. 348.

Revolution of venus about an axis according to *Caſſini*.

1059. In the year 1666 Signor *Caſſini*, then profeſſor of aſtronomy at Bologna, diſcoverd a bright ſpot in venus, not unlike ſome of thoſe that a teleſcope conſtantly ſhews us in the moon near the full; and made the following obſervations upon it in the year 1667*. The ſeveral ſemicircles *nes* repreſent the diſk of venus, as appearing about half enlightened, at her greateſt weſtern elongation from the ſun; *n* is the northern, *e* the eaſtern and *s* the ſouthern ſide of it; and *t* the places of the bright ſpot at the times of obſervation noted at the ſide of each ſemicircle. Suppoſing he conſtantly ſaw one and the ſame ſpot, adhering to the ſame part of venus, by a compariſon of theſe obſervations one with another, he concluded with ſome diffidence, that venus revolved about an axis in leſs than a day; at leaſt that the ſpot finiſhed its period either of revolution or libration

* Mem. de l'Acad. 1732 p. 197. 4to. Fig. 657.

on in that time, so, as in 23 days to return nearly to the same place in the disk at the same time of the first and last of those days. For by reason of the vicinity of venus to the sun and horizon, he could not observe her long enough to see a continued motion of the spot through a large space; and therefore durst not pronounce whether its motion were of the revolving or librating kind.

1060. To give one deduction of the spot's periodical time, on Apr. 20 the sun rose at *Bologna* at $5^h.17^m$, when, by the second observation, *st* was equal to $\frac{1}{3}$ of the diameter *sn*; and on Apr. 21 the sun rose at $5^h.15^m$, and was $6°.10'$ high at $5^h.51^m$, when, by the fifth observation, *st* was equal to $\frac{1}{2}$ *sn*. Therefore in the interval of $24^h.34^m$, supposing a continued progressive motion of the spot, it described an arch of about 20 degrees, answering to $\frac{1}{6}$ of the diameter *sn*, or else had finished one or more revolutions besides that arch. But without finishing one revolution its motion could not have appeared so evidently as it did between the third and sixth observations. Consequently as $360° + 20° : 360° :: 24^h.34^m : 23^h.16^m$ the periodical time required. These two observations are fitter for this purpose than the rest, because the motion of the spot, then nearest to the center, was apparently quickest. Fig. 657.

1061. In the 7th observation, May 9, he saw the spot move exactly parallel to the diameter *sn*, from the south towards the north, for above an hour; in the foregoing observations its course deviated a little from the north towards the west, as appears by the figures. Of this direction of motion there is no other instance in the heavens, except in the moon's libration. He likewise perceived some obscure spots upon the disk, but their motions were too irregular to conclude any thing from them.

Revolution of venus according to *Blanchini*.

1062. We hear of no other observations of this kind till the year 1726, when Sig^r. *Blanchini*, the Pope's domestick prelate, observing venus at Rome with *Campani*'s glasses, whose focal distances were from 70 to 100 Roman palmes, discovered several dark spots in her disk, as represented in the 1st, 2d and 3d plates of his book called *Hesperi & Phosphori nova Phænomena*.

1063. On Feb. 9. he saw two spots bounded on one side by the section that separates the light and dark parts of the disk. Both were shaped like a segment of a circle, and that which lay towards the northern horn of the disk was much smaller than the other, that lay towards the southern. Fig. 658.

1064. On Feb. 14, the large spot was vanished, but he thought he reconnoitred the small one, now advanced towards the southern horn, and followed by two new ones lying towards the northern horn.

1065. On Feb. 16, he believed he saw these spots again, tho' somewhat altered in shape and carried southwards.

1066. On Feb. 18, the southermost spot was quite gone; the other two

two were advanced towards the ſouth, and a new one appeared in the north.

1067. On Feb. 20, the ſpots laſt ſeen were all advanced ſtill more ſouthwards.

1068. On Feb. 24, the two ſouthermoſt did not appear; that in the north was carried ſouthwards, and was followed by two new ones in the north. On the 26th theſe three appeared a little more ſouthward.

1069. Laſtly on March 5, 24 days after the firſt obſervation, he believed he ſaw thoſe two ſpots, which he firſt diſcovered on Feb. 9, in the ſame ſituation they then had. They appeared narrower, as they ought, becauſe the phaſe of venus was now become ſharper; and of conſequence the dark part of the diſk had now encroached upon a larger portion of them.

1070. Upon theſe obſervations and many more, made in the following months of May and June 1726, of July, Auguſt and September 1727, and of Jan. 7. 1728, *Blanchini* concludes, that a revolution of venus about her axis was not finiſhed in 23 hours, as *Caſſini* imagined, but in $24\frac{1}{3}$ days; that the north pole of this revolution faced the 20th degree of aquarius, and was elevated 15 degrees above the plane of the ecliptick and that the axis kept parallel to it ſelf, during its revolution about the ſun.

Situation of the axis of venus.

1071. The motion of theſe ſpots was from the northern towards the ſouthern ſide of the viſible diſk, quite contrary to that of *Caſſini*'s ſpot; but this is a neceſſary conſequence of the different ſituation of the axis of venus to the eyes of the two obſervers. The only point to be adjuſted is their different determinations of the periodical time of the ſpots.

Fig. 659.

1072. *Blanchini*, to ſupport his opinion, alledges an obſervation made in the evening of Feb. 26. 1726, with *Campani*'s teleſcope of 88 palmes, in the preſence of ſeveral perſons, who agreed with him in the appearances of the ſpots. He obſerved them near an hour from $5^h. 25^m$ to $6^h. 15^m$, when venus being got behind the Barbarine palace, obliged him to deſiſt. About three hours after the middle of the foregoing obſervation, *i. e.* at $8^h. 40^m$, getting ſight of her again and obſerving her till 9, he found the ſpots nearly in the ſame ſituation in which they appeared at $5^h. 45^m$, as was evident by comparing a draught of the ſpots then taken, with their preſent appearance.

1073. Now, ſays he, according to *Caſſini*'s period of 23 hours, in the three hours interval between the two obſervations, the ſpots would have finiſhed above an eighth part of a revolution or above 45 degrees of their parallel circles; and thus the ſpot which at $5^h. 25^m$ poſſeſſed the middle of the diſk, at $8^h. 45^m$ would have been carried about 50 degrees from it towards the ſouthern horn, and have appeared beyond the place of the ſouthern ſpot, which then would have been nearly out of the diſk; and the

the northern ſpot would have nearly ſucceeded to the place of the middlemoſt; inſomuch that of the three ſpots, which at $5^h. 45^m$ were equally diſtributed over the diſk, two would have been found on its ſouthern ſide and not one on the northern. But all the gentlemen that obſerved with me from $8\frac{1}{2}$ till 9, ſaw plainly that the large ſpot appeared about the middle of the phaſe, and found that the ſame opening of the micrometer nearly meaſured the interval between the top of the ſouthern ſpot and the upper horn, and alſo between the top of the northern ſpot and the under horn, juſt as it did before at $5\frac{1}{2}^h$. It muſt needs be acknowledged then, ſays *Blanchini*, that in thoſe three hours the ſpots did not advance above 2 degrees, the ſpace due to the periodical time of 24 days; and that this ſmall advance did not make a perceptible change in their ſituations.

1074. This is Sigr. *Blanchini*'s concluſion, which M. *Caſſini* the ſon after due conſideration of the obſervations cannot acquieſce in*. I admit, ſays he, M. *Blanchini*'s obſervations, without the leaſt ſcruple, as himſelf has deſcribed them; namely, that at $5^h. 45^m$ they ſaw three ſpots as repreſented at *E*, *F*, *G* in the figure for Eeb. 26; and that about three hours after they likewiſe ſaw three ſpots nearly in the ſame ſhapes and ſituations as before. But it muſt be conſidered that the courſe of obſerving was interrupted from $6^h. 15^m$ to $8^h. 40^m$ by the interpoſition of the Barbarine palace, in which interval, of almoſt $2\frac{1}{2}$ hours, they could not ſee the ſpots nor the motion they might have; which, in three hours time*, might carry them through 47 degrees; ſo that the ſouthern ſpot *E*, in approaching to the ſouthern horn, might at laſt go out of the diſk, while the ſpot *F* might move from the center into the place of *E*, and likewiſe the northern ſpot *G* into the place of *F* at the center, and a new ſpot might ſucceed to the place of *G*; ſo that at $8^h. 45^m$ three ſimilar ſpots might appear in the ſame places as before at $5^h. 45^m$. For by the rules of perſpective, the ſpot *F* being tranſlated to *E*, from the center towards the limb, would become ſmaller in appearance, like that whoſe place it had taken; and on the contrary the ſpot *G*, in moving from the limb towards the center, would grow larger in appearance; and all this will be farther evident by examining the figure; where drawing lines, from the middle of the ſpots, perpendicular to a line that connects the horns, their intervals will anſwer to arches of about 45 degrees. Therefore ſuppoſing the periodical time of the ſpots to be about 23 hours, the middle of the ſouthern ſpot ought to be gone out of the diſk in three hours time, while the ſpot *F* ſucceeded to the place of *E*, and *G* to that of *F*. As to the new ſpot, ſuppoſed to ſucceed to the place of *G*, there is reaſon enough to believe it, upon conſidering M. *Blanchini*'s figures, where we ſee that in ſeveral ſucceſſive days, ſeveral ſpots ſucceeded one another; and in particular that the ſpot there called *A* ought to appear upon the diſk next after *G*, tho'

The obſervations reconciled by *Caſſini* the ſon.

* Mem. de l'Acad. 1732 p. 205. 4to.

Fig. 659.

* Art. 1073.

at a little greater diſtance than this period requires. Therefore the obſervation on Feb. 26. 1726, is not deciſive, as the author imagined, againſt the period of 23 hours.

1075. After a more critical compariſon of his father's obſervations with *Blanchini*'s, M. *Caſſini* concludes at laſt, that if we ſuppoſe the periodical time of the revolution of venus to be $23^h. 20^m$, it agrees equally with both their obſervations; but if in $24^d. 8^h$, as *Blanchini* would have it, we muſt entirely reject his father's obſervations as of no conſequence at all. He adds that M. *Maraldi* and he made a very great number of obſervations upon venus in the moſt favourable days in the year 1729, with tubes of 82 and 114 feet, but could never perceive the leaſt ſpot; ſo that *Blanchini*'s ſpots had either diſappeared or elſe the air at Paris was not ſo clear as at Rome; which might probably be the reaſon why *Caſſini* the father could never perceive any ſpots in her at Paris, even with the ſame teleſcopes that he had uſed in Italy.

CHAPTER III.

Teleſcopical diſcoveries in the Moon.

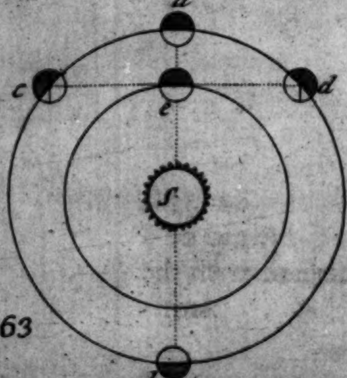
Moon's ſurface mountanous.

1076. GALILEO's teleſcope though but ſmall in compariſon to ours, was yet ſufficient to give him a juſt idea of the moon's ſurface. He immediately concluded it was not ſmooth like a ſpeculum, as many philoſophers had imagined, but rough like the earth, and diſtinguiſhed with innumerable mountains, caves and valleys. This he collected from the following appearances; that in the new moon the line which connects her horns and paſſes between the bright and dark portions of the viſible hemiſphere, was not an even, regular curve, as it ſhould be upon a ſmooth, ſpherical ſurface, but a line compoſed irregularly of many crooked turnings and windings as repreſented in the figure; that many ſmall bright ſpots appeared in the dark portion, ſtanding out at ſeveral ſmall diſtances from that common boundary of the bright and dark portions; that in a few hours they grew ſenſibly larger and approached nearer to, and at laſt united with, the bright portion; juſt as the rays of the riſing ſun ſhine firſt upon the tops of our high mountains, then deſcend gradually to their baſes, and at laſt into the valleys. On the other hand he obſerved many ſmall ſpots interſperſed all over the bright portion of the diſk, ſome of which had their dark ſides next the ſun, and their oppoſite ſides very bright and circular, which plainly ſhewed them to be round cavities, whoſe ſhadows fell within them; and that ſome of theſe were ſurrounded with ridges of mountains. Laſtly that thoſe larger and leſs luminous tracts, that are viſible to the naked eye, appeared ſmoother in the teleſcope and more depreſſed than the ambient brighter regions;

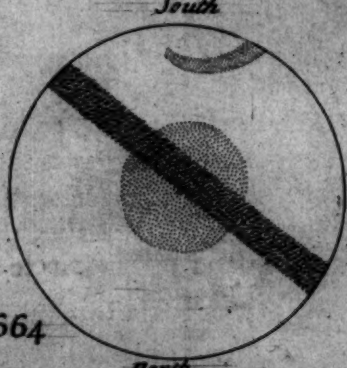
Galileo's nuncius ſidereus.

Fig. 660.

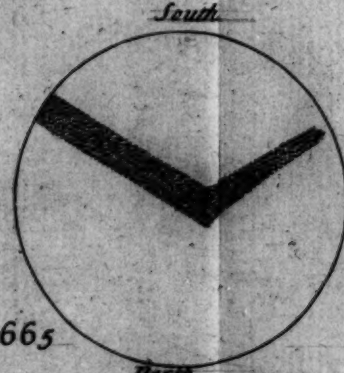
662.



663



664



665

regions; as was evident by a greater regularity and evenneſs of that part of the boundary of light and ſhade which paſſed through them at certain times, and by its protuberances at both its extremities. Nevertheleſs theſe tracts were not quite free from ſmaller inequalities, eſpecially of light and ſhade.

1077. Theſe darker regions may properly enough be compared to the receptacles of our ſeas, emptied of their water. For that they contain none is evident from thoſe permanent bright ſpots obſerved in them by *Galileo*, and becauſe larger teleſcopes plainly diſcover not only ſmall eminencies but cavities within them; which are quite repugnant to the nature of ſeas. The obſcurity of their colour may proceed from a kind of ſoil that reflects leſs light than the other regions do. No ſeas in the moon.

1078. The ſurface of the moon being ſo mountainous and irregular as has been deſcribed, *Galileo* conſidered how it came to paſs that the bright circular limb of the diſk did not appear rugged and irregular, as well as the oval boundary between the light and dark parts of it, and explained it in this manner. If the ſurface of the moon had but one row of mountains placed round the limb of the diſk, the bright part of it would appear irregularly indented; but ſince the moon's ſurface is all over mountainous, and ſince the viſible limb muſt be conſidered as a large zone, poſſeſſed by many rows of mountains lying behind one another with reſpect to the obſerver's eye, the mountains in ſome rows, being oppoſite to the valleys in others, will fill up the inequalities in the viſible limb; eſpecially being all ſo remote and uniformly enlightened, that the inequalities of their diſtances from the eye cannot be perceived: juſt as a great number of mountains upon the earth, viewed from a great diſtance, or the waves in the ſea, how great ſoever, do yet compoſe an horizon apparently ſmooth and circular. Why the moon's limb does not appear mountainous.

1079. *Galileo* next conſidered the magnitude of the moon's mountains and found them much higher than any of the earth's. He told us before, that when only the tops of them are enlightened by the ſun, they appeared as bright ſpecks in the dark part of the diſk; and he found the apparent diſtance of ſeveral ſuch ſpecks, from the limit between the dark and bright parts of the new moon, was above a 20th part of her apparent diameter; and thence he concluded that the perpendicular height of thoſe mountains was above 4 Italian miles or rather about $5\frac{1}{2}$ Engliſh. Moon's mountains higher than the earth's.

1080. *Ricciolus* found the diſtance of the top of St. *Catharine* from the bright part of the moon, to be about a 16th part of her diameter; and thence the perpendicular height of this mountain comes out 9 Engliſh miles, which is three times greater than that of our higheſt mountains.

1081. The method of computation is this. Let *e* be the moon's center, *gcd* a ray of the ſun touching the moon's ſurface in *c*, and the top of a mountain in *d*; join *ec* and *ed*, and let the circle *caf*, deſcribed with the Fig. 661.

ſemidiameter *ec*, cut *ed* in *a*; then *ad* is the perpendicular height of the mountain. Now if *ce* be divided into 8 equal parts, we have *cd* equal to 1 of them, by the obſervation of *Ricciolus*; and in the right angled triangle *ecd*, the ſquare of *ce*, which is 64, added to the ſquare of *cd*, which is 1, gives 65 for the ſquare of *de*, whoſe ſquare root $de = 8.062$; hence deducting $ae = 8$ we have $ad = 0.062$, and conſequently $ae : ad :: 8 : 0.062 :: 8000 : 62$. But the moon's ſemidiameter is known to contain about 1180 miles engliſh, and as $8000 : 62 :: 1180 : 9$, which therefore is the number of miles in the height *ad*.

Maps of the moon.

1082. The fame of *Galileo*'s obſervations ſoon excited many others to repeat them, and to make maps of the moon's ſpots. Among the reſt *Langrenus* the K. of Spain's coſmographer and *Hevelius* conſul of Dantzick were the moſt diligent. To fit their maps for aſtronomical uſes, it was neceſſary to give names to the moſt remarkable ſpots and regions. *Langrenus* called them by the names of the moſt noted mathematicians, philoſophers and patrons of learning. But *Hevelius* pretending great difficulty in a juſt diſtribution of the lands in proportion to the merits of the learned, aboliſhed their received grants and titles, and called them by the geographical names of places on earth, without the leaſt reſemblance in their ſhapes and ſituations. This vanity of his has embarraſſed the lunar regions with a double nomenclature.

1083. That thoſe who are curious in obſerving lunar eclipſes, may note the times when the ſhadow begins to touch the more remarkable ſpots, or biſect them, or wholly cover them, I have given a copy of Monſ. *Caſſini*'s map of the full moon *; in which the poſitions of the ſpots were determined by his own obſervations made at the times of ſeveral eclipſes. He has noted no other than thoſe which appear plaineſt in eclipſes, and to avoid embarraſſing the map with their names, he has only marked them with numbers, referring to the names in the following table.

* Mem. de l' Acad. 1692

Fig. 662.

1. Grimaldus.	13. Capuanus.	25. Menelaus.	37 Snellius & Furnerius.
2. Galileus.	14. Bulialdus.	26. Hermes.	38. Petavius.
3. Ariſtarchus.	15. Eratoſthenes.	27. Poſidonius.	39. Langrenus.
4. Keplerus.	16. Timocharis.	28. Dionyſius	40. Taruntius.
5. Gaſſendus.	17. Plato.	29. Plinius	A Mare humorum.
6. Schikardus.	18. Archimedes.	30. Catharina, Cyrill. Theophilus.	B Mare nubium.
7. Harpalus.	19. Inſula ſinus medii,	31. Fracaſtorius.	C Mare imbrium.
8. Heraclides.	20. Pitatus.	32. Promontorium acutum.	D Mare nectaris.
9. Lansbergius	21. Tycho.	33. Meſſala.	E Mare tranquillitatis.
10. Reinoldus.	22. Eudoxus.	34. Promontorium ſomnii.	F Mare ſerenitatis.
11. Copernicus.	23. Ariſtoteles.	35. Proclus.	G Mare fœcunditatis.
12. Helicon.	24. Manilius.	36. Cleomedes.	H Mare criſium.

How ſmall a place may be ſeen in the moon.

1084. Monſ. *de la Hire* demonſtrates that a place in the moon no bigger than Paris, is eaſily diſcernible thro' a teleſcope that magnifies about 100 times; and that Paris placed in the middle of the moon's diſk, would appear as plainly thro' ſuch a teleſcope, as *Mare criſium* does to the naked

ed eye*. Omitting his demonstration I will consider this matter in a more general way. * Mem. de l' Acad. 1706.

1085. A spot about 70 English miles in diameter, in the middle of the moon's disk, may be just discerned by the naked eye. For a degree of a great circle on the earth's surface contains about 70 miles, and the moon's distance from the earth's center contains about 60 semidiameters of the earth. Therefore a degree of the moon's orbit, supposed to be circular, contains 60 times 70 miles, and consequently a 60th part of that degree, contains 70 miles, and, by subtending an angle of a minute at the earth, is therefore visible to most people's eyes*. * Art. 97.

1086. Hence a telescope that magnifies 100 times, will just discover a spot whose diameter is $\frac{1}{100}$ part of 70 miles or $\frac{7}{10}$ of a mile; and a telescope that magnifies 210 times (as one of Sir *Is. Newton's* form of $3\frac{1}{4}$ feet focal distance will do) will just discover a spot whose diameter is $\frac{70}{210}$ or $\frac{1}{3}$ of a mile. By which we may form a judgment of the visibility of spots of any given diameters.

1087. With smaller telescopes than those, we might plainly discern more minute changes in the moon's surface than what happen to the earth's in every season of the year, through the alterations in its colour and verdure by heats, colds, snows, rains, inundations and the like. But from the time of the invention of telescopes, no visible changes at all, in the colours shapes and situations of the moon's spots, have yet been observed: their appearances are always the same, allowances being made for their different lights and shades in different ages of the moon, for the different goodness of telescopes, and the clearness of our atmosphere. Indeed as there are no seas nor rivers in the moon, (for they could not escape our best telescopes) and of consequence no clouds, nor rain, nor snow, it is hard to conceive, how any great alterations can happen to a globe of dry earth, unless by fire. No sensible alterations in the moon's spots.

1088. Nor is it probable that the moon is surrounded with air like ours, though free from vapours. For if she were, her limb would probably appear hazy, and not so distinctly terminated as we find it to be by its appulses to fixt stars; whose rays suffer no sensible refraction, nor change of colour, nor any gradual decay of their brightness, but vanish instantaneously*. Monf. *de la Hire*, observing the moon's appulse to Aldebaran, through a very good telescope of 16 feet, saw that star enter very plainly into the bright part of the disk, till the distance of the star's center from the circumference of the disk became equal to three halves of the star's apparent diameter in that telescope; after that it vanished instantaneously*. And being apprised that *P. Feüillee* had formerly observed the like phænomenon in an appulse to another star, he took particular notice that Aldebaran advanced very uniformly towards the moon's limb before its immersion; and therefore could not perswade himself that the moon had No atmosphere in the moon. Mem. de l' Acad 1720. p. 141. 4to. * Mem. de l' Acad. 1699. p. 151. 4to.

any atmosphere either rarer or denser than the ambient æther; but is of opinion that this appearance of a star upon the moon's bright disk, is owing to nothing else but the glaring brightness of the limb, which causes it to appear always bigger than it should do, even through a large telescope; and consequently, that one may see the more brillant light of the star directly through that dilated weaker light of the moon's limb; and that the star does not vanish till its rays touch the very body of the moon.

The moon's libration.

1089. Another discovery that we owe to the telescope is, that the hemisphere of the moon, visible to us, is not at all times quite the same. At one full moon we see a small goar or segment in the margin of her disk, that was quite hid at another: so that her body appears to us as if it librated to and fro; sometimes eastward or westward, at other times northward or southward, and sometimes in a direction between both. Observations upon these apparent librations were first made by *Galileo*, and afterwards continued by *Hevelius*, and described in his book *de Motu Lunæ Libratorio*; and the natural cause of them may be seen in Sir *Isaac Newton*'s Principia *.

* Lib. III. prop. 17 and 38. Cor.

CHAPTER IV.

Telescopical discoveries in Mars.

Phases of mars.

1090. THE orbit of Mars surrounds the earth's, and therefore he is nearest to the earth, when being opposite to the sun, he is seen in the meridian at midnight. This achronical situation of his, is therefore the most advantageous for observing the motion of spots on his body, his diameter and parallax; all which are above five times greater in his opposition to the sun than in his conjunction. In both these positions his enlightened hemisphere is fully exposed to the earth, as well as to the sun; but not so fully in his quadratures with the sun, where he appears through a telescope a little gibbous, like the moon about three days from the full: which plainly proves that his orbit surrounds the earth's at no great distance from it. In the figure, *s* is the sun, *e* the earth, *a* and *b* the places of Mars in opposition and conjunction, *c* and *d* his places in the quadratures with the sun.

Fig. 663.

Dark spots in mars and their periodical revolution.

1091. After *Galileo*'s discovery of the phases of Mars, some other Italians in the year 1636 had an imperfect view of a darkish spot upon his body; but in the year 1666 Dr. *Hook* at London and Sigr *Cassini* at Bologna made the first discovery of spots of determinate figures. *Hook* perceived some little motions in them, but could not determine whether they revolved or not. But *Cassini* concluded from their motions, that Mars revolved about an axis of his body in 24 hours and 40 minutes *.

* Phil. Tranf. N. 14.

1092

1092. In the year 1670 he saw the same spots again, and observed their periodical revolution to be the same as before*; which M. *Maraldi* confirmed in 1704, and again in 1719; and has given us a curious account of it*, from whence I extract the following particulars. At this time the configuration of the spots was quite different from what *Cassini* saw. One of them appeared like a joiner's square, except that the angle was a little obtuse Its angular point being pretty distinct, was the mark attended to in the following observations.

* Du Hamel. Hist. Reg. Sci. Acad. p. 97. Ed. 2.

* Mem. de l' Acad. 1720 p. 144. 4to.

Fig. 664, 665.

1093. On the 19th and 20th of August at a quarter past eleven at night, this point lay a little eastwards from the center of the disk, and at the same times of the succeeding nights it was found successively more and more eastwards; till at last it withdrew and vanished under the eastern limb of the disk; and having in like manner traversed the invisible hemisphere, on the 25th and 26th of September it returned, and appeared again in the same places of the disk as before on the 19th and 20th of August; and therefore spent 37 days in these apparent retrogradations. This number of days being divided by 36 revolutions of Mars about his axis, performed in that time, gives $\frac{37}{36}$ or $1\frac{1}{36}$ or 1 day and 40 mintues for the time of each revolution, as *Cassini* had found before: which period was also verified by many other observations upon this and some other spots.

Bright spots in mars.

1094. Besides these dark spots, situated in different places of the surface of mars, M. *Maraldi* observed a bright one near the southern pole, appearing like a polar zone. He observed it for six months together, and found it subject to many changes, appearing sometimes very bright, at others very faint; and after an entire disappearance, it revived with the same brightness as before. At every time of its bright appearance, the disk of Mars did not appear exactly round; but the bright part of its southern limb, that terminated this spot, appeared protuberant in the shape of a bright cap whose outward arch was a portion of a larger circle than what terminated the darker parts of the disk. Its appearance thro' the telescope resembled that of the new moon to the naked eye, when the bright part of the limb appears as a portion of a larger circle than the dark part. The cause of which, in both cases, M. *Maraldi* attributes to a stronger impression of the rays of the more luminous part upon the retina of the eye.

Situation of his axis.

1095. The axis of Mars is nearly perpendicular to the plane of his orbit; consequently its poles are never far from the north and south parts of the limb of his visible disk; therefore if the polar zone abovementioned was not all over uniformly bright, those variations of its appearance might result from its revolution about the axis of Mars. M. *Maraldi* collected from many observations, that a large part of that zone never failed to appear very bright, for six months together whenever it lay exposed to his view; and that the opposite part, on the other side of the pole,

was

was subject to great variations, appearing sometimes bright, at others faint and contracted in breadth till it totally vanished. Notwithstanding these changes on one side of the pole, he takes notice, that the other side had continued brighter than the rest of the disk, more or less, for near 60 years; and that it was the only permanent spot upon the whole body of Mars, the dark ones having changed their shapes and situations and totally vanished in much shorter periods of time; which also happened to another bright spot that he saw near the north pole.

Reasons for an atmosphere about mars.

1096. By the following observations it seems as if mars had a very extensive atmosphere. *Cassini* being at Briare in 1672, Oct. 1. at $2^h.45^m$. a. m, observed a star in the water of Aquarius, which at the distance of six minutes from the disk of Mars, became so faint before its occultation, that it could not be seen with the naked eye, nor with a 3 foot telescope*. The like diminution of its light, after its occultation, was also observed at *Paris* by M. *Roemer*; who could not see that star with a large telescope, in a very clear air, till its distance from Mars became equal to two thirds of his diameter, and yet stars of that magnitude are plainly visible even in contact with the moon. By a comparison of several observations then made upon that star, it was also judged that it varied its distance from the neighbouring stars.

* Du Hamel Hist. Ac. Reg. Sci. p. 107. Ed. 2.

CHAPTER V.

Telescopical discoveries in Jupiter.

Galileo's discovery of Jupiter's satellites.

1097. IN the year 1610 *Galileo* discovered the satellites of Jupiter, as follows*. At a small distance from Jupiter, which in the evening of Jan. 7, he happened to view through a better telescope than his first, he saw three small bright stars, which he took to be fixt stars, but wondered a little to see them placed exactly in a straight line, parallel to the ecliptick, and to outshine others of the like magnitude. Their situation with respect to Jupiter was this, *East* * * ○ * *West*; two of them lay eastwards and the third westwards from Jupiter; and the two outermost appeared a little bigger than the other. Taking them for fixt stars, he did not much mind their distances from Jupiter, but happening the next night to look at them again, he saw their situations quite changed. For now they all lay westwards from Jupiter, equidistant from one another and closer together than the night before, *East* ○ * * * *West*. Here again, not taking notice of their approach to one another, he began to wonder how it could be that Jupiter lay westwards from two of them the night before, and now eastwards from them all; and was afraid lest, contrary to astronomical computation, Jupiter had got before them all by a direct motion. He was therefore

* Nuncius sidereus.

fore very impatient to observe him the night after, but was disappointed by cloudy weather. On the 10th their appearance was thus, *East* * * ○ *West*; for now he saw but two, and both eastwards from Jupiter, the third being hid behind the planet, as he imagined; these two and the planet were exactly in a straight line parallel to the ecliptick as before. Seeing things were so, and that such changes could not be owing to Jupiter's motion, and that the stars were constantly the same, for no other lay near them in the ecliptick, all his scruples were changed into admiration; being satisfied at last that these changes of position were not owing to Jupiter but to the stars themselves. He therefore resolved to observe them more accurately for the future.

1098. On the 11th he saw them in this position, *East* * • ○ *West*; that is, only two stars lying eastwards from Jupiter, whose distance from the next to him was triple their distance from each other; and the eastern star was twice as large as the other; whereas they were almost equal the night before.

1099. By this time he was fully convinced that Jupiter had three stars revolving about him, as Venus and Mercury do about the sun; and the second night after he found a fourth: they all lay nearly in a line, in this manner, *East* * ○ * * * *West*, and appeared equal in magnitude; very small indeed, but exceeding bright, and much more splendid than fixt stars of that size.

1100. Thus he went on for two months together, constantly observing his new planets, which he called the *Medicean* stars, after the name of his patrons, the family of the *Medici*, and concludes his account with the following reflections. These, says he, are the observations I have hitherto made of the four Medicean planets, of which I was the first discoverer; and though I cannot yet reduce their periods to calculation, yet I have something to say of them worth remarking. First then, since they sometimes go before and sometimes follow after Jupiter, through certain small spaces lying eastwards and westwards from him, and since they always accompany him, both in his direct and retrograde motions, they must undoubtedly revolve about him, while he is revolving about the sun in about 12 years time. In the next place they revolve about Jupiter in unequal circles; because says he, I could never see two of them in conjunction at their greatest digressions from Jupiter; whereas I have often seen two or three, and sometimes all of them crowded close together near the body of Jupiter. Lastly, those that revolve in the smaller circles, have shorter periodical times; because those that appeared nearer to his body were often observed to pass from one side of it to the other in a day's time. But the outermost of all seemed to compleat its revolution in about sixteen days. Hence, says he, we are supplyed with a noble argument for the Copernican System, by removing a difficulty from some peo-

ple's

-ple's minds, who, admitting the revolution of the planets about the ſun, are yet ſo diſſatisfied with the moon's revolution about the earth, while both are moving round the ſun, as to think it impoſſible; and conſequently a ſufficient argument for rejecting that ſyſtem. But now we have ocular demonſtration, not only of one but four planets revolving about Jupiter, like the moon about the earth, while all of them are moving round the ſun in larger orbits. Our author concludes at laſt by attempting at a reaſon, why the Medicean ſtars appear at different times of different magnitudes; which we ſhall ſee, by and by, is moſt probably owing to very large, dark ſpots upon their bodies, and to their turning about certain axes of their own, as our moon does about hers.

Satel.	Periodical times.				Diſtances.
1.	1^d.	18^h.	27'.	34"	5, 667
2.	3.	13.	13.	42	9, 017
3.	7.	3.	42.	36	14, 384
4.	16.	16.	32.	09	25, 299

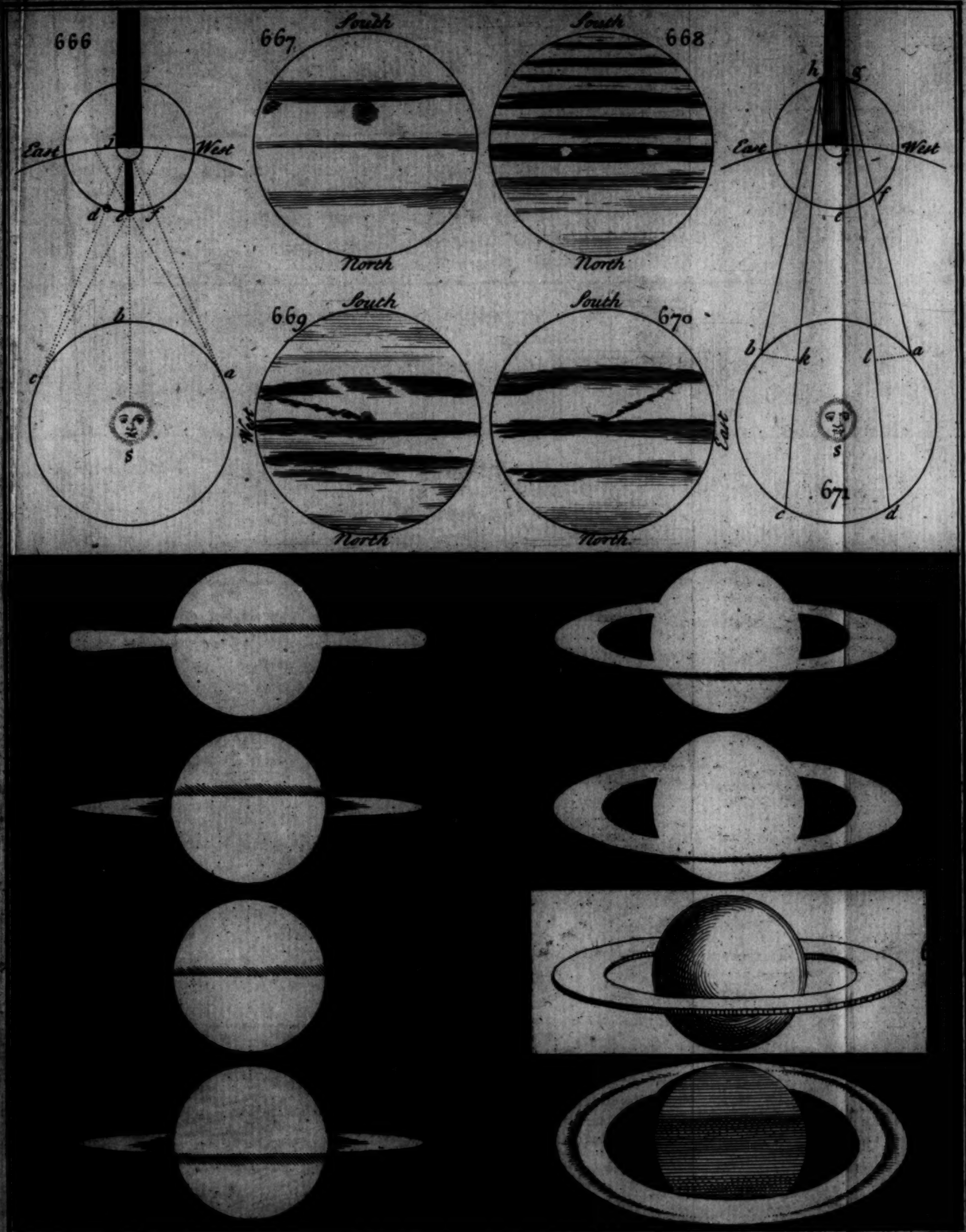
1101. By the lateſt and moſt exact obſervations, the periodical times, and diſtances of theſe ſatellites from the center of Jupiter, meaſured by his ſemidiameters, are as follow; and the analogy among them is, that the ſquares of their periodical times, are as the cubes of their reſpective diſtances from the center of Jupiter, as in the primary planets.

The earlieſt obſervations of Jupiter's belts and ſpots.

1102. About twenty years after *Galileo*'s diſcovery of theſe ſatellites, we are told that *Fontana*, *Zupus* and *Bartolus* three Neapolitans were the firſt diſcoverers and obſervers of Jupiter's belts. By that time it ſeems the teleſcope had received ſome improvement, particularly by a convex eye-glaſs firſt applyed by *Rheita* inſtead of the concave. For *Gaſſendus*, with *Galileo*'s own teleſcope, could perceive nothing of them. Theſe belts were nearly parallel to the eckliptick, in ſome years quite ſtraight, in others a little incurvated, ſometimes upwards and ſometimes downwards, which curvity *Ricciolus* ſuppoſed to ariſe from a ſlow libration of the planet, or elſe from a ſmall inclination of the axis of the belts to the plane of the ecliptick. About the year 1643 the ſame perſons diſcovered the appearance of two ſatellites, or more probably of their ſhadows, paſſing over the diſk of Jupiter; and alſo of two very large roundiſh ſpots.

Two ſorts of ſpots ſeen upon Jupiter. Mem. de l' Acad. Tom. X. p. 513.

1103. After farther improvements of the teleſcope, M. *Caſſini* in the year 1665 publiſhed a theory of two ſorts of ſpots viſible at certain times upon the diſk of Jupiter. One ſort he ſhewed to be the ſhadows of the ſatellites, paſſing over the diſk, at ſuch times as the ſatellites themſelves paſſed between the ſun and Jupiter, and made there a ſolar eclipſe; ſuch as the moon makes here in paſſing between the ſun and the earth. Theſe ſpots are therefore diſtinguiſhable from all others by the following properties; that they fall preciſely upon that part of Jupiter, where ſome ſatellite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East
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satellite would be seen by a spectator placed at the sun; that they move from the eastern towards the western limb of the disk with an uniform velocity equal to that of the satellite it self; that while the earth is approaching towards the line that connects the sun and Jupiter, the satellite, which casts the shadow, is seen eastwards from Jupiter, and westwards from him after the earth has passed by that line; and lastly that the apparent distance of the satellite from the shadow, is found proportionable to the angle which a line connecting Jupiter and the satellite, subtends at the earth. What has here been said will appear plainer by applying it to fig. 666, where *s* is the sun, *j* Jupiter, *abc* the earth's annual orbit, and *def* that of the satellite, both described according to the alphabetical order of those letters. Fig. 666.

1104. The other sort of spots has no relation to the satellites, but seems to be of the nature of the belts; for in passing from the eastern to the western limb, they move slowest at the extremities of their course, and quickest in the middle, where they appear broadest; which shews them to be thin substances in or near the surface of Jupiter, as was said above in relation to spots in the sun.

1105. M. *Cassini* has given us a draught of a remarkable round spot of this sort adjoining to the southermost of the belts that then appeared. Its nearest distance from the center of the disk was about one third of Jupiter's diameter, and its own diameter about one tenth of his. *Cassini* began to observe it in 1665 and by repeated observations found the periodical time of its revolution to be $9^h. 56^m$; and taking a certain time of its arrival to the middle of its parallel for an epocha, he calculated tables of its motion, and found them constantly agreeable to his subsequent observations, till Jupiter became immersed in the beams of the sun; but after his emergence from them, the spot was so wasted that it could hardly be seen; after which it soon vanished. Fig 667.

1106. But in the year 1672 he saw it again, as he judged by its size, shape, position and adherence to the same southern belt as before; and by comparing his present observations with those he had made six years before, he concluded, that the mean period of the spot's revolution to the middle of its parallel, was not greater than $9^h. 55^m. 58''$, nor less than $9^h. 55^m. 51''$. In the evening of Mar. 1. 1672 at $7^h. 30^m$ he saw this spot in the middle of the disk, or rather of its parallel, and next morning at $5^h. 26^m$ he saw its return to the same place; which was the first observation he could ever make of an immediate return.

1107. The same spot continued visible till the year 1674, when it wholly disappeared. In 1677 it appeared again in the same parallel to Jupiter's equator, but was soon extinct, and never seen again till Mar. 1685. From this time M. *Cassini* continued to observe it till Octob. 1687. After that it never appeared again till in 1690, and was visible but a Mem. de l' Acad. 1708. p. 235 4to.

very ſmall time, perhaps by reaſon of great changes which happened that year in Jupiter's belts. It appeared again in 1692, and diſappeared in 1693 with part of the contiguous belt. In 1694 it revived with the ſame belt, and continued viſible till Jupiter entered the beams of the ſun; but was never ſeen after till in 1708 though frequently watched.

1108. M. *Maraldi* having conſidered theſe viciſſitudes of appearance and diſappearance, concluded they had no regular relation to Jupiter's diſtances from the ſun, as ſome changes upon the earth have to the variation of ſeaſons; that they rather had a dependence upon the contiguous belt; that this belt had ſometimes appeared interrupted, and that the breach widened more and more, till the whole belt vaniſhed together with the contiguous ſpot; that the ſpot had never appeared without the belt, though the belt had without the ſpot; and laſtly that the ſpot was probably ſome effuſion of the matter of the belt into a determinate place of Jupiter's body; its latitude from his equator being conſtantly the ſame, and its longitude too, becauſe it returned to the middle of the diſk at the times calculated by the tables of its motion, conſtructed from the obſervations made upon its firſt appearance. In the year 1713 M. *Maraldi* ſaw this ancient ſpot again, and made ſome more obſervations upon it*, confirming what has been ſaid.

* Mem. de l' Acad. 1714.

Changes in Jupiter's belts and ſpots Mem. de l' Acad 1692. Fig. 668. 669. 670.

1109. In the largeſt of Jupiter's northern belts, lying next his center, in Oct. 1691, *Caſſini* ſaw two bright ſpots almoſt as broad as that belt, as in fig. 668; and at the end of that month, two more oppoſite to one another. They revolved in $9^h. 51^m$. He alſo perceived that this broader belt grew narrower, while thoſe on each ſide of it grew broader, till in December following they all became nearly of the ſame breadth, as if the intermediate belt diſcharged it ſelf into the collateral ones, which he thought not improbable from the appearance of ſome dark parts between them like tracts of communication, as expreſſed in fig 669 and 670; that the largeſt of the northern and ſouthern belts were frequently interrupted, and that their broken ends generally revolved in the ſame time with the ſpots in thoſe belts; that at certain times in Oct. 1691, ſeven or eight dark belts appeared very near to one another, moſt of them upon the ſouthern part of the diſk; that ſome late ſpots near Jupiter's equator revolved quicker than the old ones, their periodical times being but $9^h. 50^m$; and in general, that all ſpots near the equator revolved quicker than thoſe that lay farther from it; which he thought might be owing to the ſun's greater heat upon the equator; whoſe plane is parallel to that of the belts and to the courſe of the ſpots, and differs but little from the plane of Jupiter's motion round the ſun; that ſeveral ſpots which appeared round at firſt, grew oblong by degrees, in a direction parallel to the belts, and ſometimes ſplit themſelves into two or three rounder ſpots. Laſtly, that he never ſaw ſo many new ſpots as about

Sept.

Sept. 1690, perhaps, ſays he, becauſe Jupiter was then the neareſt poſſible to the ſun and the earth too, which ſituation gave an advantage to the view, and returns but once in 83 years.

1110. We have juſt taken notice that Jupiter's axis is nearly perpendicular to the plane of his orbit, ſo that the difference of ſeaſons there is very ſmall. In 1691 *Caſſini* made obſervations upon the ſpheroidical figure of Jupiter, and found the ratio of his axis to his equatorial diameter as 14 to 15. In 1719 Mr. *Pound* found it as 12 to 13 with an excellent micrometer applyed to the Hugenian teleſcope of 123 feet, as Sir *Iſaac Newton* informs us, who deduced the latter ratio from the conſideration of its cauſes *, that is, from Jupiter's denſity and periodical time of revolving round his axis, compared with thoſe of the earth. Spheriodical figure of Jupiter.

* Princip. philoſ. p. 415.

1111. In the year 1707 Mar. 26. M. *Maraldi* perceived a round black ſpot upon Jupiter's diſk; which at $6^h. 50^m$.-p. m. had paſſed by the middle of it, and in approaching towards the weſtern limb, did not alter its ſize, ſhape or velocity; by which and the ſlowneſs of its motion he concluded that it did not adhere to the body of Jupiter. Neither was it the ſhadow of a ſatellite upon the diſk, though round and black, as they uſually appear. For, of the three innermoſt ſatellites, which he then ſaw, the firſt and third he knew were in the remoter halves of their orbits; and though the ſecond was in its nearer half, yet it had paſſed by its conjunction with the ſun and Jupiter. As to the fourth, he knew alſo by the tables and ſoon after by obſervations, that it then was at *d* between the earth at *c* and Jupiter at *j*, and conſequently that its ſhadow was projected ſo far eaſtwards from Jupiter, that it could not arrive at the place of the ſpot in leſs than 7 hours. Beſides, had the ſpot been a ſhadow of this fourth ſatellite, the ſatellite it ſelf would have been ſeen weſtwards from Jupiter, at the diſtance of about two of his diameters. Spots upon Jupiter's ſatellites Mem de l' Acad. 1707 p. 289. 4to. Fig. 666.

1112. Hence he was perſwaded that the dark ſpot was in the ſatellite it ſelf, and was confirmed in his opinion by the following arguments; that its motion was agreeable to the known motion of the ſatellite; that in going out of the diſk it grazed upon the weſtern end of one of the belts, in the very place where he ſaw the ſatellite it ſelf emerge from the diſk ſome minutes after the vaniſhing of the ſpot; that this interval of time might ariſe from the difficulty of diſcerning the ſpot to the very edge of the diſk, or, perhaps, from the ſituation of the ſpot upon the body of the ſatellite; that if the ſpot lay on the weſtern ſide of the ſatellite, it ought to diſappear before the brighter eaſtern ſide could emerge from the diſk of Jupiter; that the ſpot began to emerge from the diſk at $7^h. 49^m$, and the ſatellite at $8^h. 6^m$, when it appeared very ſmall; that this interval of 17 minutes, agreed well enough with the time that the diameter of this ſatellite ſhould take up in emerging from the diſk; ſo that the ſpace between the antecedent limb of the ſpot and the ſubſequent limb

of the satellite answered to the whole diameter of the satellite. The observations were made with a 34 foot tube.

1113. On April 4. he observed a like transit of the third satellite, in the form of a spot, over the disk of Jupiter. But at the next conjunction, on the 11th, though he saw the ingress and egress of the same satellite, yet during its passage, in three hours and a half, he could not perceive any spot at all; and therefore concluded, that the spot had vanished some time between this and the foregoing transit; and adds, that a visible transit of a satellite in the form of a spot on the disk is a thing that happens very rarely. He also observes, that the satellites appear sometimes like little bright spots upon the margin of Jupiter's disk, and disappear near the middle of it, where we can only distinguish them when they happen to have large dark spots upon the hemisphere next us; and that *Cassini* the Father had observed the like phænomena in every one of these satellites.

1114. It is highly probable, says he, that the satellites turn about certain axes of their own, and have permanent spots like those in the moon, or variable ones like those in the sun and primary planets; whose changes of magnitude and position may occasion their visible and invisible transits over the disk of Jupiter. For the apparent magnitudes and degrees of brightness of the same satellites are very different at different times; the fourth appears generally the smallest of all, but sometimes the largest, and, though its shadow on Jupiter's disk ought to be rather less in diameter than the satellite it self, and be still diminished a little by the apparent incroachment of the ambient light, yet it appears larger than the satellite seen at the same time on the outside of the disk. The third satellite is also variable in apparent magnitude, being generally larger than the rest, though sometimes equal to them and sometimes less. The like happens to the other two.

1115. But none of them appear large enough out of the disk, to distinguish their spotted parts from the bright ones, as it happens also to certain fixt stars, which yet increase and decrease in apparent brightness; and while the satellites are entering into the shadow of Jupiter, we plainly perceive a gradual decay of their brightness, but cannot discern the phases of the eclipse even with the best telescopes. M. *Maraldi* concludes at last that *Cassini*'s hypothesis of spots upon the satellites is farther confirmed by the outermost of saturn's, which, for many years after *Cassini* had first discovered it, could never be seen on the eastern side of saturn, till it became visible even through the same telescopes in Sept. 1705, and and continued so till Jan. 1706.

Transit of the body and shade of Jupiters 4th. satellite over his disk. Phil. Transf. Jones's abr. Vol 4, p. 307.

1116. In 1719 Feb. 16^{d}. 9^{h}. 45^{m}, Mr. *Pound*, through the Hugenian telescope of 123 feet, saw the outermost satellite of Jupiter in the middle of his disk, and its shadow near the eastern limb; the satellite appeared

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to him almoſt as black as its ſhadow, but ſomewhat leſs and a little more northerly. For, at the ſame time he ſaw the three innermoſt ſatellites to the eaſt of Jupiter, and the times, $9^h\ 45^m$ and $11^h.\ 45^m$, at which the outermoſt and its ſhadow arrived at the middle of the diſk, were agreeable to the times, found by calculation, in which they ought to be there. He adds, that at other times he had ſeen the firſt and ſecond ſatellites, appearing not as dark ſpots, but as bright ones, ſomewhat different from the brightneſs of Jupiter, for ſome little time after they entered his diſk, but as they approached nearer the middle he loſt ſight of them; that he had frequently ſeen the ſame ſatellites appearing brighter at ſome times than at others; that when one of them ſhined with its utmoſt ſplendor the light of the other was conſiderably diminiſhed; that from theſe obſervations it was very probable at leaſt, not only that the ſatellites revolve upon their proper axes, but alſo that ſome parts of their ſurfaces do very faintly, if at all, reflect the ſolar rays to us.

1117. Tables of the motions of Jupiter's ſatellites were firſt publiſhed by Sig. *Caſſini*, and were afterwards epitomiſed, corrected, and reduced to our ſtyle and meridian by Dr. *Halley* * and laſt of all by Mr. *Pound* *; who applyed himſelf particularly to rectify the motion of the innermoſt ſatellite, and to facilitate the calculation of its eclipſes in the ſhadow of Jupiter, as affording the beſt and moſt frequent opportunitys of determining the longitudes of places at land, by clocks and teleſcopes of a convenient length, thirteen of theſe eclipſes happening every twenty three days

Tables of the eclipſes of Jupiter's ſatellites.

* Phil. Tranſ. abrid. Vol. 1. p. 409.

* Phil. Tranſ. abrid. Vol. 4. p. 308.

1118. The application of this moſt uſeful invention, to the purpoſes of geography and navigation was firſt ſuggeſted by *Galileo* and firſt put in practice by the members of the Royal Academy of Sciences, ſent by order of *Lewis* the 14th into many diſtant parts of the world; and if duly proſecuted, might contribute more to the improvement and perfection of geography and navigation in a few years, than all other methods can do in as many ages. And till the geography of the ſea-coaſt be duly ſettled, the very beſt methods of determining the place of a ſhip at ſea, at her approach towards land, where the danger is greateſt, will be ſtill defective.

Applyed to find the longitude of places.

1119. From the conſideration of the eclipſes of theſe ſatellites Monſ. *Roemer* made a noble diſcovery of a method for determining the velocity of light; by which it was concluded and confirmed by long experience that light is ſhot from the ſun to the earth in 7 or 8 minutes of time; a diſtance ſo great, that a cannon ball cannot deſcribe it in leſs than 25 years, at the rate of 500 feet in every ſecond of time.

Velocity of light.

1120. *Roemer*'s method is this. Let *s* be the ſun, *abc* the annual orbit of the earth, *j* Jupiter, *efgh* the orbit of the innermoſt ſatellite, the propereſt for this enquiry by reaſon of the quickneſs of its revolution; and

Fig. 671.

let

let this ſatellite enter the ſhadow of Jupiter at *g* and emerge from it at *h*. Now ſuppoſing the earth at *b*, ſometime before the laſt quadrature, let an emerſion of this ſatellite be obſerved at *b*; then if the earth continued in the ſame place, we ſhould ſee the next emerſion at the end of 42 hours and a half, this being ſuppoſed the exact time in which this ſatellite revolves to the ſhadow. Likewiſe if the earth continued at the ſame place *b* during any number of revolutions, ſuppoſe 30, we ſhould ſee an emerſion at the expiration of 30 times $42\frac{1}{2}$ hours. But the earth in that time being really transferred from *b* to *c*, farther from Jupiter, it follows, if light requires time for its propagation, that this emerſion will be perceived later at *c* than it would have been at *b*, and that to 30 times $42\frac{1}{2}$ hours, we muſt add the time which the light takes up in deſcribing the ſpace *kc*, the difference of *bh* and *hc*.

1121. On the contrary towards the other quadrature, whilſt the earth in going from *d* to *a*, is approaching towards Jupiter, the immerſions of the ſatellite at *g* ſhould be perceived ſooner at *a* than they would be, had the earth remained at *d*.

1122. Now theſe retardations of the emerſions in going from Jupiter and accelerations of the immerſions in going towards him, have been often found to amount to above ten minutes; and from the moſt accurate conſideration of them it is concluded that light deſcribes a line *kc* equal to the ſemidiameter of the earth's annual orbit, nearly in half a quarter of an hour. The motion of Jupiter in his orbit during the earth's paſſage from *b* to *c* and from *d* to *a*, is conſidered in that concluſion; and it is at laſt agreed by aſtronomers that theſe equations, of the times of theſe eclipſes, cannot be accounted for either by any inequality in the motion of the ſatellite, or by any excentricity or inclination of its orbit; and laſtly that the three other ſatellites require the ſame equations of the times of their eclipſes *.

* Phil. Tranſ. Vol. 1. abrid. p. 422.

Chapter VI.

Teleſcopical diſcoveries in Saturn.

The firſt teleſcopick obſervations upon Saturn.

1123. GALILEO was the firſt perſon that obſerved any thing extraordinary in Saturn, which at that time he took for three globes, a larger between two ſmaller. They appeared almoſt to touch one another, and their centers lay exactly in a ſtraight line, which he judged nearly parallel to our equator. In the year 1610 he advertiſed this diſcovery by the letters of this ſentence tranſpoſed; *Altiſſimum Planetam tergeminum obſervavi*; and afterwards added this caution, that unleſs the teleſcope was good and magnified at leaſt 30 times in diameter, Saturn would not appear through it as three diſtinct globes, but as one,

only

only lengthened like an olive. He obſerved much the ſame appearance for above two years together, till in 1612 he was amazed to find the middle globe left quite alone. But ſometime after that, the collateral globes appeared to him again, and being viewed by different perſons, through teleſcopes of different goodneſs, they ſeemed to ſome to ſtick to the middle globe, to others to be ſeparated from it, and in proceſs of time to put on various ſhapes, ſometimes round, ſometimes oblong like acorns, ſometimes ſemicircular, then lunar, with horns pointing towards the globe in the middle, and growing by degrees ſo long and ſo wide as to encompaſs it, as it were with an oval ring.

1124. Theſe ſtrange appearances were obſerved for forty years together, and many hypotheſes were invented to ſolve them, as by *Hevelius* in his book *de Saturni nativa facie*, by *Roberval*, *Hodierna*, *Gallet* and others, an account of which is given us by *Hugenius* in his *Syſtema Saturnium* publiſhed in 1659. This judicious perſon, finding no ſatisfaction in what had been offered to the publick, and judging the chief impediment to lye in the ſmalneſs and other imperfections of the beſt teleſcopes then made uſe of, reſolved in the firſt place to improve the art of grinding object-glaſſes; in which he ſoon ſucceeded ſo far as to make ſome teleſcopes that magnified two or three times more than ordinary. With theſe he diſcovered a ring about Saturn, which he ſhewed was the true cauſe of all thoſe ſurpriſing phænomena, and beſides that a ſatellite, revolving round Saturn every 16 days, or thereabouts, in the plane of the ring produced.

Hugenius's obſervations upon Saturn.

1125. In March 1655, he ſaw Saturn through a 12 foot tube, as repreſented in fig. 672, with two Arms oppoſitely extended from the globe. They appeared a little thicker towards their extremities than near the diſk, and kept this ſhape till Saturn became immerſed in the beams of the ſun. But when he had loſt theſe arms in 1656 and recovered them again ſome time after, they appeared through the 12 foot tube in the ſame ſhape as before, but through another tube of 23 feet, which he had prepared in that interval of time, they appeared ſlenderer towards their extremities, as in fig. 673. He therefore concluded they would have appeared of this truer ſhape before, had they been ſeen through the longer tube. A darkiſh zone or line paſſing ſtraight over the diſk, between the upper ſides of the Arms was viſible even through the 12 foot tube. Fig 672. Fig. 673.

1126 In Jan. 16. 1656, after his return from his travels, he ſaw Saturn quite round as in fig. 674, but others had obſerved this phaſe before about the latter end of November; which continued round when Saturn entered the ſun's beams a ſecond time. In every obſervation of this phaſe, that darkiſh line paſſed through the center of the diſk, and was conſtantly directed to the ſatellite, and was alſo in the direction of the diurnal motion of the planet, and conſequently parallel to our equator. Fig. 674.

1127.

Fig. 675. 1127. In Octob. 1656, Saturn began to appear again, with his arms as in fig, 675, of the same shape as in the year before, but more distinctly defined through better telescopes. The darkish line now joined the undersides of each arm, and was still parallel to the diurnal motion of the planet, and likewise ever after.

1128. On Novem. 26. The arms grew a little broader and appeared not so bright near the body as at their extremities.

1129. In 1657. Decem. 17. As soon as Saturn had emerged from the sun, both the arms were divided into two parts by a dark space next the body, and appeared like *Ansæ* as represented in fig. 676. The darkish zone was now carried farther downwards; and the same appearance continued till Saturn's heliacal setting.

Fig. 676.

1130. In 1658. Novem. 10. After his heliacal rising, the *Ansæ* seemed wider than before, but were not distinct by reason of the vapours near the horizon. But on Feb. 6. they appeared very distinct, as represented in fig. 677, which phase continued till the latter end of March, when the author left off observing. I have omitted an account of his other observations, as relating only to the motion of the satellite.

Fig. 677.

Dimensions of Saturn and his ring.

1131. In the year 1656. before he had gone through all these observations, he advertized his discoveries in Saturn in the letters of this sentence transposed; *Annulo cingitur*, *tenui*, *plano*, *nusquam cohærente*, *ad eclipticam inclinato*; and afterwards he described the shape of the ring as in fig. 678, where he makes the space between the globe and the ring equal to, or rather bigger than, the breadth of the ring; and the greatest diameter of the ring in proportion to the diameter of the globe as 9 to 4. But the truer proportion is 7 to 3, as Mr. *Pound* determined it by an excellent micrometer applyed to the *Hugenian* glass of 123 feet. That there is empty space between the ring and the Body is evident not only from its colour and other optical appearances, but also from what Mr. *Whiston* relates in his Memoirs of Dr. *Clarke*'s life; viz. that the Doctor's father once saw a fixt star through the dark space of one of the *Ansæ*.

Fig. 678.

The ring seems to be double.

1132. In the year 1675, after Saturn had emerged from the sun's rays, Sig. *Cassini* saw him, in the morning twilight, with a darkish belt upon his globe parallel to the long axis of his ring, as usual. But what was most remarkable, the broad side of the ring was bisected quite round by a dark elliptical line, dividing it as it were into two rings, of which the inner one appeared brighter than the outer, with nearly the like difference in brightness as between that of silver polished and unpolished; which though never observed before, was seen many times after with tubes of 34 and 20 feet, and more evidently in the twilight or moon light, than in a darker sky*.

* Phil. Transf. Abr. Vol. I. p. 367. Item Recueil d'Obser. de l'Acad. Tom. X. p. 582.

1133. Mr *Hadley* with his 5 foot Reflecter took notice*, that the upper part of the ring seemed to be narrower than the lower or inner part next the

* Phil. Transf. Abridg. Vol. 6. p. 153.

the body, and that the dark line which ſeparated them was ſtronger next the body and fainter towards the upper edge of the ring as in fig. 679. Within the ring he alſo diſcerned two belts, one of which croſſed Saturn cloſe to its inner edge, and ſeemed like the ſhade of the ring upon the body of Saturn; but when he conſidered the ſituation of the ſun in reſpect to the ring and Saturn, he found that belt could not ariſe from ſuch a cauſe. The edge of the ſhadow of Saturn caſt upon his ring was alſo viſible, as repreſented in the figure, Fig. 679.

1134. Agreeable to theſe are Mr. *Pound*'s obſervationes made with the Hugenian glaſs *; who beſides the ring on the ſouth ſide, ſaw a zone on the north ſide, not ſo far from the center as the ring, and not unlike the ſmalleſt of Jupiter's belts.

* Phil. Tr. Abr. Vol. 4. p. 321.

Inclination of the ring to the eclіptick.

1135. *Hugenius* was of opinion at firſt, that the plane of the ring was conſtantly almoſt parallel to the plane of the earth's equator, and conſequently inclined to the ecliptick in an angle of about 23 $\frac{1}{2}$ degrees. Becauſe by ſeveral methods, mentioned in his *Syſtema Saturnium* p. 51, he found that the long axis of the ellipſis, which the ring reſembles, was at all times nearly parallel to the plane of our equator. But in 1668 Meſs^{rs}. *Roëmer*, *Picard* and himſelf meaſured diverſe ways the inclination of the greater diameter of the oval to our equator, and found it 9 degrees, and thence concluded its inclination to the ecliptick was about 31 degrees, eſpecially becauſe in the beginning of July 1664 Sig. *Campani* had obſerved, that the ellipſis of the ring appeared broader than was expected, its long axis being double the ſhort one *; for as the long axis is to the ſhort one, ſo is the radius to the ſine of the ring's inclination to a line connecting its center with the eye of the obſerver; which inclination was therefore 30 degrees, and Saturn's latitude being then about one degree, gave 31 degrees for the ring's inclination to the ecliptick.

* Phil. Tr. N. 45. p. 900.

1136. But unleſs *Campani*'s teleſcopes were then very excellent, or at leaſt much better than *Huygen*'s, no certain concluſion can be drawn from his obſervation upon the ſhape of the ellipſis. For at the mean diſtance of Saturn from the ſun or earth, the long axis of his ring, according to *Hugenius* *, appears under an angle of 64 ſeconds, but according to Mr. *Pound*, of but 42 ſeconds *; and there is much the ſame difference of 22″ in their meaſures of Jupiter's apparent diameter, ariſing from the different goodneſs of their teleſcopes, the better of the two ſhewing the object diſtincter and freer from a falſe border of dilated light, and conſequently under a ſmaller angle. Now this border being ſeen of the ſame breadth quite round the ring, makes the ellipſis appear broader than it ſhould do in proportion to its length, and conſequently would give too great an inclination of its plane to the viſual rays. But what to allow for that border is uncertain.

* Syſtem Sat. p. 77.
* Newton's Princip. p. 392.

Saturn's phases explained.

1137. *Hugenius* explains the phases of Saturn, during his whole revolution about the sun, by the following principles. First, that the plane of the ring keeps constantly parallel to it self. Secondly, that the convex edge of the ring reflects too few of the sun's rays to render it visible; the foundation of which shall be considered by and by.

Fig. 680.

1138. Admitting these two principles, let *abcd* be Saturn's orbit, *s* the sun in its focus, *asc* a line drawn parallel to the plane of the ring.

1139. First then, whenever Saturn arrives at *a* or *c*, he will appear round, (as if he had no ring,) to an eye in any place of the earth's orbit *txuy*. For though the distance between the planes of the ring, produced through the sun, be so small that the outward segments of the sun's body will send some rays upon those planes, yet by falling too obliquely they will be scattered too thinly to render them visible. And though they fall more directly upon the outward edge of the ring, yet in fact it is invisible, either because it reflects too few of those rays, according to the principle premised, or because its thickness subtends too small an angle at our eyes as Dr. *Jurin* has observed in his curious *Essay on distinct and indistinct vision*, at the end of the Remarks upon this Book.

1140. Secondly, for the reasons last mentioned, the ring is also invisible whenever its plane produced passes through the earth: as when Saturn is at *e* and the earth at *t* or *u* in the line *etu* parallel to *asc*.

1141. Thirdly, the ring continues invisible all the time that its plane produced passes between the sun and the earth: as at *e*, where its dark plane is exposed to the eye placed any where in the arch *txu* opposite to the sun, and its light plane is turned from the eye; the edge being constantly invisible as before.

1142. Draw *bsd* perpendicular to *asc*, and while Saturn is moving from *a* to *b*, or *c* to *d*, a spectator at the sun would see the elliptical figure of the ring grow broader continually, till it arrives at *b* or *d*, where it would appear broadest; the plane of it being then inclined to the visual rays in the greatest angle.

1143. And to a spectator at the earth, the ellipsis will appear broadest, when a plane passing through the centers of the Earth and Saturn, and standing upright on the plane of his orbit, becomes perpendicular to the line *asc*: for then it is also perpendicular to the plane of the ring, above which the eye is then elevated nearly as much as possible.

1144 The figures in Saturn's orbit, represent the ring in its parallel positions, the shaded half being below the plane of the orbit and the other above it; and the collateral figures shew the corresponding phases of Saturn at those places, as seen from the sun, or even from the earth. For the radius of the earth's orbit being between $\frac{1}{9}$ and $\frac{1}{10}$ of Saturn's, can scarce ever

ever ſubtend a greater angle at Saturn than 6 degrees: ſo that the elliptical ſhapes of the ring, ſeen from the ſun, or from the earth, are not much different.

1145. *Hugenius* grounds the neceſſity of admitting his ſecond principle upon the appearance of that darkiſh belt, which he conſtantly ſaw upon that part of Saturn's body where the outward edge of the ring was optically projected*. Firſt becauſe the edge was as fully expoſed to the ſun as the plane of the ring. And ſecondly becauſe that dark belt could not be the ſhadow of the ring caſt upon the body. For in all his obſervations, his eye happened to be more elevated than the ſun above the enlightened plane of the ring; and conſequently could ſee nothing of the ſhadow of its outward edge. He therefore concluded that in theſe caſes the dark liſt that he ſaw was only the optical projection of the outward edge; which being thick enough to be viſible upon a light ground, would be alſo viſible upon a dark one, or againſt the ſky, if it reflected light enough; and conſequently that the Arms vaniſhed, in the caſes above mentioned, for want of a reflective power of the edge. In the round phaſe indeed, the dark plane of the ring was expoſed to his eye in a very oblique angle; and its optical projection upon Saturn together with that of the edge it ſelf, made up the whole darkiſh liſt ſeen as in fig. 674. In fig. 680 the numbers 1, 2, 3, 4, 5 ſhew the places of Saturn, and 1, 2, 3, 4, 5 the correſponding places of the Earth, when Saturn appeared to him as repreſented in fig. 673, 74, 75, 76, 77 reſpectively. And the angle 5 5 *z*, (for example) which the viſual ray makes with 5 *z*, the ring's interſection with the plane of the orbit, being greater than the angle *s* 5 *z*, which the ſun's rays make with it, ſhews that the eye was more elevated than the ſun above the enlightened plane of the ring. The ſhort axis of the ellipſis ſeen from the ſun, is as the angle *s* 5 *z*, but ſeen from the earth, as the angle 5 5 *z* very nearly, becauſe the earth deviates very little from the plane of Saturn's orbit.

The grounds of the ſecond principle.

* Art. 1125.

1146. The longitude of the line *a s c*, to which the interſection of the plane of Saturn's ring and orbit is conſtantly parallel, was determined by *Hugenius* to be in 20°. 30′ of Virgo and Piſces about the year 1656. But by ſome better obſervations made in 1685, M. *Maraldi* placed it ſcarce ſo forward as 19°. 55′ of thoſe ſigns; and by comparing an obſervation of *Caſſini*'s made in 1671, with another of his own in 1715, he ſettled it more exactly in 19°. 45′ of thoſe ſigns*.

The longitude of the nodes of the ring. Fig. 680.

* Mem. de l'Acad. 1715.

1147. He determined theſe places of the nodes of the ring, by biſecting the interval between Saturn's heliocentrick places, given by the obſerved times when he loſt his Arms and regained them, or when they ſeemed to be of the ſame ſize before and after their diſappearance; or elſe by biſecting the interval between one heliocentrick place and the point oppoſite to the other, when the ring was obſerved in oppoſite parts of the orbit.

1148. By obſervations in the former caſe he found that interval to be 3 degrees, and in the latter but little more than one degree; which ſhews that leſs than half a degree's elevation of the ſun's central rays above the plane of the ring, is enough to make it viſible to an eye ſufficiently raiſed above that plane, and aſſiſted with as good glaſſes as were uſed by *Caſſini* and *Maraldi* at the Royal Obſevatory.

Times of the round phaſes how found.

1149. Therefore ſince Saturn, in this part of the zodiack, deſcribes a degree in a month, it follows that his ring is inviſible for want of the ſun's rays, no more than 15 days before, or 15 days after, its arrival at 19°. 45′ of Virgo or Piſces. The times when this round phaſe ſhall happen may therefore be readily found by the tables of Saturn's motion, or even by an Ephemeris; and ſo may the times when the plane of the ring produced will paſs through the earth, by ſeeking when Saturn's geocentrick place ſhall be in 19°. 45′ of Virgo or Piſces; and likewiſe the times when the ring will appear moſt open, *viz* when its geocentrick place is in 19°. 45′ of Gemini and Sagittarius, that is, 90 degrees from its nodes.

And which plane of the ring is enlightened.

1150. It may not be amiſs to obſerve, that the ſun enlightens the northern plane of the ring during Saturn's heliocentrick motion from 19°. 45′ of Virgo to the oppoſite point, and the ſouthern plane during its paſſage through the other ſix ſigns.

A ſuſpicion that the ring turns round an axis. Mem. de l'Acad. 1715.

1151. In Oct. 1714 when the earth was very near to the plane of the Ring produced, and was moving towards it, M. *Maraldi* obſerved, while the Arms were decreaſing night after night, both in length and breadth, that the eaſtern Arm appeared a little larger than the other for three or four nights, and yet vaniſhed firſt. For after two nights interruption by clouds, he ſaw the weſtern Arm alone, but never after, becauſe the next night was cloudy, and on the night after that (viz. the 14th of Octob.) Saturn appeared quite round and continued ſo ſome months.

1152. This inequality of the arms gave him grounds to ſuſpect, that ſome parts of the ring or even the whole were not bounded by planes exactly parallel; and that the larger Arm did not really diſapear firſt, but in the interval between the obſervations, was transferred into the place of the ſmaller, by a circular motion of the ring about an axis perpendicular to its planes. But whether the globe of Saturn turns round an axis or not, it does not appear by any obſervations that I have yet ſeen.

Diſcovery of 4 ſatellites more.

1153. Every one knows that Saturn has five ſatellites; that which *Hugenius* diſcovered in 1655, is the outermoſt but one; all the reſt were diſcovered by Signor *Caſſini:* the outermoſt of all, and alſo the middlemoſt, or the next within the Hugenian, in 1671, and the two inner ones in 1686, which he diſcovered with tubes of 100 and 136 feet, but afterwards could ſee all five with a 34 foot tube. He called them *Sidera Lodoicea* in honour of *Louis le Grand*, in whoſe Reign and Obſervatory they were firſt diſcovered.

In

1154. In 1659 M. *Huygens* publiſhed, in his *Syſtema Saturnium* a table of the mean motion of his own ſatellite, which Dr. *Halley* found by his own obſervations in 1682 to be conſiderably run out; and therefore preſented the publick with a new table computed from correcter Elements *. He alſo reformed *Caſſini*'s tables of the mean motions of all theſe ſatellites *, and about the year 1720 he publiſhed them a ſecond time, farther corrected from Mr. *Pound*'s obſervations made with the Hugenian teleſcope *. Here he takes notice that the four inner ſatellites deſcribe their orbits very nearly in the plane of the ring produced, which he affirms to be parallel to our equator as to ſenſe, *quicquid in contrarium proferant nonnulli*; and that the orbit of the 5th ſatellite is ſituated a little wide of the reſt, according to ſome late obſervations by M. *Caſſini* the ſon.

Tables of their motions.

* Phil. Tranſ. abrid. Vol. I. p. 370.
* Ibid p. 376.
* Ibid. Vol. 4. p. 223.

1155. The diſtance of the Hugenian ſatellite from Saturn's center, has been commonly obſerved to contain 8 ſemidiameters of the ring, but Mr. *Pound* with his micrometer applyed to the Hugenian teleſcope, found it to be 8,7 *. Hence the diſtances of all the reſt, in the table annext, were deduced from their periodical times, according to the known relation among them, (that the cubes of the diſtances are as the ſquares of the periodical times,) and are found agreeable to the obſerved diſtances. So much for Saturn.

Satel.	Periodical times				Diſtances.
1.	$1^d.$	$21^h.$	$18'.$	$27''.$	2, 10
2.	2.	17.	41.	22.	2, 69
3.	4.	12.	25.	12.	3, 75
4.	15.	22.	41.	14.	8, 70
5.	79.	07.	48.	00.	25, 35

* Newt. Princip. p. 392.

1156. The following proportions of the mean diſtances of the earth and planets from the ſun, were alſo deduced from their periodical times; according to the law juſt mentioned, diſcovered by *Kepler* and demonſtrated by Sir *Iſaac Newton*.

Periodical times in days and decimal parts of a day.

Saturn.	*Jupiter.*	*Mars.*	*the Earth*	*Venus*	*Mercury.*
10759,275.	4332,514.	686,9785.	365,2565.	224,6176.	87,9692.

Mean diſtances from the ſun.

954006.	520046.	152369.	100000.	72333.	38710.

1157. By comparing theſe proportions of the diſtances of the ſun and planets, with the proportions of their apparent diameters meaſured by a micrometer, we have the proportions of their real diameters, as follow.

The

Real diameters of the Sun and Planets.

The Sun.	*Saturn.*	*Jupiter.*	*Mars.*	*the Earth.*	*Venus.*	*Mercury.*	*Moon.*
10000.	790.	996.	57.	109.	112.	40.	30.

1158. For, the Sun's apparent diameter at his mean distance from us is 32′, 12″ = 1932.

1159. And the Earth's apparent diameter, supposed to be seen from the sun at the same mean distance, is 21″, as being double the sun's horizontal parallax of $10''\frac{1}{2}$, which by many repeated observations Dr. *Halley* found to be not greater than 12″ nor less than 9″*. Therefore the sun's real diameter is to the earth's as 1932″ : 21″ : : 10000 : 109.

* Art. 875.

1160. Sir *Isaac Newton* collected * from Mr. *Pound's* observations made with a micrometer applyed to the Hugenian telescope of 123 feet, that an observer at the sun would see Saturn at his mean distance, under an angle of 16″, and Jupiter under an angle of 37″; and consequently would see Saturn, if brought down to the earth's mean distance, under an angle of $\frac{954006}{100000} \times 16''$* $= 152,64096''$. Therefore the Sun's real diameter is to Saturn's, as 1932″ : 152, 64096″ : : 10000 : 790.

* Philos. Princip p. 392.

* Art. 60.

1161. By the like reduction the Sun's real diameter is to Jupiter's, as 1932″ : 192, 417″ : : 10000 : 996.

1162. When Mars was nearest to the earth, *Hugenius* found his apparent diameter did not exceed 30″, and that the distance of Mars from the earth, was then to the sun's mean distance from us, as 15 to 41 *. Consequently Mars removed to the sun at that distance, would have appeared under an angle of $\frac{15}{41} \times 30'' = 10,9756''$. Therefore the sun's real diameter is to that of Mars, as 1932″ : 10, 9756″ : : 10000 : 57.

* Systema Saturnium p. 79.

1163. Dr. *Halley* collected * from the appearance of Venus and Mercury in the sun's disk *, that Venus seen from the sun at her mean distance, would appear under an angle of 30″, and Mercury at his mean distance, under an angle of 20″. Consequently Venus removed to the Earth's mean distance from the sun, would appear under an angle of $\frac{72333}{100000} \times 30'' = 21,6999''$. Therefore the Sun's real diameter is to that of Venus, as 1932″ : 21, 6999″ : : 10000 : 112.

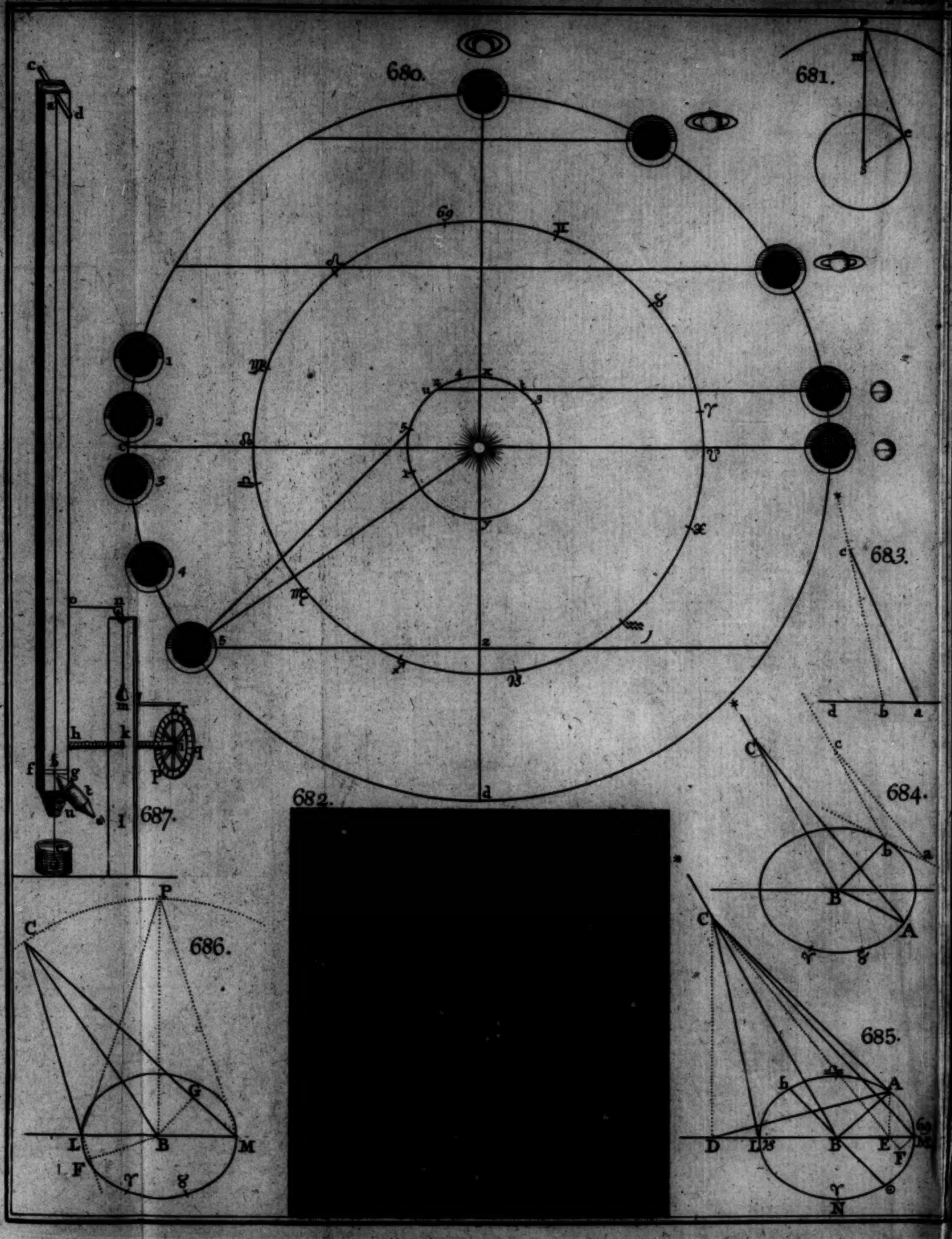
* Phil. Transf. N. 348. abrid. by Jones Vol. 4. p 213.
* Art. 1054. &c.

1164. By the like reduction the Sun's real diameter is to Mercury's as 1932″ : 7, 742″ : : 10000 : 40.

1165. The apparent diameters of the planets, as seen from the sun at their mean distances from him, were collected from their observed diameters in this manner. Let the sun be at *s*, the earth at *e*, the planet at *p*, when its diameter was observed at the distance *pe*. By astronomical tables the angles of the triangle *sep*, and consequently the proportions of its sides are given; and also that of *sp* to *sm*, supposed to be the planet's mean distance from the sun; and from these proportions those of its apparent diameters at the distances *ep*, *sp*, *sm*, are deduced by art. 60.

Fig. 681.

1166.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687.

1166. From the sun's horizontal parallax of $10''\frac{1}{2}$, it follows that his distance from us is 9962 diameters of the earth. Because in a right angled triangle, in which the earth's semidiameter subtends an angle of $10''\frac{1}{2}$ at the sun's center, the distance between the centers of the sun and earth, is to the earth's semidiameter, as the radius to the tangent of $10''\frac{1}{2}$, that is, as 19924 to 1. Distances of the Planets from the Sun.

1167. Hence the real distances of all the planets from the sun, are given by the table of their proportions in Art. 1156.

1168. The moon's apparent mean semidiameter is $15'.38''=938''$, and at that time her mean horizontal parallax is $57'.12''=3432''$. Therefore the earth's real semidiameter is to the moon's, as $3432'':938''::109:30$ almost, which expresses her real diameter with respect to those of the sun and planets in Art. 1157. Moon's real diameter and distance.

1169. Hence the moon's mean distance from us is a little above 60 of the earth's semidiameters. Because the radius is to the tangent of $57'.12''$ as $60\frac{1}{10}$ to 1 nearly.

CHAPTER VII.

Telescopical discoveries in the Fixt Stars.

1170. THAT the fixt stars have no sensible parallax, or, which is the same thing, that the earth's annual orbit (whose diameter a cannon ball could not describe in less than 50 years,) would appear of no sensible magnitude through a telescope placed at a fixt star, is such an amazing conclusion as could not be believed, were it not supported by undeniable evidence. But as this is the case, it is no longer a wonder why the best telescopes don't at all magnify the apparent diameters of the fixt stars, though they discover vast multitudes that are quite imperceptible to the naked eye; and the more of them as the aperture is more enlarged to take in more light, and the eye-glass made flatter to render it distinct*. The Milky Way, which had puzzled the ancient philosophers for many ages, was found at last to be nothing else but a prodigious number of very minute stars, so close to one another that the naked eye can only perceive a whitish mixture of their faint lights. This was *Galileo*'s discovery, who found also that those faint stars, which Astronomers call *Nebulosæ*, appeared through his telescope to be small clusters of very minute stars. Multitudes of Telescopick Stars.

* Art. 360.

1171. *Hugenius* in the year 1656, looking by chance through a large telescope, at three small stars very close to one another in the middle of Orion's sword, saw several more as usual. But the three little stars very near one another (marked θ by *Bayer*), together with four more, shone out as it were through a whitish cloud, much brighter than the ambient sky: which being very black and serene caused that lucid part to appear like Lucid spots among the fixt Stars. Fig. 682.

like an aperture, that gave a prospect into a brighter region. He viewed it many times, and found it continued in the very same place, and of the same shape as the figure represents, and called it *Portentum cui certe simile aliud nusquam apud reliquas fixas potuit animadvertere* *.

* Systema Saturnium p. 8.

1172. But in the Philosophical Transactions *, there is an account of a later discovery of five more such lucid spots, though less considerable than this of *Hugenius*; the middle of which, we are there told, is at present in ♊. 19°. 00'. with south latitude 28°. 45'; and that it sends forth a radiant beam into the south east, as another in the girdle of *Andromeda* seems to do into the north east. It is also there remarked, that "though these spots are in appearance but small, and most of them but a few minutes in diameter, yet since they are among the fixt stars, as having no annual parallax, they cannot fail to occupy spaces immensely great, and perhaps not less than our whole solar system; in all which spaces it should seem, that there is a perpetual uninterrupted day".

* N. 347. Jones's abr. Vol 4, p. 224.

New Stars.

1173. It is to the Author of these reflections, if I mistake not, that we owe another curious account of what is principally remarkable in the new stars that have appeared and disappeared for 150 years last past*. I will mention but one or two. " That in the chair of *Cassiopeia* was not seen by *Cornelius Gemma* on the eighth of November 1572, who says, he that night considered that part of the Heaven in a very serene sky, and saw it not: but that the next night, Novemb. 9, it appeared with a splendour exceeding all the fixt stars and scarce less bright than Venus. This was not seen by *Tycho Brahe* before the 11th of the same month; but from thence he assures us, that it gradually decreased and died away, so as in March 1574, after 16 months, to be no longer visible; and at this day not the least signs of it remain. The place thereof in the sphere of the fixt stars, by the accurate observations of the same *Tycho*. was 0s. 9°. 17'. from the first star of Aries, with 53°. 45' north latitude". To this account Sir *Isaac Newton* adds *, that in November, when it first appeared, it seemed equal to Venus in brightness, in December to Jupiter, in January 1573 less than Jupiter but bigger than Sirius, and equal to him in February and March; in April and May equal to the stars of the second magnitude, in June, July and August to those of the third, in September, October and November to those of the fourth, in December and January 1574 to those of the fifth, in February to those of the sixth, and in March it vanished. That its colour was at first clear, white and splendid, afterwards yellow, and in March 1573 red and fiery like Mars or Aldebaran, in May of a pale livid colour like that of Saturn, which grew fainter and fainter till it vanished.

* Phil. Transf. N. 346. Jones's Abr. Vol. 1 p. 222

* Philos. Princip. p. 526.

1174. " That such another star was seen and observed by the scholars of *Kepler* to begin to appear on Sept. 30. *St. Vet. Anno* 1604, which was not to be seen the day before; but it broke out at once with a lustre greater

greater than that of Jupiter; and like the former, died away gradually, and in much about the same time disappeared totally, there remaining no footsteps thereof in January 160$\frac{5}{6}$. This star was near the ecliptick, following the right leg of *Serpentarius*; and by the observations of *Kepler* and others, was in 7ˢ. 20°. 00′. from the first star of *Aries*, with north latitude 1°. 56′."

1175. Lastly, that the sudden eruption of such another star, shining out more than usual, engaged *Hipparchus* to make the first catalogue of the fixt stars; that posterity might know what changes might happen among them.

1176. We observed above*, from Sir *Isaac Newton*, that those comets which approach so near the sun as to pass through his atmosphere, may be so much resisted and retarded after several revolutions, as at last to fall down upon the sun with a vast force. And from thence he conjectures, that the stars we have mentioned, which suddenly shine out with very great splendor and then decay gradually till they vanish out of sight, may now and then be stirred up and blaze out again by the shock of a comet falling down upon them. But those other new stars, which appear and disappear periodically, which increase by very slow degrees, and seldom exceed the stars of the third magnitude (several of which may be seen in the history I mentioned) he takes to be of another sort, or at least in another state; which revolving about their axes, like the sun, may expose their light and dark parts to us successively. For the fixt stars are undoubtedly self-shining bodys of the same kind as the sun, and therefore equally subject to large dark spots or crusts upon their surfaces. Because the light of the sun propagated to those vast distances, and reflected back from opake bodys of no sensible apparent magnitudes, would be too much rarified to affect our senses; as *Galileo* collected from the fainter lights of the remoter planets from the sun, compared to the lustre of the fixt stars.

The Origine of new Stars.
* Art. 1049.

1177. After several attempts by Dr. *Hook*, Mr. *Flamstead** and others*, to determine the annual parallax of the fixt stars, the Honourable *Samuel Molyneux* Esq; in the year 1725, erected at *Kew* a very accurate instrument, in order if possible to arrive at some degree of certainty in this difficult inquiry: in the prosecution of which he followed Dr. *Hook* in some respects, as in taking the zenith distances of the brightest star in the Dragon's head at the times of its transits over the meridian, and also in the form of his instrument, constructed almost upon the same principles with the Doctor's, but executed to a degree of exactness vastly greater, and chiefly owing to the care and contrivance of Mr. *George Graham*.

An inquiry into the annual parallax of the fixt stars.
* Wallisii opera Vol. 3. p. 701,
* Phil. Trans. N. 364. Abridg. Vol. 6. p. 165.

1178. The Rev. Mr. *Bradley* Professor of Astronomy at Oxford, who all along assisted Mr. *Molyneux* in the prosecution of this noble design,

 has

has obliged the publick with a very accurate hiſtory of it, in a letter to Dr. *Halley* *; containing not only an account of ſeveral new and ſurpriſing phænomena that attended the obſervations, (which he therefore continued and repeated after Mr. *Molyneux*'s deceaſe,) but alſo a compleat diſcovery of the true cauſe of them; which at laſt enabled him to ſettle the point in queſtion, and to draw from it ſome admirable conſequences relating to the propagation of light. As I look upon theſe diſcoveries to be ſome of the fineſt that we have had ſince the invention of teleſcopes, I will endeavour to give the ſubſtance of them in as clear a manner as I can.

* Phil. Tranſ. N 406. Abridg Vol. 6. p 167.

Some account of the obſervations and phænomena.

1179. The reſult of the obſervations upon the bright ſtar in the dragon's head, marked γ by *Bayer*, was this.

Beginning from December 3. 1725, its diſtance from the zenith being taken ſeveral days, at the time of its *tranſit* over the meridian, there appeared no material difference in the obſervations.

1180. On Decem. 17. it paſſed a little more ſoutherly from the zenith than before, and ſtill more on the 20th; which was matter of ſurpriſe, both becauſe no ſenſible alteration of parallax could ſo ſoon be expected in this ſtar at that time of the year, and becauſe it was the contrary way to what it would have been, had it proceeded from an annual parallax.

1181. About the beginning of March 1726, the ſtar was found to be 20″ more ſoutherly than at the time of the firſt obſervation, and ſeemed to have arrived at its utmoſt limit ſouthwards.

1182. By the middle of April it appeared to be returning back again towards the north, and about the beginning of June it paſſed at the ſame diſtance from the zenith as it had done in December when it was firſt obſerved.

1183. From that time it continued to move northwards till September following, when it again became ſtationary, being then near 20″ more northerly than in June, and no leſs than 39″ more northerly than it was in March.

1184. From September it returned towards the ſouth, till it arrived in December at the ſame ſituation it was in at that time twelve months, allowing for the difference of declination on account of the preceſſion of the equinox.

1185. By the like obſervations made upon a ſmall ſtar almoſt oppoſite in right aſcenſion to γ *Draconis*, and at about the ſame diſtance from the north pole of the equator, it appeared to change its declination 19″, that is about half as much as γ *Draconis* did in the ſame time. Which plainly proved, as Mr *Bradley* obſerves, that theſe apparent changes were not owing to a nutation of the earth's axis, ſince the changes on this account would have been nearly equal in theſe ſtars, as lying near the ſolſticial colure.

Upon

1186. Upon comparing the obſervations with each other it was diſcovered in both theſe ſtars, that the apparent difference of declination, reckoned from the limits above mentioned, was always nearly proportionable to the verſed ſine of the ſun's diſtance from the equinoctial points.

1187. And that the whole difference of declination in theſe ſtars, was as the ſine of the latitude of each reſpectively.

1188. After a year's obſervations upon many other ſtars, in different parts of the heavens, made with a new inſtrument ſet up at *Wanſted* in 1727, Mr. *Bradley* found out ſome other properties of their apparent motions; and after examining and rejecting two or three hypotheſes, by which he attempted to ſolve them, at laſt he conjectured that all theſe phænomena proceeded from the progreſſive motion of light and the earth's annual motion in her orbit. For he perceived, that if light was propagated in time, the apparent place of a fixt object would not be the ſame when the eye is at reſt, as when it is moving in any other direction than that of a line paſſing through the eye and object; and that when the eye is moving in different directions, the apparent place of the object would be different. I will firſt deduce ſome conſequences from this hypotheſis and then compare them with the phænomena. Mr. *Bradley's* hypotheſis to ſolve theſe phænomena.

1189. If an eye moves uniformly in a ſtraight line from *a* to *b* in the time that the light of a fixt ſtar deſcends uniformly in a ſtraight line from *c* to *b*, the ſtar will appear in a direction conſtantly parallel to *ac*. Some conſequences from the hypotheſis Fig. 683.

For conceiving the eye to carry the line *ac* parallel to itſelf, its interſection with the fixt line *bc* will move uniformly* from *c* to *b*, and will therefore accompany a particle of light deſcending uniformly from *c* to *b*; and becauſe this interſection is a moving point, not only in the fixt line *bc*, but alſo in the moving line *ac*, it is plain that the particle which accompanies the interſection *c*, moves relatively in the moving line *ac*. In like manner a particle of every other ray, parallel to *cb*, which the moving line *ac* ſucceſſively meets with, moves alſo in the moving line *ac*; and thus a ſucceſſion of theſe particles, moving along *ac*, conſtitute a viſual ray in whoſe direction the ſtar appears.

* Euclid VI. 2.

1190. Hence ſuppoſing the earth's center *b* to move uniformly in a circular orbit ♈ ♉ *Ab*, round the ſun in its center *B*; if in a line *BC* drawn towards a fixt ſtar, ſuppoſed infinitely diſtant, you take a diſtance *BC* in proportion to *Bb* or *BA*, as the velocity of light to the velocity of the earth's center, an obſerver upon the earth at *b*, will conſtantly ſee that ſtar in a direction very nearly parallel to a line *AC*, connecting the point *C* with a point *A* in the orbit conſtantly 90 degrees behind the earth. Fig. 684.

For drawing *ba* and *bc* parallel to *BA* and *BC* reſpectively, and tending the ſame ways from *b* and *B*, and alſo any line *ac* parallel to *AC*; by the ſimilar triangles *bca*, *BCA*, we have *bc* : *ba* :: *BC* : *BA*, as the velocity of light to the velocity of the earth. Conſequently if an eye

be ſuppoſed to move along the tangent *ab* with this latter velocity, it will ſee the ſtar in a direction conſtantly parallel to *ac** or *AC*. But the eye in the orbit moves with that velocity, and paſſes by the point *b* in the direction of that tangent, and therefore at that paſſage it ſaw the ſtar in the ſame direction in which the other eye in the tangent ſees it conſtantly. The earth's diurnal motion alters this concluſion ſo little that I need not here conſider it.

* Art. 1189.

1191. Therefore the apparent parallax of the ſtar to an obſerver at *b*, is conſtantly meaſured by the angle *ACB*, if the point *A* be always 90 degrees behind the earth at *b*, and conſequently 90 degrees before the ſun's apparent place ⊙ in the ecliptick.

1192. Hence the apparent latitude of any ſtar, ſuppoſed infinitely diſtant, will be leaſt of all when the ſun's place in the ecliptick is 90 degrees forwarder than the ſtar's; and from that time it will increaſe for half a year, and then decreaſe for the next half year, and its increment reckoned from theſe limits will be conſtantly as the verſed ſine of the ſun's longitude reckoned from his place before mentioned.

Fig. 685. For drawing *CD* perpendicular to the plane of the orbit, join *AD*, and draw *DB* cutting the orbit in *L* and *M*. The point *L*, neareſt to *D*, is the ſtar's place in the ecliptick, and the point ⊙, oppoſite to *b*, is the ſun's place therein. Now when the point ⊙ was at *N*, 90 degrees forwarder than *L*, the point *A*, being always 90 degrees forwarder than ⊙ *, was at *M*, the fartheſt point from the perpendicular *CD*; and conſequently the ſtar's apparent latitude, always meaſured by the angle *CAD*, was then the leaſt poſſible.

* Art. 1191.

Draw *AE* perpendicular to *LM*, and joining *CL*, *CE*, *CM*, draw *MF* perpendicular to *CE* produced. Then conceiving the point *A* to move in the perpendicular *AE* towards *E*, the angle *CAD* will approach to a *maximum* *CED*, and therefore will increaſe very little, eſpecially as the angular approach *ACE* is exceeding ſmall. Therefore inſtead of the apparent latitude *CAD* we may take *CED*, and conſequently the angle *ECM* for the increment of the leaſt latitude *CMD*: now this ſmall angle *ECM* is as its ſine *MF* or (becauſe the ratio of *MF* to *ME* varies very little) as *ME* the verſed ſine of the arch *MA* equal to *N* ⊙, the ſun's longitude from *N*, 90 degrees forwarder than the ſtar's place *L*.

1193. When a ſtar is ſituated any where in the ſolſticial colure, the increments or decrements of its latitude and declination are the very ſame quantities; and therefore if the ſtar be ſuppoſed infinitely diſtant, and its longitude be in the beginning of Capricorn, with north declination, its apparent declination will be the leaſt at the time of the vernal equinox, and the greateſt at the autumnal; and its increments and decrements reckoned from theſe limits, will be proportionable to the verſed

ſine

ſine of the ſun's longitude reckoned from the equinoctial points: which agrees with the phænomena in Art. 86.

1194. The whole apparent parallax *LPM* of a ſtar in the pole of the eclíptick, is to the whole apparent parallax *LCM* in the latitude of any other ſtar, as the radius to the ſine of the latitude *CBL*. For ſince *BP* equals *BC*, drawing *BF* and *BG* perpendiculars to *CL* and *CM*, the ſmall angle *BPL* is to *BCL* as *BL* to *BF*, that is, as the radius to the ſine of the angle *BLF* or of the latitude *CBL**. Again, the ſmall angle *BPM* is to *BCM* as *BM* to *BG*, that is, as the radius to the ſine of *BMG* or of the latitude *CBL*, as before. Therefore the whole angle *LPM* is to *LCM* as the radius to the ſine of the latitude *CBL*. Fig. 686. * Art. 204.

Hence, from the obſerved parallax in latitude, or in declination of ſuch ſtars as lye in or near the ſolſtitial colure, we have the parallax that would belong to a ſtar in the pole of the ecliptick, which is plainly the greateſt of all. Thus in γ *Draconis* whoſe latitude $CBL = 74^{\circ}. 58'. 20''$, the obſerved parallax *LCM* was $39''$*, and thence the greateſt parallax *LPM* comes out $40'',4$. Likewiſe in the little ſtar above mentioned, whoſe diſtance from the north pole of the equator is $38^{\circ}. 28'. 35''$*, and conſequently its latitude a little above $28^{\circ}. 02'. 25''$, as being almoſt oppoſite in right aſcenſion to γ *Draconis*, the obſerved parallactick angle *LCM* was $19''$*, and thence *LPM* comes out $40'',4$ as before. * Art. 1183. * The 36th. Star *Camelopardi Hevelii* in *Flamſtead's* Catalogue. * Art. 1185.

1195. Mr. *Bradley* having applyed his obſervations upon the parallax in declination of ſtars in any ſituation whatever, to his theory farther purſued, aſſures us they all conſpire to prove, that the greateſt parallax *LPM* is about 40 or 41 ſeconds, and thinks the *medium* $40''\frac{1}{2}$ cannot differ ſo much as one ſecond from the truth. The greateſt apparent parallax.

1196. Hence the velocity of ſtar-light comes out 10210 greater than the velocity of the earth's mean motion round the ſun. For the former velocity is to the latter as *BP* to *BL* or *BM**, that is as the radius to the tangent of *BPL* or $BPM = 20''\frac{1}{4}$ as above determined. The Velocity of Star-light. * Art. 1189.

1197. From what has been ſaid Mr. *Bradley* infers. 1. That the lights of all thoſe ſtars arrive at the earth with equal velocities. 2. That unleſs their diſtances from us are all equal, (which for other reaſons beſides that of their different luſtre, is highly improbable) their lights are propagated uniformly to all diſtances from them. 3. That the velocity of ſtar-light is ſuch as carries it through a ſpace equal to the ſun's diſtance from us in $8'. 13''$, this time being to the time in which the earth might deſcribe that diſtance, with the velocity of her mean motion round the ſun, as 1 to 10210, and this latter time, to half a year, as the diameter of a circle to its circumference.) 4. That the time ſo determined can ſcarce differ 5 or 10 ſeconds from the truth, which is ſuch a degree of exactneſs, as can never be expected from the eclipſes of Jupiter's ſatellites. 5. That as this determination of the velocity of ſtar-light, comes out a *medium* a- Some properties of Light.

mong

mong ſeveral determinations of the velocity of the ſun's light reflected from thoſe ſatellites, we may reaſonably conclude that the velocities of theſe lights are equal. And laſtly, ſince it is highly probable that the velocity of the ſun's emitted light is alſo equal to that of ſtar-light *, it is equally probable that its velocity is not altered by reflection into the ſame medium.

* Art. 1176.

1198. From Art. 1190, 1194. &c, it follows plainly, that a ſtar placed in the pole of the ecliptick would appear in a years time, to deſcribe about the pole a little circle whoſe apparent ſemidiameter is $20'' \frac{1}{4}$; and that any other ſtar will appear to deſcribe, about its true place, an ellipſis whoſe long axis is at right angles to the circle of longitude paſſing through the ſtar's true place, and equal to the diameter of the little circle juſt mentioned, and whoſe ſhort axis is to the long one, as the ſine of the ſtar's latitude to the radius.

Real parallax of the Stars inſenſible.

1199. Upon this theory farther purſued Mr. *Bradley* proceeds ſynthetically, by aſſuming the *maximum* of apparent parallax as determined above, and calculating tables of the differences in declination of *γ Draconis* ſituated near the ſolſtitial colure, and of *γ Urſæ Majoris* nearer to the equinoctial than the ſolſtitial colure; and by comparing the tables with his obſervations, he found they agreed together throughout the year, ſometimes in the very ſame number of ſeconds, and that in 50 or 60 obſervations of each ſtar, they never differed ſo much as two ſeconds; allowing for the variation of declination cauſed by the regreſſion of the equinoctial points: which amounts to a phyſical demonſtration of the truth of his theory, and does in conſequence afford a very ſatisfactory anſwer to the point in queſtion, concerning the real parallax and diſtance of the fixt ſtars. As to which he believes he may venture to ſay, that the real parallax in either of the ſtars above mentioned does not amount to 2″, being of opinion that if it were 1″ he ſhould have perceived it in the great number of obſervations that he made eſpecially upon *γ Draconis*; which agreeing with the theory, without allowing any thing for a real parallax *, nearly as well when the ſun was in conjunction with, as in oppoſition to this ſtar, it ſeemed to him very probable, that its real parallax is not ſo great as one ſingle ſecond; and conſequently that it is above 400000 times farther from us than the ſun.

* Art. 1189. 1190. &c.

Some account of Mr. *Molyneux*'s parallactick teleſcope.

1200. As the agreement of the obſervations with one another and with the theory, to the degree of exactneſs here mentioned, may ſeem incredible to perſons unacquainted with the ſtructure of the inſtruments here made uſe of; having by me a memorandum that I took of Mr. *Molyneux*'s when I ſaw it at *Kew*, I will endeavour to give the reader a general Idea of it; ſuch at leaſt as may ſatisfy his curioſity till Mr. *Bradley* ſhall be pleaſed to favour us with a fuller deſcription both of this and his own.

1201. The teleſcope *ab* was ſuſpended in a vertical poſition by two poliſhed

polished cylinders *c*, *d*, fixt near the top of the tube, so that their common axis, if produced through the tube, would pass at right angles to its axis, through a point near the center of the object glass. When the telescope was turned upon these cylinders, its axis of vision moved like a pendulum in the plane of the meridian; while a fine long wire *abe*, whose loop was put over one of the cylinders, hung down by the side of the tube, being gently stretched by a plumbet immersed in a vessel of water, designed to retard its vibrations. The lower end of this wire played gently against the smooth side of a slender brass plate *fg*, fixt transversely to the side of the tube, so as to point northwards and southwards; and in the middle of this plate was punched a very fine round hole at *b*, rather broader than the thickness of the wire *ab*. The telescope was gradually moved upon its axis of suspension by the pressure of a long skrew *hi*, in a direction parallel to *fg*; the skrew worked in a hole of a plate *kl*, fixt to the wall of the house, and the tube was made to bear against the end of the skrew by a weight *m* fixt to the end of a string *mno* passing over a pulley *n*, and having its other end tyed to a hook, fixt at *o* in the side of the tube. The opposite end of the long skrew *hi* was fixt, like an axis, in the center *i* of a brass wheel *pqr*, whose circumference was divided into a convenient number of equal parts, while an index *rs* fixt to the board *kl*, pointed to the divisions of the wheel. Fig. 687.

1202. Things being thus prepared, while the wheel *pqr* was gently turned, the part of the wire near *b*, which played against the transverse plate *fg*, was viewed through a double microscope *tu*, till the little hole in this plate appeared to be bisected by the wire. The telescope being thus rectified immediately before the beginning of every observation of the transit of γ *Draconis*, (which passes very near the zenith of *Kew*,) and the division of the wheel over against the index *rs* being then noted, the wheel was turned again, till the intersection of the wires in the focus was brought to touch the star at the instant of its transit. Then by the number of the revolutions of the wheel and the parts of a revolution that had passed by the fixt index *rs*, the angular motion of the axis of the telescope was easily collected, from a proper table of minutes and seconds answering to those revolutions. Now the differences of these angles found at different observations, are the differences of the star's declination. And the instrument being rectified every time as above, it is easy to understand that these angular differences could not be altered by any warping, shrinking or swelling of the materials of the instrument.

1203. Mr. *Bradley*'s instrument has the addition of a divided arch of $12\frac{1}{2}$ degrees in the place of the plate *fg*; and though its radius is but $12\frac{1}{2}$ feet, which is but half that of Mr. *Molyneux*'s, yet he is satisfied, from all the trials he has made, that when it is carefully rectified as above, its situation may securely be depended on to half a second.

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pendicular cylinders, c, fixt near the top of the tube, so that their common axis, if produced through the tube would pass at right angles to its axis, through a point near the center of the object glass. When the telescope was turned upon these cylinders, as an axis on which it moved like a pendulum in the plane of the meridian: while a fine long wire $a\,b$, whose loop was put over one of the cylinders, hung down by the side of the tube, being gently stretched by a plummet immersed in a vessel of water, designed to retard its vibrations. The lower end of this wire played freely against the smooth face of a thin brass plate $f\,g$, fixt transversely to the side of the tube, so as to point northwards and southwards; and in the middle of the plate was punched a very fine round hole at b, rather broader than the thickness of the wire $a\,b$. The telescope was gradually moved upon its axis of suspension by the pressure of a long screw $k\,l$, in a direction parallel to $f\,g$; the screw worked in a hole of a plate n, fixt to the wall or wainscot, and the tube was made to bear against the end of the screw by a weight w, tied to the end of a string m passing over a pulley, o, and having its other end tyed to a hook, fixt at p the side of the tube. The opposite end of the long screw $k\,l$ was fixt, like an axis, in the center r of a brass wheel $q\,r$, whose circumference was divided into a convenient number of equal parts, while an index $s\,t$ fixt to the board $q\,r$, pointed to the divisions of the wheel.

Things being thus prepared, while the wheel $q\,r$ was gently turned, the part of the wire near b, which played against the transverse plate $f\,g$, was viewed through a double microscope $x\,y$, till the little hole in the plate appeared to be bisected by the wire. The telescope being thus adjusted immediately before the beginning of every observation of the star γ Draconis, [which passes very near the zenith of Kew,] and the position of the wheel observed; the index $s\,t$ being then noted, the wheel was turned again, till the intersection of the wires in the focus was brought to touch the star at the passing of its transit. Then by the number of the revolutions of the wheel and the parts of a revolution that had passed by the fixt index, the angular motion of the axis of the telescope was easily collected from a proper table of minutes and seconds answering to those revolutions. Now the differences of these angles found at different observations, are the differences of the star's declination. And the like constantly rectified every time as above, it is easy to understand that these angular differences could not be altered by any irregular trembling or swelling of the materials of the instrument.

Mr. Bradley's instrument has the tenth part of a divided arch of a degree in the place of the plate $f\,g$, and though its radius is but $12\frac{1}{2}$ feet, which was but half that of Mr. Molyneux's, yet he is satisfied, from all the trials he has made, that when it is carefully rectified as above, its situation may safely be depended on to half a second.

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AUTHOR'S REMARKS

UPON THE WHOLE WORK.

TO shorten the way to the principal conclusions in the foregoing work, I forbore to mention many things not unworthy of notice; but being not necessary to the train of reasoning, for greater perspicuity I chose to separate them from it; and to put them together under the title of Remarks, to be perused or passed by at the pleasure of the reader.

Having begun a new numeration of the additional figures for the Remarks, I quote them thus, Fig. 1. 2. &c. and the former figures thus, FIG. 1. 2. &c. which I omit to quote whenever the Remark relates to the figure belonging to the article it self.

UPON BOOK I.

Chapter 1. Concerning Light.

Upon ART. 4.

1. By some authors the angle ACP is called the angle of inclination, and ECQ the refracted angle, and their difference ECG the angle of refraction; which for greater distinction may be called the angle of deviation.

Upon ART. 13.

History of refractions. 2. *Hugenius* gives us the following history of the discovery of this law [a]. "The refraction of rays at the surfaces of transparent bodies, is a thing taken notice of by the ancients. For *Aristotle* has a problem concerning the apparent curvity of an oar in water; and *Archimedes* is said to have written a book about the appearance of a ring or circle under water; in which no doubt he considered the refraction of the rays and the fallacy of sight thence arising. *Albazen* the Arabian and *Vitellio* tell us that the Angles of incidence and refraction are in a given ratio, and thought they had pretty well proved it by experiments; but this proportion being found erroneous in large angles, the moderns began to examine the matter more strictly. *Kepler* among the rest made several experiments about it, but missed his aim [b]; nevertheless his conjectures and attempts became useful to others. After the invention of telescopes, the subject of this enquiry, being thought more valuable than before, was farther pursued; and *Willebrordus Snellius* after many troublesome experiments and attempts, at last found out the truth; but still he did not throughly comprehend his own invention.

a Dioptr. page 1.

b Paralipomena ad Vitellionem.

Fig. 2. 3. It was this. Supposing the surface of water to be AB, and an object under it at D, which to the eye at F appeared as it were in the line FC; he produced this FC till it met in G with the perpendicular DA to the surface AB. Then he affirmed that the image of the object D appeared at G; and that CD was to CG in a certain given ratio, as of 4 to 3, if the fluid was water. This is very true and agrees perfectly with our 5th law. Because, by the known properties of the triangle CDG *, the side CD is to CG, as the sine of the angle DGC or AGC or HCF, to the sine of CDG or DCE; which are the angles of incidence and refraction. * Art. 221.

4. But yet *Snellius* never imagined that this was the ratio of the sines. For being prepossessed with a notion that all depended upon the apparent image; even of the perpendicular HC he thought there was a refraction, or as he falsly calls it a decurtation or shortening of the visual ray: being led into the mistake by observing that the whole bottom of a vessel, filled with water, seemed to rise up when viewed from above. But the true cause of this appearance is to be gathered from the tendency of the rays to both the eyes. [Here our author himself has made a slip as well as *Snellius*. For the bottom of the vessel seems as much elevated to one eye as to both, and the true cause of its elevation is explained in Art. 139, 145 and 146.] I have seen the whole book which *Snellius* wrote upon this subject, though not yet published; and am told that *Des Cartes* saw it too; and perhaps it was from hence he found that the true measure of refractions was to be taken from the sines of the angles: and this he applyed very successfully to the explication of the rain-bow, and the determination of proper figures of glasses to refract rays to a given point. So far *Hugenius*.

Upon Chapter 2. Concerning Glasses.

Upon Art. 42.

5. I forgot to observe that Fig. 59 and 74 represent the refractions of a ray passing through a sphere placed within a medium denser than it self.

Upon Art. 48.

6. line 1. read thus. On the contrary, if rays be returned directly back from the focus F in convex glasses, and towards it in concaves, the &c. See the end of that article.

Upon Art. 60.

7. Read the title of this article thus. Small angles subtended by the same perpendicular, are reciprocally as its distances from the angular points.

Upon Chapter 3. Concerning the eye and manner of vision.

Upon Art. 85.

Variations of figure in perfect eyes.

8. If an object be viewed distinctly and successively at three different distances from the eye; the first of which may be the least distance at which it can be viewed distinctly, the second double the first, and the third infinite; it is remarkable that as great alterations in the figure of the eye are necessary for seeing the object distinctly at the first and second distances, whose difference is but small, as at the second and third, whose difference is infinite. For let $BCDE$ be the axis of the eye infinitely produced; BC, BD, BE, the three distances of the object from the cornea AB; and CA, DA, EA, three rays falling upon any given point of the cornea; whereof EA is parallel to the axis. (Fig. 2.)

9. Now to procure distinct vision of the points C, D, E; it is plain that every one of the rays CA, DA, EA must be successively refracted to the same point F upon the retina, where it is cut by the eye's axis. At first let us suppose the point F to be given, or the length of the axis BF to be immutable; and then the quantity of the refraction of each ray must be varied. And because the distance CD is supposed equal to CB or CA, the angle CAD is equal to CDA and consequently to DAE. Therefore conceiving each ray to come back again from the fixt point F, to the points C, D, E successively; the whole quantity of its refractions must first be lessened by the angle CAD, and then by the equal angle DAE; and so the changes of the figures of the refracting surfaces must be much the same when the object is removed from C to D, as when it is removed from D to E.

10. Secondly, let all the refracting surfaces be supposed to keep their figures immutably; and let F be their principal focus, that is, of rays falling parallel upon the cornea after refraction through them all; and likewise G their other principal focus, that is, of rays falling parallel upon the backside of the chrystalline and refracted through them all. I find by computation that BG is but 5 or 6 tenths of an inch, and therefore if we make GC equal to CD, reckoning the distances of the object from G instead of B, the present case will not be much different from the former. Now let a pencil flowing from C be refracted towards c, and another from D towards d; and by art. 373 we have Fc reciprocally as GC; (as if the refractions were only made through a lens [a];) that is, $Fc : Fd :: GD : GC :: 2 : 1$; that is, the variations cd and dF of the places of the retina are equal to each other while the distance GC varies from single to double and then to infinite. (Fig. 3.)

a Art. 240.

11. Lastly if we suppose distinct vision to be successively procured by a variation partly of the place of the retina, and partly of the figures of the refracting surfaces, it is easy to apprehend that the variations of all these parts taken together must still be equal in both the cases abovementioned. As to Mr. *Huygens*'s opinion that the crystalline may approach towards the cornea for seeing near objects distinctly; I formerly computed, from such measures of the eye as I then had, that if the crystalline could move so far as to touch the cornea, even this motion would be too little for the purpose aforesaid. But even this is obstructed by the Uvea, which lyes much nearer to the crystalline than to the cornea, according to M. *Petit*; who finds also that the surface of the uvea in human eyes, is not spherical but plane. See Mem. de l' Acad. 1728. p. 206. 4to.

In short-sighted eyes.

12. Hence if short sighted persons can read a small print distinctly at two different distances, whereof the larger is but double the lesser, which I believe most of them can do; it follows that as great alterations of figures are made in their eyes as in perfect eyes, that can see distinctly at all intermediate distances between infinity and the larger of those two. And this is the reason that a short sighted person can see distinctly at all distances with one single concave of a proper figure; which otherwise must have been differently figured for different distances.

13. It follows then that the cause of short sightedness, is not a want of power to vary the figure of the eye, and the quantity of refractions;

but that this whole quantity is always too great for the distance of the retina from the cornea.

Dr. Pemberton's opinion upon these variations in the eye.

14. The ingenious and learned Dr. *Pemberton* upon taking his degree at *Leyden* has printed a dissertation upon this subject; and in the dedication of it to Dr. *Mead*, has declared his opinion to be this. That the crystalline changes its figure in such a manner, that as one surface becomes rounder, the other becomes flatter; but which of the two surfaces becomes rounder for viewing near objects, and then flatter for remote ones, he says, he had not then discovered. But I could never satisfy my self of the certainty of the phænomenon he builds upon. For though the experiment was carefully made in the manner he directs, both by my self and one or two of my friends, yet none of us met with the expected success. Possibly I might have repeated it again, had I not since been apprised that the Doctor's conclusion could not stand good, although the experiment were to succeed ever so well; inasmuch as the whole tenour of his reasoning depends upon this fundamental supposition: viz. That in viewing an indistinct object the pupil of the eye is always contracted to the least size of which it is capable. For the contraction of the pupil does not depend upon the indistinctness of the object only, but also upon the degree of light.

Mr. de la Hire's opinion. Dissertation sur les differens accidens de la Vüe. Mem. a Paris. 1694.

15. Mr. *de la Hire* is the only writer I have ever seen who maintains, that for viewing objects distinctly at different distances from us, no variations are ever made in any part of the eye except in the size of the pupil. The experiment he builds his conclusion upon may be made in the following manner.

Fig. 4.

16. Let two very small holes, *a*, *b*, be made with a pin in a card or in paper, so near to each other, that being held close to the eye, the rays that come through both, may enter the pupil. Then let a small black spot *c* upon a white paper, be viewed through them: and if the experimenter be short sighted, let the paper be placed at such a distance from his eye, as he usually sees an object at with most distinctness and most ease, as suppose at the distance of six inches; and in looking through the holes the spot will appear distinct and single. Then let the paper be removed to a greater distance, suppose of ten inches, such as that the same eye may be able to see an object without any apparent indistinctness. Then let the spot be attentively viewed by the naked eye, in order to make such a change in its conformation, as is usually supposed necessary to see an object distinctly at such an increased distance. Now, the eye being supposed to have taken the necessary conformation for seeing that spot distinctly at that distance, it may in consequence of this supposition be expected, that upon clapping the card before the eye and looking through the two holes, the spot should appear single, as it did at the former distance of six inches. But experience shews the contrary: for it appears double like two distinct spots, *d*, *e*; whose interval *de* is so much the greater as the distance of the paper from the eye is greater, as in fig. 5. Fig. 5.

17. If the experimenter be long sighted, let the paper, likewise, be first placed at such a distance from the eye, as he usually sees an object at with most distinctness and most ease, as suppose at the distance of fifteen inches; and in looking through the holes the spot will appear distinct and single. Then let the paper be brought nearer, suppose to the distance of seven inches, at which distance the same eye is able to see the object without any apparent indistinctness. And let the spot be viewed attentively by the naked eye, in order to make such a change in its conformation, as is commonly supposed necessary in order to see an object distinctly upon so lessening the distance. Now, the eye being supposed to have taken the necessary conformation for seeing the spot distinctly at that distance, it may, in consequence of this supposition, be expected, that upon clapping the card before the eye and looking through the two holes, the spot should appear single, as it did at the former distance of fifteen inches. But experience shews the contrary: for it appears double like two distinct spots, *d*, *e*, whose interval *de* is so much the greater as the distance of the paper from the eye is less, as in fig. 7. Fig. 6. Fig. 7.

18. Now the spot appears single in the first case of each of these two experiments, because the rays *ca*, *cb*, fig. 4 and 6, are united at a single point *f* exactly upon the retina. But when the paper is remoter, fig. 5, *ca* and *cb* diverge less than before, and therefore are reunited at *f* before they arrive at the retina; then crossing each other they fall upon it in two distinct points *g*, *h*, which occasion the double appearance at *d* and *e*. For if the holes be moved upwards, the upper spot first disappears at *d*; because the upper ray *ca*, first misses the pupil. And when the paper is brought nearer, as in fig. 7, the rays *ca*, *cb* diverge more than before, and therefore tend to reunite in the point *f* behind the retina, upon which they fall in two distinct points *g*, *h*, which occasion the double appearance at *d* and *e*. For if the holes be moved upwards, the under spot first disappears at *d*, because the upper ray *ca* first misses the pupil.

19. We come now to consider the consequence which Mr. *de la Hire* draws from these experiments. His argument runs thus. It is commonly believed, that an eye which is so formed, as naturally to unite the rays upon the retina, when the object is at six inches distance, can make such a change in its conformation as still to unite

them exactly upon the retina, when the object is removed to a greater distance, as that of ten inches. If this opinion were true, the eye of the observer in the second case of the first experiment*, must have made such a change in its conformation. But the experiment shews that this eye was not in such a conformation as to unite the rays exactly upon the retina; for upon clapping the card before it, the appearance was of two distinct spots, not of one only, as it ought to have been, if the eye had had the supposed conformation. And just after the same manner he reasons upon the second experiment.

* Rem. 16.

20. In order to make this reasoning conclusive, Mr. *de la Hire* ought to have proved, that whatever conformation the eye had, in viewing the spot without the holes, the same conformation must necessarily have continued, when the spot was seen through the holes.

21. But we take the contrary to be highly probable. For when the spot was viewed at the distance of six inches, the eye was then in its natural conformation. It received the rays in the same manner as an artificial eye of the same dimensions might have done, without any the least strain, *nisus* or endeavour. But when the spot was viewed at the distance of ten inches, it must at the first instant have appeared indistinct, and in order to remedy that indistinctness, the eye may be supposed to have exerted some force in order to change its conformation so as to suit it self to that distance. If so, this forced conformation will continue while the occasion remains, and no longer. While the eye is viewing the object at ten inches distance, if it happens in the least to relax and unbend it self, a sense of indistinctness will immediately begin to arise, which will serve as a monitor to return exactly to the necessary conformation; but the moment the eye ceases to view the object at that distance, it will probably depart from this forced conformation, and return to its natural conformation suited to the object at six inches distance. Therefore when the card is clapped before the eye, as it must necessarily then lose sight of the spot, before it comes to see the spot through the two holes, it may then probably depart from the forced conformation, and return to its natural state, or near it; the consequence of which is, that the rays will not now unite upon the retina, but will therefore exhibit the appearance of two spots.

22. I might here observe that Mr. *de la Hire* himself must necessarily admit one alteration in the eye at this instant of time, namely the dilatation of the pupil. Why then may not the conformation of the coats and humours as well be supposed to change at the same time.

Upon Art. 87.

History of the causes of vision.

23. Great disputes have been among the ancient philosophers, whether vision be caused by reception of rays into the eye, or by emission of them from the eye to the object. At last the latter opinion was generally received and followed by *Euclid*, *Ptolemy*, *Alhazen* and other ancient opticians; who thought it more agreeable to nature that certain Emanations, by them called visual rays, should flow from an animate substance to an inanimate one, rather than on the contrary.

24. *J. Baptista Porta* is said to be the first discoverer of those pictures of external objects that appear upon the wall of a dark room, and are made by rays coming through a small hole in the opposite wall. This he describes at large in his book *de Magia Naturali* printed in 1560, and shews by what methods the distinctness of these pictures may be promoted: and then concludes that he had not only decided the grand dispute about the reception and emission of rays, but had also found out the true cause of vision. For, says he, the image is let in through the pupil, and is painted upon the surface of the crystalline humour, which answers to the wall in the dark room, as the pupil does to the hole in the window-shutter. And here he followed the opinion of *Vitellio* and others, who imagined we began to perceive, when the crystalline was enlightened, but that the perception was not compleat till it was propagated from thence and united as it were in the optick nerve; that vision was then distinct when the rays fell perpendicularly upon the crystalline; and confused, when coming from collateral objects, they fell obliquely upon it.

25. These were the received notions about vision till about the year 1600, when *Kepler* made the grand discovery*, and shewed by his geometry in what manner the rays were refracted through all the humours of the eye, and formed a distinct picture upon the retina; in like manner as pictures are formed by a glass globe full of water. He also discovered the constitution of defective eyes; that is, how the pictures became confused, and shewed in what manner they were rendered distinct by spectacles and concave glasses, whose effects had been so much admired for two or three hundred years, and had so long perplexed the greatest wits to account for them.

* Paralipomena ad Vitellionem.

Inverted pictures in the eye considered.

26. The vulgar account of objects appearing erect notwithstanding the inversion of the pictures upon the retina, is also *Kepler*'s; who tells us that the mind perceiving an impulse of the ray Pp (in Fig. 153) on the lower part of the retina, considers this ray as directed from an higher point of the object; and likewise perceives the impulse of the ray Rr upon the higher part of the retina, to be directed from the lower part of the object. *Des Cartes* illustrates this solution, by conceiving a blind man to hold in his

his hands two sticks crossing each other, and to push the top and bottom of an upright object, with their extremities; and observes that this man will judge that to be the upper part of the object, which he pushes with the stick held in the lower hand, and that to be the lower part of the object which he touches with the stick in his upper hand. But we shall shew hereafter that all this is quite foreign to the purpose.

Spots in the eyes.

27. People in growing old, are often troubled with the appearance of dark, irregular spots continually flying before their eyes, like flies; especially in looking at bright objects, such as white paper or the sky-light. M. *de la Hire*'s account of them is this. They are of two sorts, some permanent, which in fixing the eye upon a point of an object, appear always fixt in the same situation to that point; others seem to fly about, and to change their situation though the eye be fixt. The shapes of both sorts are changeable; the first sort are commonly like a dark spot upon a white ground, the second sort appear like the knots of a deal board; some parts of them being very clear and surrounded with dark threads; they are alſo attended with long fillets of irregular shapes, which are bright in the middle and terminated on each side by parallel black threads. Sometimes after shaking the head suddenly and fixing the eye upon an object, they appear to descend gradually.

28. The spots that appear fixt in respect to the axis of the eye, for that reason must be caused by some disorder in a corresponding fixt part of the retina, or in some part of the vitreous humour lying pretty close to the retina. For an opacity of the coats or humors in any part remoter from the retina, by intercepting some part of the rays of every pencil, could only cause an uniform obscurity or faintness of light in every place of the retina, and not a total defect of it in any particular place[a]. An instance of this kind may be seen in the Phil. Trans. N°. 384, of a woman who in looking with the left eye only, at three short words in print, could see the extreams but not the middlemost; and in looking with the right eye only, at the middle between four short words, could see but three of them; one of the two middlemost being covered by a dark round spot; but in looking with both eyes she could see them all.

a Art. 281.

29. M. de la *Hire* attributes the cause of these permanent spots to small drops of extravasated blood upon the retina. But he finds it more difficult to account for the volatile spots. When the rays of the sun are transmitted through a piece of bad glass, and fall upon white paper, the shadows of the little sands, veins and irregularities in it, appear not unlike these spots. He therefore imagines the aqueous humor is sometimes troubled with some little mothery, ropy substance; some parts of which, by the figures of their little surfaces or by refractive powers, different from the humor it self, may cast their distinct images upon the retina He supposes them in the aqueous humor rather than the vitreous, because of its greater fluidity for a freedom of descent, and because they will then appear to descend; as being situated before the pupil or at least before the place of intersection of the pencils. But if this heterogeneous mother be in the vitreous humor, it must be lighter than this humor, so that after a sudden shake of the head, it may first descend a little, and then ascend gradually, to cause the gradual apparent descent abovementioned. These spots are observed to change their figures, sometimes in two or three hours, at other times not in two or three days; and to appear more numerous at one time than another. From *de la Hire*'s *differens accidens de la vue*.

Opacity of the cornea.

30. It sometimes happens that the cornea grows opake; which I am told may be cured by glass ground to an impalpable powder and blown every day into the eye; provided the opacity does not quite go through the thickness of the cornea.

Opacity of the crystalline.

31. An opacity of the crystalline is called a Cataract and is cured by couching; that is by piercing the side of the cornea with a sharp needle and by forcing the crystalline out of its place into one side of the eye. And then the patient is obliged to use a very convex glass, to supply the want of the crystalline.

Part of the retina insensible of light.

32. Monsr. L' Abbe *Mariotte* is the author of a curious experiment, shewing that any object, whose picture falls upon the base of the optick nerve, where it enters the bottom of the eye, is not perceived while the other eye is shut. He describes this experiment in a letter to M. *Piquet*, as follows[b]. "Having often observed in dissections of men as well as brutes, that the optick nerve does never answer just to the middle of the bottom of the eye, that is to the place where the picture of the objects we look directly upon is made; and that in man it is somewhat higher and on the side towards the nose; to make therefore the rays of an object to fall upon the optick nerve of my eye, and to to find the consequence thereof, I made this experiment.

b Phil. Trans. N°. 35.

33. I fastened on an obscure wall, about the height of my eye, a small round paper to serve me for a fixt point of vision; and I fastened such another on the side thereof towards my right hand, at the distance of about two feet, but somewhat lower than the first, to the end that it might strike the optick nerve of my right eye, while I kept my left shut. Then I placed my self over against the first paper, and drew back by little and little, keeping my right eye fixt and very steady upon the same; and being about ten

ten feet distant, the second paper totally disappeared.

34. That this cannot be imputed to the oblique position of the second paper to my eye, is hence evident, that I can see other objects farther extant on the side of it. So that one would believe the second paper was taken away by a slight, if one did not soon find it again by the least stirring of ones eye. This experiment I made often, varying it by different distances, and by removing or approaching the papers one to another proportionably. I made it also with my left eye by keeping my right shut, the second paper being fastened on the left side of my point of vision. So that by the situation of the parts of the eye, it cannot be doubted but that this defect of vision is upon the optick nerve; and the same experiment succeeds in other persons though not exactly at the same distances.

A remarkable inference from it.

35. This experiment hath given me cause to doubt, whether vision be indeed performed in the retina, (as is the common opinion,) or rather in that other membrane, which at the bottom of the eye is seen through the retina and is called the Choroïdes. For if vision were made in the retina, it seems that then it should be made wherever the retina is; and since the retina covers the whole nerve, as well as the rest of the bottom of the eye, there appears no reason to me, why there should be no vision in the place of the optick nerve where it is. On the contrary if it be in the choroïdes that vision is made, it seems evident that the reason why there is none in the nerve, is that the choroïdes parts from the edges of it, and covers not the middle thereof, as it does the rest of the bottom of the eye.

Disputed.

36. *Piquet*'s objections to this inference are specified in *Mariotte*'s reply[a], from which I have extracted what follows. "You say in your first objection that if the sclerotis and choroïdes be taken away from a fresh eye, and the retina be left distended on the vitreous humor, one shall not be able to see well through this membrane; whence you conclude it is not transparent enough to let so much light pass upon the choroïdes as is sufficient for vision. In my opinion there is reason to doubt of this consequence. There being a great difference between the retina of a dead animal and a living one. Different dispositions change the qualities of things. Fat, which is trasparent when melted, grows opake when cold. The tunica cornea of an eye being held some hours in ones hand, in a hot air grows thick and a little after opacous. Take the eye of an ox new killed, while it is hot, and cut it in two, in such manner that a good part of the vitreous humor may remain extended upon the retina; then you shall see distinctly the colours of the choroïdes, the base of the optick nerve, the trunk of the little vessels that proceed from thence, and their disposition through the thickness of the retina; with so much perspecuity that you cannot even discern whether there be a retina beyond the vitreous humor or no. The transparency of the retina will also be evident by this other experiment. Place by night a lighted candle near your eyes, and cause a dog, distant from the candle 8 or 10 paces, to look upon you; then you shall see in his eyes a light sufficiently bright. Which I hold to proceed from the reflection of the candle; whose image is painted on the choroïdes of the dog; which having much whiteness and lustre, causes this strong reflection. For if it proceeded from the crystalline or the retina [or the far surface of the vitreous] the same appearance would be seen in the eyes of men, birds and other animals, who have the choroides black; which is not found so by us. It is therefore manifest by this experiment that the rays do pass with much force as far as the choroïdes. The like experiment may be made with cats, in whose eyes this light appears blueish, which shews that it proceedeth from their choroïdes which hath much of this colour. But this colour or any other, which may be in the choroïdes, brings no confusion to the sense of seeing. For the senses receive no impression from their own organs.

a Phil. Transf. N°. 59.

37. Black bodies grow hotter in the sun and take fire sooner than white ones; and therefore the light acts more upon them. And this is the cause why men and birds see better and more distinctly than most other animals. For their choroïdes being black and by consequence very sensible of light, they contract much their pupil or sight-hole of the eye, which makes the rays which pass there from each point of the object, to be all very near the axis of the crystalline and to reunite more exactly in a point[a], than in the eyes of most other animals which have their choroïdes white towards the axis of the eye and by consequence less sensible of light: who in recompence can very much dilate the pupil of their eyes when they stand in need of a great light. But then their sight is not so distinct, because the rays which fall towards the edges of the crystalline, do intersect the axis too near the crystalline in their refractions[b].

a Art. 81.

b Art. 73.

38. It is true that to supply this defect in some sort, they have a little crystalline in the middle of the great one; and this little crystalline being of a more spiss consistence than the great one, its refraction is also more strong[c]; and makes the rays, which come from a single point in the axis produced, and pass near the center of the crystalline, to be more refracted than if there had been but one crystalline; and brings their intersections with the axis nearer to the intersections of the rays that fell more remote[d].

c Art. 5.

d Art. 73.

39. Fishes also have a double crystalline. For other-

otherwise their sight would be more confused than that of other animals who live in the air. For their crystalline being spherical, if it was homogeneal, the rays of a pencil would cut the axis in points at greater distances from one another, than if it had been lenticular, and its convexities were of greater spheres.

40. I come now to a more particular proof of the want of vision on the base of the optick nerve. It is agreed that in this experiment most men lose sight of an entire circle of white paper, whose diameter is about the 9th or 10th part of its distance from the eye. Now the visual triangle whose base is the diameter of this circle and vertex is near the center of the eye, is nearly milar to the triangle, whose base is the diameter of the picture of that circle upon the retina and vertex the same center, where the extream rays intersect each other: the which center being 6 or 7 lines from the basis of the optick nerve [c], whose breadth is about $\frac{3}{4}$ of a line, this basis will also be about the 9th or 10th part of its distance from the said center in the eye. And so the picture of the white paper will precisely cover the base of the optick nerve: and because the paper then wholly disappears, it follows that the basis of the optick nerve is insensible of light. Whence I conclude more accurately than before, that the choroïdes is the principal organ of sight, and that the retina is not; seeing the retina is placed in that basis, and is there apparently disposed in like manner as the rest of it upon the bottom of the eye.

c Art. 90.

41. You alledge that the trunks of the vessels which proceed from the basis of the nerve, may be the cause of this defect of sight. Yet you cannot deny but that they are very small; and that it is very hard to discern the little holes through which they pass, when the nerve is cut off above its insertion into the eye. Now because they come out of the basis by two distinct little holes, the diameter of each of which does not take up above an eigth part of the diameter of the basis, it follows, if the rest of the basis was sensible of light, that we should not lose sight of a paper of 2 inches diameter at most, at 10 feet distance; and sometimes in fixing one eye upon a little piece of paper, two other very little ones, separated one from the other, would disappear; which is contrary to experience: for the default of vision is continued.

42. To confirm my opinion the more, I will add some other observations. The first is that the pupil dilates it self in the shade and contracts in a great light: and it is hard to account for this unvoluntary motion, but by the hypothesis that the choroïdes is sensible of light. For then it is easy to conceive that being hurt by too much light, it may dilate or contract its fibres, which have one continuity with those of the fore part of the uvea: and thus it can contract its aperture; and when it is not hurt, relax it again. Whereas if the retina be supposed the organ of sight, it will be very difficult to explain how this contraction is made. The author's other arguments are taken from the structure of the eyes of birds.

43. So far Mr. *Mariotte*; whose experiment I have tryed in a chamber from which all sensible light was excluded, except what came into it through a key-hole; and this also disappeared totally when it fell upon the base of the optick nerve; which shews it to be totally insensible of light. Yet in looking at objects of an uniform colour with one eye, we are not sensible of any such defect or dark round spot, as the woman was, whom I mentioned in the 28th remark. This defect of sensation having been constantly supplyed by the other eye, is now supplyed by the imagination only.

44. M. *Picard* has devised a way by which an object is lost keeping both eyes open [a]. "Fasten against a wall a round white paper of the bigness of an inch or two; and on the sides of this paper put two marks, one on the right, the other on the left side, each about two feet distant from the paper, and somewhat higher. Then place your self directly before the paper at the distance of nine or ten feet, and put the end of your finger over against both your eyes, so that it may hide from the right eye the left mark, and from the left eye the right mark. If you remain firm in that posture, and look steadily with both eyes on the end of your finger, the paper which is not at all covered thereby, will totally disappear. Which is the more surprising, because without this particular encounter of the optick nerves, where no vision is made, the paper will appear double; as you will find when the finger is not rightly placed. See FIG. 190.

a Phil. Transf. N°. 35.

45. As to the dispute whether the retina or the choroïde be the principal organ of sight. I will subjoin a remark taken from M. *de la Hire*'s dissertation abovementioned. "To clear up this difficulty in some measure, we must consider what relates to our other organs of sense; and I think by this comparison it will be evident enough, that the retina is the principal organ of sight, although a certain part of it be not susceptible of immediate impressions from outward objects. I say then the retina is the principal organ of sight, as being an expansion of the optick nerve; because we must not seek for sensations any where else but in the nerves. But yet this organ may receive the impressions of light from an intermediate organ, which receives them immediately from the object it self. For this is the case of the organs of our other senses. Whence it is evident that this intermediate organ is the choroïde, because it touches and supports the retina

The dispute adjusted.

tina every where but at the base of the nerve; and being of a dark colour it is more capable of being agitated by the impressions of light, than if it was white or transparent. Nature acts in the same manner in the organ of hearing. For the labyrinth has a proper disposition to receive the tremblings of the air, and to communicate them to the ramifications of the auditory nerve expanded within it. The case is the same in the other senses, as M. du *Verney* has observed in the 96th page *de l' organe de l' oüie*. The constitution of the nerves is too tender and delicate to be exposed to the naked impressions of external bodies; and therefore the coats of the nerves receive these impressions, and communicate them to the nerves themselves in a proper manner for exciting sensations.

Beams of light shooting from candles.

46. This Author accounts for those beams of light which appear to shoot upwards and downwards from a lighted candle, as follows. The circumstances of them are these: while you look at the candle if you bend your head downwards, you will only see the upper beams, and in lifting your head upwards you will only see the under ones; and to see both the upper and under beams at once, you must hold your head upright and almost close your eye-lids. To shew the cause of these appearances, it is to be observed that the eye is always moistened with a tenaceous sort of water, which is gathered more plentifully to the edges of the eye-lids than any where else, because they rub the cornea. This water sticking to each eye-lid, is there formed into a concave surface, like that of water adjoining to the sides of a drinking glass: and then the rays which flow from the candle *B*, in passing through this concave at the upper eye-lid *H*, (which when the head is bowed downwards, is below the upper margine of the pupil) are there refracted upwards from the thinner towards the thicker part of the water[a]; and therefore in entering the pupil they diverge from one another[b], and fall upon the upper parts of the retina, as if they had come from a long beam *BN* below the candle *B*. But if the head be bowed too low, the projection of the upper eye-lid and eye-lash, may hinder the rays from falling upon this watery surface; and then the beam will disappear, though the candle may still be visible, by rays passing as usual through the lower part of the pupil; which is also agreeable to experience. In this position of the head no beam can appear above the candle, because the lower eye-lid *I* is below the pupil.

Fig. 8.

a Art. 40.
b Art. 41, 42.

47. But when the head is lifted up, the eye being fixt upon the candle, the under eye-lid is also lifted above the lower margine of the pupil; and then the rays which pass through this watery concave at *I* are bent downwards and are directed through the pupil towards the lower part of the retina; and cause the appearance of a beam above the candle. But no beam can appear below it because the upper eye-lid is now above the pupil.

48. Therefore holding the head upright, and closing the eye-lids till their interval be less than the diameter of the pupil, it is plain that the beams will shoot both upwards and downwards. And these explications are confirmed by intercepting the rays that fall upon these watery concaves at *H* or *I*, with any obstacle *P* held close to the eye. For when the beam *BN* appears below the candle if the obstacle be raised gradually from below towards the pupil, it will have no effect; but if from above the obstacle be brought downwards, the beam below the candle will disappear before the candle it self does.

49. Though M. *Rohault*'s sentiments upon this subject cannot be supported[c], yet I do not deny but some rays are reflected from the thickness of the eye-lids into the pupil, in some positions of the eye and candle: but these rays cause a different appearance from what I have been speaking of. As soon as I had invented this solution I intended to have printed it, but I found soon after that M. *Briggs* an English physician, had in general given the same solution in his *Opthalmography*. So far *de la Hire*.

c System of Nat. Phil. Part. I. Chap. 35.

Their true cause.

50. The manner in which these gentlemen have explained these appearances, would have been very just, had the extremities of these beams appeared coloured. But since they do not, they cannot be caused by such great refractions[d], though they may by inflections of the rays[e] upwards and downwards at the edges of the upper and under eye-lid, when over against the pupil.

d Art. 172.
e Art. 185.

Upon Art. 88, 89.

Glasses for defective eyes determined.

51. In order to determine the properest glasses for defective eyes, the limits of confused and distinct vision, or the distances of those places from the eye, where an object begins to appear confused, may be found by measuring the least distance from which a long-sighted person can see a pretty large print distinctly and read it readily: and likewise by measuring the greatest and the least distances, from which a short-sighted person can see a small print distinctly and read it readily: or still more exactly by placing the end of a long ruler close to the eye, or rather a little under it, and by observing the greatest and least distances at which the lines drawn lengthways along the ruler, begin to appear confused. I shall call those glasses the properest for defective eyes, which are the least concave or the least convex of any that will answer the purpose of distinct vision; for a reason to be mentioned hereafter.

Fig. 9.

52. Let *Eq* be the least distance from which any small objects appear distinct to the eye of a long-

long-sighted person; and EQ the least distance from which he wants to see them distinctly. Towards q take QF to QE as QE to Qq, and EF will be the focal distance of a convex lens, which being put close to his eye, will make him see an object distinctly at any place between Q and F, and possibly beyond F. For the rays that flow from Q will emerge from the glass and will enter the eye as if they had come directly from q, to the naked eye[a]; and supposing Q to recede from the eye, q will also recede from it to infinity through places where the naked eye can see distinctly. And therefore the refracted rays, diverging as from these places, will also produce distinct vision of the object Q as far as to F; and still farther if the person can see distinctly by converging rays.

[a] Art. 239.

53. Therefore if he wants to see distinctly from no less a distance than half Eq, that is, only as near again as with his naked eye, a convex lens whose focal distance is Eq will be the properest; and will make him see an object distinctly at any distance not less than half Eq. For supposing Qq and QE to be equal, the point F will fall upon q by the foregoing proportion.

Glasses for short-sighted persons. Fig. 10.

54. Let EF be the greatest distance from which an object at F appears distinct to the eye of a short-sighted person; and it will also be the focal distance of a concave lens, which being put close to his eye at E, will be the properest for seeing remote objects distinctly. Because the rays of a pencil, which come from any remote object, and consequently fall parallel upon the lens, will emerge from it to the eye, as if they had come directly to the naked eye from an object at F. And consequently the picture of the remote object formed upon the retina by rays refracted through the lens, will be as distinct as the picture of an object at F seen by unrefracted rays.

Fig. 11.

55. Let Eq be the least distance from which the same person can see an object distinctly with his naked eye; then say as QF to QE so QE to Qq, and placing Qq towards F, the point q will be the nearest point, which he will be able to see distinctly through the lens abovementioned. For by art. 239, the rays of a pencil which fall upon the lens converging towards Q, will after refraction converge to q; and on the contrary, the rays which flow from q will emerge from the lens diverging from Q; and supposing the point q to recede from the eye, the point Q will also recede from it to such places where the naked eye can see distinctly. But if the point q approaches towards the eye, the point Q will also approach towards it, to such places where the naked eye cannot see distinctly, by the supposition.

56. Consequently if QF, the space between the limits of confused vision, be not less than QE, that one glass whose focal distance is EF, will make all objects appear to him distinct which are any where placed beyond F, the reach of his naked eye. For in this case Qq cannot be greater than QF, as is manifest by the proportion above.

Fig. 12.

57. But if he wants a pair of concave spectacles to read or write with, let the distance Eq be no greater than what is convenient for that purpose; and let QF be the limits of confused vision as before; and towards q take FG to FE as FE to Fq, and a concave lens whose focal distance is EG will be the properest for this purpose. For by art. 239, the rays of a pencil which fall on this glass converging towards F will converge to q after refraction, and on the contrary; and therefore he will see an object distinctly as far off as q; and also nearer than F, if QF be but half of EF. For supposing rays to fall on the lens converging towards Q, say as QG to QE so QE to QH, and the refracted rays will converge to H and consequently H will be the nearest point that can be seen distinctly through this glass. But if Q bisects EF it is manifest that QH is less than QF; because QG, QF, QH are now continual proportionals.

Directions for the choice of convex and concave spectacles.

58. Thus any person may be fitted with the properest glasses though he lives at a distance from the shops where they are sold, by sending the workman their focal distances computed by the foregoing rules. But if choice of glasses be at hand, he may be better fitted by trial; observing only to use those glasses which are the least concave or the least convex of any that will fit the eye. These are the glasses which I have computed and called the properest. For since they cannot be put quite close to the eye, the less any glass is concave, the less it diminishes the pictures of objects upon the retina. It will also accustom the eye to that conformation of its coats and humors, which is proper for seeing objects as far off as it can; and consequently may prevent the eye from growing more and more short-sighted. On the other hand, the less any glass is convex, the less it magnifies the pictures of objects upon the retina; and also obliges the eye to that conformation, which is requisite for seeing objects as near as it can. Both which may prevent the eye in some measure from growing more and more long-sighted. For when the picture upon the retina is very large, it need not be quite so distinct, as when it is smaller, to convey an idea of the same number of parts of an object; and consequently the eye will be more at liberty to recede from that conformation, which is proper for the glass; and to relapse into that to which it inclines, and which is only proper for seeing remote objects.

Directions to prevent short-sightedness.

59. It is generally observed that those persons whose eyes are mostly employed upon remote

mote objects, as countrymen, sportsmen, sailers, travellers and the like, want spectacles the soonest; and on the other hand, that the greatest numbers of short-sighted persons, are found amongst scholars and mechanicks, who are daily conversant with books and other near objects; so that the eye it seems, like every thing else, is inclined to keep to that conformation, to which it is most accustomed.

60. Out of so great a number of short-sighted persons as are daily to be met with, it is probable there are but few that were born so, even of short-sighted parents. For it generally grows upon young people at the age of 20 or 25, and therefore may possibly be prevented by using their eyes while young to all sorts of conformations; that is by often looking through glasses of all sorts of figures, and by reading, writing or working with spectacles of several degrees of convexity. For whatever be the powers by which the eye conforms it self to distinct vision, they may possibly grow weak and lose their extent one way or other, for want of variety of exercise. I have heard of persons who have had so great a latitude of sight, as to be able to see distinctly through glasses of all manner of shapes; and possibly most children may have the same faculty, and by practice may preserve it. To imagine that such an exercise of the eyes can any way injure them, seems to me to be an opinion without any foundation; provided due care be taken to avoid looking at objects that are too bright.

Short sightedness comes by accidents.

61. Dr. *Briggs* mentions a person above seventy years old who had used spectacles for ten years, and yet by catching cold by reading in the winter time too near a window, he suddenly became so short sighted that he could not distinguish objects 3 feet off; and after his cold and defluxion was cured, he continued to read the smallest prints without spectacles for many years[a]. I know a young gentleman who became short-sighted immediately after coming out of a cold bath, in which he did not totally immerse himself; and ever since he has used a concave glass for many years. It is commonly thought that short-sightedness wears off in old age, but I question whether this be matter of fact or an hypothesis only.

a Opthalmographia p. 34.

Properties of short-sightedness.

62. It is remarkable of short sighted persons, that they write a small hand and love a small print, because they can see more of it at a view; that they do not look at the person they converse with, because they cannot see the motion of his eyes and features, and therefore are attentive to his words only; that they see more distinctly and somewhat farther off by a strong light than a weak one; because a strong light causes a contraction of the pupil, and consequently of the pencils both here, and at the retina; which lessens their mixture and consequently the apparent confusion; and therefore to see more distinctly they almost close their eye-lids; for which reason they were anciently called *Myopes*.

63. To these persons a lighted candle at a great distance appears very large and round, because the retina cuts the pencils at a good distance from the image; and consequently this section partakes of the figure of the pupil as well as of the image of the candle. On the other hand dark objects appear smaller to them, because the contiguous pencils that come from the contiguous brighter objects, are diffused into the pictures of the dark ones.

Spectacles for diving.

64. *Hugenius* has determined the proper convexity of spectacles for the use of Divers in the Sea, and has found that if the convexities of the glasses be made equal on both sides, they must be the same as the convexity of the cornea; whose diameter is about $\frac{4}{5}$ of an inch. "It is certain says he[b], that neither fish out of water, nor other animals within water, can see any object distinctly. Divers see things in water much in the same manner as an old man sees things through a very concave glass put near to his eye. For since it is found by experiments, that the aqueous humor, lying next to the cornea, has nearly the same refractive power as water; it follows that when the eye is under water, there can be no refraction of the rays at their first entrance into it. For though the cornea may have a different refractive power from water, yet being but thin and terminated by parallel surfaces, adjoining to fluids equally refractive, it will transmit all the rays in a manner straight[c] through it. Therefore parallel rays which, by the refraction at the cornea, when out of water, were made to fall converging upon the crystalline, will now fall parallel upon it: and therefore the crystalline will not be able to collect them to a point upon the retina, but only to direct them to a point beyond it: and so the vision will be confused. On the other hand the refractions at the cornea of a fish out of water will be very great, whereas under water there is none at all or much smaller; and so the rays will cross one another before they fall upon the retina, and cause a confused appearance.

b Dioptric. pag. 118.

c Art. 37.

65. But to correct the confusion in human eyes under water, it is requisite to find the convexity of a lens, which being applyed to the eye, shall transmit the rays to the crystalline, with the same degree of convergency, as they usually have when the eye is out of water. The sine of incidence is to the sine of refraction out of water into glass as 9 to 8, and the surface of the cornea is a portion of a sphere whose diameter is $\frac{4}{5}$ths of an inch of our Rheinland foot or of the old Roman foot. Let *AC* be a section of this surface through its center *B*; and the sine of inci- Fig. 131.

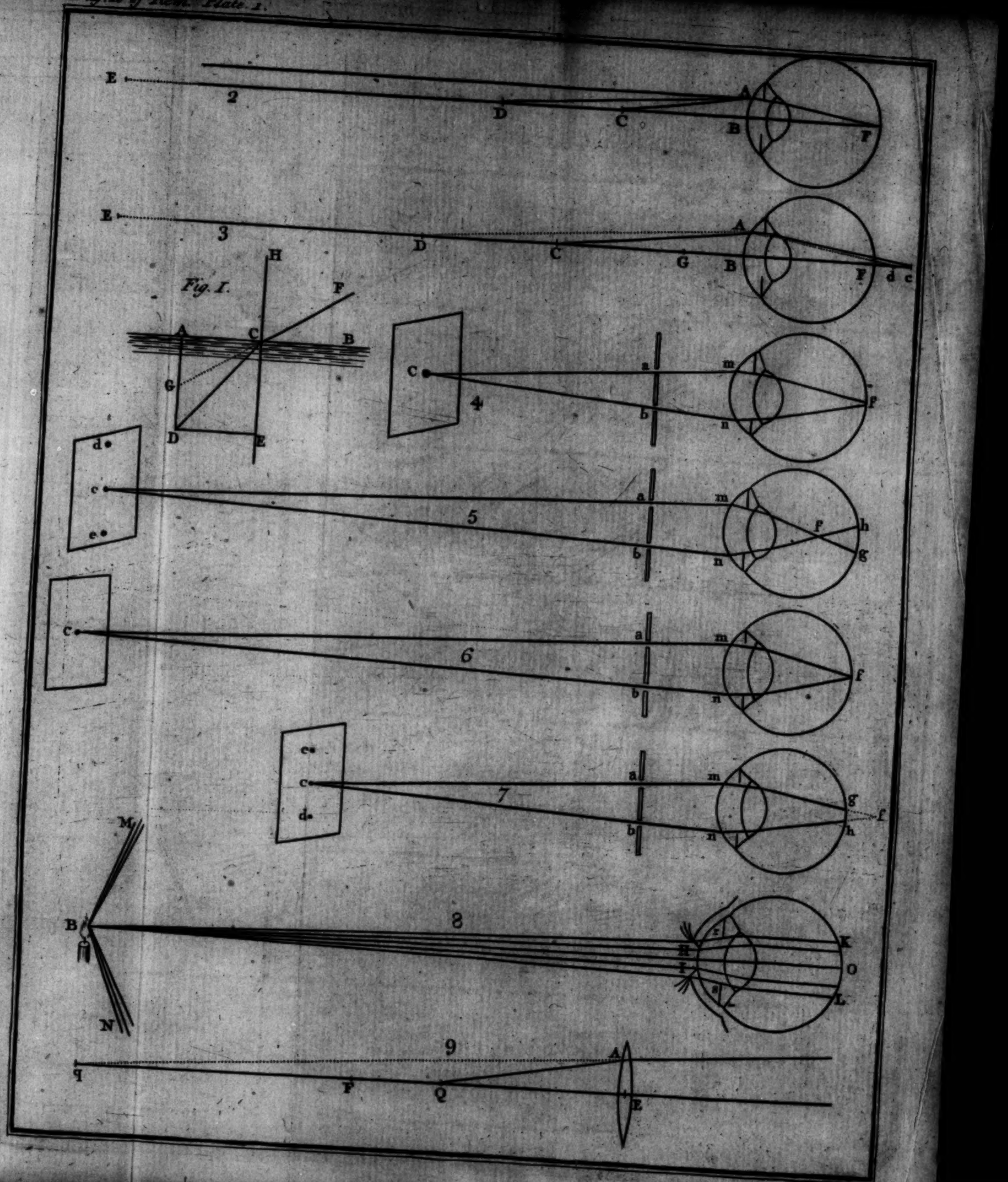
Fig. I.
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incidence be to the sine of refraction, out of air into the aqueous humor, as 4 to 3. Therefore taking BD triple of the semidiameter BA, it is certain that parallel rays in air, will be refracted by the aqueous humor, towards the point D *. But when the eye is under water this refraction at AC will be nothing; therefore a convex lens must now be applyed to AC which shall collect parallel rays to the same point D. Let EAF be this lens, having one side plane, and the other convex towards the eye, and let its semidiameter be AH. Therefore since parallel rays are collected to D, we have HD to DA as 9 to 8, that is, in the ratio of refraction of glass under water [a], and disjointly HA to AD as 1 to 8. But we had AD to AB as 4 to 1 or as 8 to 2; therefore by a regular equality of ratio's [b], we have HA to AB as 1 to 2. But AB is $\frac{1}{10}$ inch and therefore AH is $\frac{1}{20}$ inch; which was to be found. Now if instead of this plano-convex lens you would have a lens of equal convexities, they must each be the same as that of the cornea [c]; that is they must be portions of a spherical surface whose diameter is $\frac{1}{5}$ inch. So far Mr. *Huygens*.

* Art. 224.

a Art. 225.

b Eucl. V. 22.

c Art. 235.

A perfection in the eyes of fishes.

66. It is a common opinion that the want of refraction of the visual rays at the cornea of the eye of a fish, is compensated by the sphericity of the crystalline humor; so that the distance of the retina from the cornea need be no greater than in other animals whose crystallines are lenticular. But this is a mistake. For conceiving a lens to be made of two equal small segments of the spherical crystalline, its focal distance would be shorter than that of the whole sphere, measured from its remoter surface, by three quarters of its diameter, whatever be the refractive power of the ambient medium. This may be easily collected from Art. 227 and 232. Accordingly it is observed that fish have larger eyes than land animals in proportion to their bodies. There may be this advantage indeed in a spherical crystalline, that the eye will take in more objects at one view, provided the cornea be sufficiently protuberant and the pupil large, as is usual in fish's eyes. The reason is that the rays from collateral objects fall perpendicularly upon the sides of a spherical crystalline, and obliquely upon a lenticular one. Consequently the pictures of collateral objects upon a retina concentrick to a spherical crystalline, will be as distinct as those of objects placed directly before the eye [c]. By this means animals whose eyes are placed on each side of their heads, will have the advantage of seeing quite round them almost at one view; which is a great perfection in vision and preservative of their lives; and in fishes it may compensate for their want of hearing.

c See FIG. 124, 153.

The antiquity of spectacles.

67. Mr. *William Molyneux* has given us the fullest account of the antiquity of spectacles, and places the invention about the year 1300 *. "Were there no other use of dioptricks than that of spectacles for the help of defective eyes; I should think the advantage that mankind receives thereby, inferior to no other benefit whatsoever, not absolutely requisite to the support of life. For as the sight is the most noble and extensive of all our senses; as we make the most frequent and constant use of our eyes in all the actions and concerns of human life; surely that instrument that relieves the eyes when decayed, and supplies their defects, rendering them useful, when otherwise almost useless, must needs of all others, be esteemed of the greatest advantage. How melancholy is the condition of him who only enjoys the sight of what is immediately about him? with what disadvantage is he ingaged in most of the concerns of human life? Reading is to him troublesome, war more than ordinary dangerous, trade and commerce toilsome and unpleasant. And so likewise on the other hand, how forlorn would the latter part of most mens lives prove, unless spectacles were at hand to help our eyes, and a little formed piece of glass supplyed the decays of nature? The curious mechanick, engaged in any minute work, could no longer follow his trade, than till the 50th or 60th year of his age; the scholar no longer converse with his books, or with an absent friend in a letter. All after would be melancholy idleness, or he must content himself to use another man's eyes for every line. Thus forlorn was the state of most old men, and many young, before this admirable invention; which on this very account can never be prized too highly."

* Dioptric. p. 207. and 251.

Optick-glasses unknown to the ancients.

68. That the ancients had no knowledge of optick-glasses, is most evident from their universal silence in this matter: their most learned and inquisitive philosophers making no mention, or the least hint thereof, in their writings. And doubtless a contrivance of that universal use, beneficial to all old men, both in reading and writing, could never have been so concealed, as that not the least footsteps thereof should remain to posterity. The only relief they had for their decayed sights were certain *Collyria* or eye-salves; and when these failed them, they were left almost in the dark for minute and close objects.

69. We hear indeed mighty stories of *Archimedes* burning the ships of *Marcellus*, at a great distance from the walls of *Syracuse*. But whether the matter of fact be true or false (as I am very inclinable to believe it false) yet there is no mention of his performing this admirable effect by optick-glasses. Perhaps, if there were any such thing done at all; it was performed by concave speculums: and no one denies the an-

cients the knowledge of catoptricks. For *Archimedes* himself writ a book, as it is said, *De Speculis Ustoriis Parabolicis*; but it has never yet seen the light.

70. And yet there are in the world a sort of men, so devoted to the past ages, that they will not allow any improvemens of arts in the modern generation, unknown to the ages some centuries before us. Of this class was he, (whoever he was) that rather than the ancients should be ignorant of optick-glasses, would forge a passage in *Plautus*, which really is not at all to be found in him, for confirmation of his opinion.

Pretended passage in *Plautus*.

71. *Pancirollus* (who surely was too candid a person to be the first author of this fiction) in the second book *De Rebus Inventis, Tit.* 15, quotes this passage from *Plautus, Cedo Vitrum, necesse est Conspicilio uti*; which, says he, cannot possibly be meant of any other thing but of the glasses which we call spectacles. And his commentator *Salmuth* takes some pains to cite *Christianus Becmannus* (I suppose in his *Oratio de Barbarie & Superstitione superiorum Temporum)* for clearing this passage of *Plautus*: but yet he is so hard pressed with it, that by no art, but by main strength he breaks through it, and says, that notwithstanding that passage, yet certainly optick glasses are a modern invention.

72. Whereas had he been aware, that that quotation from *Plautus* is a mere fiction a; and that no such passage can be found in all his writings; he might easily have avoided its force, without all that stir. For so we shall find it answered in the *Lettere Memorabili del Abbate Michele Giustiani, Parte Terza, Let.* 16.

a Vossius de Scient. Math Cap. 26. S. 10.

Passage in *Pliny*.

73. Another place cited for the antiquity of optick-glasses, is that of *Pliny*, Lib. 7. Cap. 53. *Hist. Nat.* wherein we find the word *Specillum*. To this passage we have this answer in the forementioned letters of *Giustiani*; that *Specillum* cannot possibly be here meant of a spectacle glass, seeing we find the expression, *Inungit Specillum*; which, says he, cannot be understood of spectacles, which we rather wipe and cleanse, than anoint and grease. But this construction of the learned authors is much forced and unnatural: for the plain sense of that passage in *Pliny* is this. *Pliny* in that chapter is giving instances of the sudden deaths of many men; and telling how they were seized, whilst they were doing so or so, and wholly thoughtless of that fatal moment. Amongst many other examples, he has this; *Super omnes C. Julius medicus dum inungit, Specillum per oculum trahens.* The meaning whereof is no more, than that the physician C. *Julius* was on a sudden seized by death, whilst he was applying an unguent to his patient's eye, and drawing his probe (called *Specillum*) through it. Whereas to join *inungit* and *Specillum*, spoils the grammatical sense of the whole, and renders it unintelligible.

74. It is evident therefore, that from neither of these passages can we draw any argument for the antiquity of optick-glasses Wherefore seeing we must necessarily allow this invention due to the modern age of the world; our next enquiry shall be, where first to fix it. But herein we shall find but faint traces to direct us.

Menage's opinion.

75. *Monsieur Menage* a learned and ingenious Frenchman, in his *Origini della Lingua Italiana, Genev*, 1685, commenting on the word *Occhiali del Galilai*, discourses there of the time of the invention of spectacles: and after relating the known story of *Frier Jordan*, of which more anon, he has this notable passage; that *Monsieur du Cange* had told the author *(Monsieur Menage)* of a greek poem, the manuscript whereof is now in the French King's Library, wherein the poet, who lived An 1150, jesting on the physicians of those times, says of them to this purport in french, *Qu'ils tatent le poux, & qu'ils regardent les excremens du malade avec une Verre.* That they observe the excrements of their patients with a glass. But Monsieur *Menage* is of opinion, that this was a transparent glass, whelmed over the vessel, more for the relief of their nose against the stench, than of their eyes.

Frier *Spina*'s pretense.

76. But however we may doubt of spectacles being so ancient as 1150, we may be certain that about the thirteenth century, they were commonly known and used. For (besides what we shall say hereafter of our country-man *Frier Bacon)* the most learned Monsieur *Spoon* in his *Recherches Curieuses d'Antiquitè, Dissert.* 16. inserts a letter of Signior *Redi* to *Paulus Falconerius*, concerning the time when spectacles were invented; and this he fixes between 1280 and 1311, from the testimony of a manuscript chronicle in latin, in the library of the Friers Preachers of St. *Katherine* at *Pisa*, Fol. 16. Wherein it is said, that *Frater Alexander de Spina, Vir modestus & bonus, quacunque vidit aut audivit facta, scivit & facere. Ocularia ab aliquo* primo *facta, & communicare nolente, ipse fecit & communicavit corde hilari & volente.* And this *Alexander de Spina* was a native of *Pisa*, and dyed there, An. 1313.

Another authority.

77. Signior *Redi* has in his library a manuscript written An. 1299. *Di Governo della Famiglia de Scandro di Pipozzo.* In which there is this passage; *Mi truovo cosi gravoso di anni, che non arei valenza di leggere e scrivere senza vetri appellati Okiali, truovati novellamente per commodità delli povveri veki, quando affiebolano del vedere.* Thus in english, I find my self so pressed by age, that I can neither read or write without those glasses they call spectacles,

lately

lately invented, to the great advantage of poor old men, when their sight grows weak.

Frier Jordan's authority.

78. The Italian Dictionary *de la Crusca*, on the word *Occhiale*, makes this remark, that *Frier Jordan de Rivalto*, who dyed at *Pisa*, An. 1311, in a book of sermons which he writ An. 1305, tells his auditory in one of them, that it is not twenty years since the art of making spectacles was found out, and is indeed one of the best and most necessary inventions in the world.

Gordon's and Chauliac's.

79. About the same time *viz.* 1305. *Bernard Gordon* a famous Physician of *Montpelier*, in his *Lilium Medicina*, thus commends a certain eye-salve; *Et est tanta Virtutis, quod decrepitum faceret legere Literas minutas absque Ocularibus.* And An. 1633, *Guido de Chauliac*, in his book entituled *Grand Chirurgery*, after proposing several *Collyria*, saith; if these or the like will not do, you must make use of spectacles.

Spectacles first used about An. 1300.

80. From all which we may be pretty certain, that spectacles were well known in the 13th century, and not much before. But who the happy man was, that first hit upon this lucky thought, may yet be questioned. It is true indeed, if we credit the forementioned Chronicle of the Convent at *Pisa*, Frier *Spina* makes as fair a challenge to the invention, as the first author, who refused to communicate it. But I am apt to believe, that, whoever this close man was that would not impart to *Spina*, he was a Frier; and that these monkish men, and *Jordan* amongst the rest, had this invention whispered amongst themselves, before it was publick; and that they all had the first hint thereof from our Country-man *Frier Roger Bacon*.

Frier Bacon's pretense to this invention.

81. That this learned *Frier Bacon* who dyed An. 1292, and lyes buried at *Oxford*, did perfectly well understand all sorts of optick glasses, shall be plainly made out, from the natural and easy sense of his own words, in his book of Perspective: whereby we shall find, that he not only understood the effects of single convex and concave glasses; but knew likewise the way of combining them, so as to compose some such instrument as our telescope. This perhaps will be looked upon as a great paradox, and as great partiality in an english author to his country-man; especially considering, how universally the contrary has prevailed; the votes of most learned men having conferred the honour of this invention on other pretenders. But if from the unconstrained words of his books, we plainly make out this assertion, I hope the attempt may not be counted unreasonable or partial.

82. And first in his book of perspective Part III. Distinction 2. Ch. 3. he has these words; *Si vero Corpora non sunt plana* (having treated of them before) *per qua visus videt, sed spharica; tunc est magna diversitas, nam vel Concavitas Corporis est versus oculum, vel Convexitas, &c.* By which it is manifest, he knew what a concave and convex glass was. Moreover, in the same place Distinct. ultim. he proceeds thus; *De Visione fracta majora sunt; nam de facili patet, maxima posse apparere minima, & è contra; & longè distantia videbuntur propinquissimè, & è converso. Sic etiam faceremus Solem & Lunam & Stellas descendere secundum Apparentiam hic inferius, &c.* Thus in english; greater wonders than all these are performed by refracted vision; for thereby it is easily made appear, that the greatest object may be represented as very little, and contrarily; and so likewise, the most distant objects as just at hand, and contrarily. Hereby also we may bring the sun and moon and stars down here below in appearance, &c. This I think is so express in the point, that it leaves no room to doubt, but that he had some admirable secret in optick-glasses. Add to this what he has in his epistle *ad Parisiensem*, of the Secrets of Art and Nature, Cap. 5. *Possunt etiam sic figurari Perspicua, ut longissimè posita appareant propinquissima, & e contrario. Ita quod ex incredibili distantia legeremus literas minutissimas, & numeraremus res quantumcunque parvas, & stellas faceremus apparere quò vellemus.* Glasses or diaphonous bodies, says he, may be so formed, that the most remote objects may appear as just at hand, and contrarily; so that we may read the smallest letters at an incredible distance, and may number things though never so small, and may make the stars appear as near as we please.

83. And that these things may not seem incredible of this great man; who, in that dark, ignorant age could be master of these admirable inventions; I shall refer the reader, for a more compleat account of him, to *Ant. a Wood Hist. & Antiquit. Universit. Oxoniensis*, Lib. 1. Pag. 136. and to Dr. *Plott's Nat. Hist. of Oxford-shire*, Cap. 9. Sect. 2, 3. &c. and Sect. 39, 40, 41. Where we may find, how he was persecuted by the ignorant malicious Friers of his order, as practising magick and necromancy: for which they cast him into prison, and there detained him for a long time, some say to his death, in the 78th year of his age.

84. Mr. *Molyneux*'s quotations from Frier *Bacon* being imperfect, by reason of his absence from books, as he informs us, I will here supply this defect. This author having described several canons, as he calls them, for shewing the visual angle under which an object appears by refractions through a plane and a spherical surface, and also the place of its image; proceeds immediately to apply them to the solution of several appearances. As why an oar appears crooked in water; why a piece of money in the bottom of a bason, becomes visible by pouring in water, when before that it could not be seen

A more ample passage from Frier *Bacon*.

over

over the side of the bason; why the sun and moon appear sometimes larger than ordinary through dense vapours in the horizon. And at last he adds [a], "*Si vero homo aspiciat literas & alias res minutas per medium crystalli, vel vitri, vel alterius perspicui, suppositi* [i. e. *superimpositi*] *literis; & sit portio minor sphæræ, cujus convexitas sit versus oculum; & oculus sit in aere; longe melius videbit literas, & apparebunt ei majores. Nam secundum veritatem canonis quinti de spharico medio infra quod est res, & citra ejus centrum, & cujus convexitas est versus oculum; omnia concordant ad magnitudinem: quia angulus major est sub quo videtur, & imago est major, & locus imaginis est propinquior, quia res est inter oculum & centrum: & ideo hoc instrumentum est utile senibus & habentibus oculos debiles. Nam literam quantumcunque parvam possunt videre in sufficienti magnitudine. Si vero sit portio major sphæræ vel medietas, tunc secundum canonem sextum accidit majoritas anguli, & majoritas imaginis, sed propinquitas deest, quia locus imaginis est ultra rem, eo quod centrum sphæræ est inter oculum & rem visam; & ideo non ita valet hoc instrumentum sicut si esset minor portio sphæræ. Et instrumenta planorum corporum crystallinorum secundum primum canonem de planis, & sphæricorum concavorum secundum primum canonem & secundum de sphæricis, possunt facere hoc idem. Sed inter omnia portio minor sphæræ, cujus convexitas est versus oculum, evidentius ostendit magnitudinem, propter tres causas simul aggregatas, ut notavi.*" If the letters of a book or any minute objects, be viewed through a lesser segment of a sphere of glass or crystal, whose plane base is laid upon them, they will appear far better and larger. Because by the 5th canon about a spherical medium whose convexity is towards the eye, and the object is placed below it, and between the convexity and its center; all things concur to magnify it. For the angle under which it is seen is greater, and its image is also greater, and nearer to the eye than the object it self; because the object is between the center and the eye. And therefore this instrument is useful to old men and to those that have weak eyes. For they may see the smallest letters sufficiently magnified. But if the medium be the larger segment of a sphere or but half of one, then by the 6th canon, the apparent visual angle will be greater than the true, and the image also greater than the object; but the place of it will be beyond the object; because the center of the sphere is between the eye and the object. And therefore this instrument is not so powerful in magnifying as a lesser segment of a sphere. Also instruments made of crystal bodies with plane surfaces, by the first canon about planes, and with concave surfaces, by the first and second canons about spherical surfaces, will perform the same thing. But the lesser of two segments of a sphere magnifies more manifestly than any of them all, by reason of the concurrence of all the three causes, as I said before.

[a] Rog. Bacon Opus Majus Lond. 1733 pag. 352.

85. This is a translation of the whole passage, and in his figures the center of the convexity op is d; the object is fg; the incident rays fo, gp; the refracted ones oa, pa to the eye at a; the image at bc, terminated by the lines df, dg produced till they cut the visual rays produced in b and c. Fig. 14, 15.

His pretense examined.

86. To find an author speaking of a small segment of a sphere of glass, of its magnifying the letters of a book, of its being a proper instrument for helping decayed sight, and to say he was not possessed either of the theory or the use of spectacles, may appear to be a paradox; but I hope to satisfy my readers it is not a mistake. First then our author plainly proposed to lay the flat base of his segment upon the letters. For the word *Suppositi* must have been a contraction in his writing of the word *Superpositi* or rather *Superimpositi*; as appears by the sequel and by the canon he quotes. Besides, he says not a word of holding the segment at a distance from the letters; nor could he indeed because he has not treated of a double refraction at both its surfaces, without which he could conclude nothing at all about its effects when raised from the book; for he argues from nothing but theory throughout the whole chapter.

His mistakes.

87. In the next place I observe that he is quite mistaken in asserting twice together, that the lesser segment of a sphere magnifies the letters more than the larger. The contrary to which is true; for I shall prove by and by, that when the thickness of the segment is very small, it magnifies the letters insensibly; and that as its thickness is increased, it magnifies them more and more, and most of all when it becomes an entire sphere. But it is no wonder that he concludes wrong from a wrong principle; namely that the letters appear less in the greater segment than in the smaller, because their image is beyond them in the greater segment and before them in the lesser. The only just consequence that can be drawn from these different distances of the image, is, that to an old man's eye the letters will appear distincter, by rays diverging somewhat less from the remoter image, and more confused by rays diverging somewhat more from the nearer, than if he viewed them with his naked eye. The effect of the lesser segment is therefore contrary to the design of spectacles; which is not to magnify the letters, but to make them appear distinct, by causing the rays to fall upon the eye less diverging, or parallel, or even somewhat converging, according to the different age or constitution of the eye; and therefore it cannot be performed, but by a very nice and determinate degree of convexity.

88.

did not ... spe... ...les.

88. Hence it is plain that our author tryed no experiments with a greater and a lesser segment, to compare their effects together. For then he must have found out his mistake, and must rather have preferred the larger segment for magnifying more, which is all he contends for. However let us suppose him to have followed his own doctrine, and to have tryed a lesser segment only. It could not be a thin segment of a large sphere, like one of our spectacle-glasses. For this could not sensibly magnify the letters underneath it, as he says it did. The most convex spectacles now made, when laid upon a book, have not this effect; because they are too thin. It follows then, if he tryed any segment at all, that it must have been a segment of a small sphere, sufficiently thick to magnify the letters underneath it; and therefore it must have been thicker than our deepest spectacles for the oldest mens eyes; and consequently when applyed to their eyes, it must have made the letters appear confused by too great a quantity of the refractions. And this confusion our author could not correct by theory and reason, because he knew not the cause of it; and it is plain he made not many experiments. The discovery of this cause, together with the manner of vision by pictures upon the retina, was first made by *Kepler* [a], above 300 years after our author's time, and also after spectacles were in common use. It was impossible then for any man before *Kepler*, even to explain the effect of spectacles, (that is, how they correct the confusion in the picture upon the retina,) and much more to invent them by theory and reason; and of consequence they must have been the result of some lucky accidents among a multitude of tryals and attempts, begun perhaps upon this hint of our author's; which is all the honour that can justly be paid him. What he knew of telescopes shall be considered in its proper place.

a ...em. 25.

He borrowed from *Alhazen*.

89. As to his theory and applications abovementioned, they are all taken from *Alhazen* whom he frequently mentions upon other occasions. *Alhazen* is reckoned to have lived about the year of our Lord 1100. Among his experiments made to confirm his theorems, he expresly mentions, that if an object be applyed close to the base of a larger segment of a sphere of glass, it will appear magnified [b]; which, as I observed, was fitter for *Bacon*'s purpose than his own lesser segment. He also treats about the appearance of an object through a globe; and says he is the first that found out the refraction of rays into the eye.

b Optic. lib. 7. th. 48.

Demonstration of Remark 87. Fig. 16, 17, 18.

90. The proof that I promised of the 87th remark is this. Let O be the place of the eye, E the center of the spherical surface AC, and $OCED$ its axis, $OAPB$ any ray supposed to flow from the eye, and to fall upon any point P of the object PQ, supposed always to be a part of the base of the segment $RACS$; and $\varpi\kappa$ a line parallel and equal to PQ, and subtending the visual angle AOC. The object PQ appears by refraction to be of the same magnitude as if it was removed to the place $\varpi\kappa$ and viewed there by the naked eye [b]; and therefore the apparent magnitude is to the true, as OQ to $O\kappa$ [*]. Suppose now the arch AC to be very small and the incident and refracted ray OAB to keep fixt, while the segment $RACS$ increases from nothing till it becomes a whole sphere; and the ratio of OQ to $O\kappa$ will at first be a ratio of equality, and then will increase perpetually [c]. The exact quantity of this ratio may be computed from this theorem deduced from Art. 208, $O\kappa = OQ - OE \times \frac{CQ}{CF}$, wherein F is the focus of rays that go parallel within the sphere.

b Art. 104.
* Art. 99. 100.
c Art. 12...

Burning-spheres known to the ancients.

91. The famous M. *de la Hire* has endeavoured to raise the antiquity of lenses or lenticular burning-glasses to a very great height [d], imagining he has found them in the *Clouds* of *Aristophanes*, Act II. Sc. 1. towards the end. *Strepsiades* an old stupid fellow tells *Socrates* he had found out an excellent contrivance against paying his debts. Strep. *Have you never seen at the apothecaries that fine transparent stone with which they kindle fire?* Socrat. *Do you mean the glass?* Strep. *Yes.* Socrat. *Bring it; what then?* Strep. *When the attorney hath written an action against me, I will take this glass, and standing at a distance, in this manner against the sun, I will melt down the letters of my action.* Socrat. *Cunningly done by the Graces.* The Scholiast upon this place says it was a round [e] thick glass made on purpose for this use; that they rubbed it with oyl and heated it, and then they fitted to it, or brought near, a match (for the greek word here is equivocal) and after this manner the fire was lighted. *De la Hire* cannot understand what the oyl was for, unless it was to polish the glass. But be that as it will, he says the Scholiast conceived it was convex, which shews that in his time, though later than *Aristophanes*, they used such glasses to kindle a fire.

d Hist. de l'Acad. R. des Sci. 1708.

e Τροχοειδὴς round like a wheel.

92. He argues also from the words Ἀποστὰς ἐγὼ, *standing at a distance*, that this glass was lenticular rather than spherical; because a sphere burns an object only at a very small distance from it. But Mr. *Waller* observes [f] that considering the whole design of this Play was to ridicule *Socrates*, it was proper enough to bring in an old coxcomb boasting of an invention for doing what indeed it could not perform. To this remark Mr. *Waller* has added others upon some curious passages from the ancients, concerning their burning glasses by refraction; from which he concludes they had no other than whole spheres, and that these were chiefly used by Surgeons

f Philos. Exp. and Obs. by Hook &c. 8o. p. 348.

to

to burn the flesh of sick persons that needed cauterizing, and by the Vestal virgins to kindle the sacred fire. I believe *Vitellio* was the first person that demonstrated the reason of this effect. Optic. Lib. 10. Th. 48.

Their mistake about burning with speculums.

93. Considering that catoptricks was known and cultivated by the ancients long before dioptricks, it is surprising they could not account for burning by reflection from a concave metal. *Euclid* in his catoptricks says its center is the burning point[b], because all the rays which pass through it are returned directly back to it. But as the sun's diameter is so small, these rays are but very few; and the consequence would be that a very broad speculum would burn no better than a very narrow one, which is contrary to experience. It is evident from this and many other blunders in that book, that *Euclid* the geometer was not the author of it, and also that the ancients made very gross experiments.

b Prop. ult. & *Vitellio* Lib. 9. Prop. 37.

Philos. Exp. and Obs. by Hook &c. p. 351.

94. But since the effects of burning with solid spheres or glass bottles filled with water, was so well known to the ancients, how came it to pass that they did not also know their effects in magnifying objects. Had the greek and latin philosophers known this augmentation of objects, would they not have mentioned it frequently; and would not several metaphors and allusions to it have been brought into their language? Mr. *de la Hire* accounts for this oversight of theirs partly from their false notions about the manner of vision, viz. by certain whimsical emanations from the eye, that went out in quest of objects, or else by little representations in miniature, which came from them and sought out for our eyes; so that having no suspicion of pencils of rays nor of our focuses, they could see no analogy between a burning-glass and the manner of vision. This indeed is a sufficient reason for their not discovering its magnifying power by theory; but can it be supposed they never looked through these spheres? To this our author answers; that the focus of a sphere of glass is at the distance of half the radius from the nearest surface; so that if these spheres had been six inches in diameter, which is the most they can be supposed to be, the object in view must have been placed at one inch and a half from the sphere to be seen distinctly. But it is natural and almost necessary that when any one had looked through these spheres, the objects in view would have been farther off, which instead of appearing bigger would only have looked confused. A defined or distinct augmentation of distant objects, requires either very large spheres, which is impracticable, or portions of large spheres, as is now practised with great success. And besides they must have known how to have wrought and ground their glasses as we do; whereas in all probability the ancients knew only how to blow their glass and make vessels of it.

Upon Art. 94.

95. M. *Bouguer* Professor of Hydrography at *Croisic*, has found by experiments that the light of the moon is frequently 2000 times weaker in the horizon, than at the altitude of 66 degrees. And that the proportion of her lights at the altitudes of 66 and 19 degrees is about 3 to 2. And the lights of the sun must bear the same proportion to each other at those heights; which he made choice of as being the meridian heights of the sun at the summer and winter solstices in the latitude of *Croisic*[c]. His manner of tryal was this. When the moon was 19 degrees high, he received her light perpendicularly upon a white paper, and also the light of 4 candles perpendicularly upon another paper hard by; then he ordered the distance of the candles to be varied, till he judged their light upon the paper to be equal to that of the moon upon the other paper; and found the distance of the candles from the paper to be 50 feet. Then he repeated the same experiment when the moon was 66 degrees high, and found her light equal to that of the same candles at the distance of 41 feet from the paper. The proportion of these lights is therefore as the square of 50 to the square of 41 *, or in round numbers about 3 to 2. The horizontal light of the moon was found in the same way, but is subject to greater variations, by passing through more vapours.

Much light stopt by the atmosphere.

c Essai dioptrique sur la gradation de la lumiere pag. 22.

* Art. 58.

Upon Art. 95.

96. By the like experiments this ingenious author finds the light of the full moon to be about 300000 times weaker than that of the sun, at a medium of several tryals. I found it by theory not much above 90000 times weaker; the difference may arise chiefly from the loss of light in the moon's body, which could not be allowed for in theory. M. *Bouguer* proceeded thus. He received the light of the sun when 31 degrees high, into a dark room through a concave lens, placed against a round hole, one line in diameter, made in the window-shutter; and at a certain distance between 5 and 6 feet from the hole, the diameter of the transmitted light upon a white paper was 108 lines. The direct light of the sun was therefore weakened in the ratio of the square of 108 to 1 *, that is 11664 times; and then it appeared equal to the light of a candle upon a paper held 16 inches from it. At another time when the full moon was also 31 degrees high, and the paper was placed at such a distance from the same lens, that the diameter of the transmitted light was 8 lines, he judged it equal to the light upon a paper coming from the

The proportion of day light and moonlight by experiment. ibid. p. 28.

* Art. 58.

the same candle 50 feet from it. To find the result of these two experiments, it is to be considered that the moon's light was weakened but 64 times by the concave lens; and therefore to weaken it 11664, (that is as much as the sun's light was weakened by the same concave,) or to weaken the equal candle light 11664 times, the candle must be removed from the said distance of 50 feet to 675 feet[a]. Because the squares of 50 and 675 are as 64 to 11664. But the candle whose light equalled that of the sun weakened 11664 times, was but 16 inches or $1\frac{1}{3}$ foot from the paper. Therefore the squares of the distances 675 and $1\frac{1}{3}$ give the ratio of two degrees of candle light equal respectively to those of the sun and moon. This ratio is as 256289 to 1. But by taking a medium among several experiments he concludes upon 300000 to 1, when the moon is at her middle distance from the earth. For when she is in perigee and apogee the proportion of her lights is about 4 to 3. So far M. *Bouguer*.

a Art. 58.

The proportion of day-light to moon-light demonstrated by theory. Fig. 19.

97. My rule may be thus demonstrated. Let the little circle *cfdg* represent the moon's body half inlightened by the sun, and the great circle *aeb*, a spherical shell concentrick to the moon and touching the earth; *ab* any diameter of that shell perpendicular to a great circle of the moon's body, represented by its diameter *cd*; *e* the place of the shell receiving full moon-light from the bright hemisphere *fdg*. Now because the surface of the moon is rough like that of the earth, we may allow that the sun's rays, incident upon any small part of it, with any obliquity, are reflected from it every way alike, as if they were emitted. And therefore if the segment *df* shone alone, the points *a*, *e* would be equally illustrated by it; and likewise if the remaining bright segment *dg* shone alone, the points *b*, *e* would be equally illustrated by it. Consequently if the light at the point *a* was increased by the light at *b*, it would become equal to the full moon-light at *e*. And conceiving the same transfer to be made from every point of the hemispherical surface *bblk* to their opposite points in the hemisphere *kaeb*, the former hemisphere would be left quite dark, and the latter would be uniformly illustrated with full moon-light; arising from a quantity of the sun's light which immediately before its incidence on the moon, would uniformly illustrate a circular plane equal to a great circle of her body, called her disk. Therefore the quantities of light being the same upon both surfaces, the density of the sun's incident light, is to the density of full moon light, as that hemispherical surface *bek* is to the said disk; that is, as any other hemispherical surface whose center is at the eye, to that part of it which the moon's disk appears to possess very nearly, because it subtends but a small angle at the eye: that is, as the radius of the hemisphere to the versed sine of the moon's apparent semidiameter[b], or as 10000000 to $1106\frac{2}{3}$ or as 90400 to 1; taking the moon's mean horizontal diameter to be 16′. 7″.

b Selecta ex Archimede Theoremata sub finem Euclid. Cantab. 1712. p. 304. prop. 27.

98. Strictly speaking this rule compares moon-light at the earth with day-light at the moon; the medium of which at her quadratures, is the same as our day-light; but is less at her full in the duplicate ratio of 365 to 366 or thereabout, that is, of the sun's distances from the earth and full moon: and therefore full moon-light would be to our day-light, as about 1 to 90900, if no rays were lost at the moon.

99. Secondly I say that full moon-light is to any other moon-light, as the whole disk of the moon, to the part that appears inlightened, considered upon a plane surface. For now let the earth be at *b*, and let *dl* be perpendicular to *fg* and *gm* to *cd*; then it is plain that *gl* is equal to *dm*; and that *gl* is equal to a perpendicular section of the sun's rays incident upon the arch *dg*, which at *b* appears equal to *dm*; the eye being unable to distinguish the unequal distances of its parts. In like manner conceiving the moon's surface to consist of innumerable physical circles parallel to *cfdg*, as represented at *A*, the same reason holds for every one of these circles as for *cfdg*. It follows then that the bright part of the surface visible at *b*, when reduced to a flat as represented at *B*, by the crescent *pdqmp*, will be equal and similar to a perpendicular section of all the rays incident on that part, represented at *C* by the crescent *pgqlp*. Now the whole disk being in proportion to this crescent, as the quantities of light incident upon them; and the light falling upon every rough particle being equally rarified in diverging to the eye at *b*, considered as equidistant from them all; it follows that full moon-light is to this moon-light, as the whole disk *pdqc*, to the crescent *pdqmp*.

Fig. 20.

100. Therefore by compounding this ratio with that in the former remark, day-light is to moon-light, as the surface of an hemisphere whose center is at the eye, to the part of that surface which appears to be possessed by the inlightened part of the moon.

Upon Art. 97.

101. This magnitude of a sensible point upon the retina was computed by the rule in art. 374 with the numbers in remark 22.

Upon Chapter 4. Concerning vision with glasses.

Upon Art. 118.

Single and double microscopes compared.

102. M. *Huygens* observes that a convex lens has some advantage over a little globule that magnifies as much; because the distance between the object and the lens is three times greater than between the object and the nearest part of the globule [a]; and so the lens allows more room for a side light to fall upon the object; by which its colours may be observed; otherwise, if transparent, it must be viewed by light transmitted through it. This indeed is some advantage in glasses that magnify but little, in others none at all; that distance from the lens being so very small. But opake objects may be better observed by double microscopes, in which they lye farther from you, and also farther from the object-glass; and this indeed is one of their chief advantages above single microscopes; which generally exceed them in the power of magnifying.

a *Hugenii* dioptr. prop. 59.

Upon Art. 120, 121, 122, 123.

History of the invention of telescopes. Dioptric. pag. 163.

103. We are obliged to *Hugenius* for the most judicious history of the invention of telescopes and microscopes; which I will here translate, and make some additions to it. "The most valuable and principal of all the subjects that are handled in dioptricks, is the theory of telescopes. The worth and excellency of this great and noble invention, can never be sufficiently declared and commended. For to say nothing now of its other uses, how great is this one, that it has opened a way to various subjects of contemplation upon the heavens, which could not be investigated by any other means. Hence many wonderful things in nature, and in a word the whole system of the universe, is laid open to our view: so that in what region of it this earth of ours and we its inhabitants are placed, is now no longer doubtful but indisputably certain. And in my opinion the wit and industry of man has not produced any thing so noble, and so worthy of his faculties, as this sort of knowledge: insomuch that if any particular person had been so diligent and sagacious, as to invent this instrument from the principles of nature and geometry; for my part I should have thought his abilities were more than human. But the case is so far from this, that the most learned men have not yet been able sufficiently to explain the reasons of the effects of this casual invention.

The inventors.

104. There are some that give the praise of the first invention of this instrument, though certainly casual as I observed, to *James Metius* a Dutchman and citizen of *Alcmaria*. But to my certain knowledge, telescopes were made before this, at *Middleburgh* in *Zeland* about the year 1609, by a certain spectacle-maker in that city; either by *John Lippersheim*, whom *Sirturus* mentions, or else by *Zacharias* whom *Borellus* makes the first inventor of them, in his book *de Vero Telescopii inventore*. The telescopes then made by these two artists, were but a foot and a half long. Nevertheless it is certain that *John Baptista Porta* had delivered some sketches of this art, in his books upon Dioptricks and *Natural Magick*, printed 15 years before any telescopes were made by either of these Dutchmen. In these books he speaks of certain instruments of his that shewed very distant objects as if they were near at hand; and of combining concave and convex lenses [b]. But that he made no great progress in this art, is hence evident, that in all that time it did not become famous; and that he did not discover any of those things in the heavens, that were observed afterwards. And this shews that his invention was not owing to his skill, but to some accidental experiments. For though he had some degree of knowledge in mathematicks, yet he did not understand those fundamental principles and abstruse theorems in dioptricks, that are necessary to invent a telescope by reason; and much less did those illiterate mechanicks that I mentioned before. But no wonder that good luck and casual experiments should produce this effect; after spectacles and concave glasses, for defective eyes of both sorts, had been commonly used for above 300 years [c]. It is rather surprising that so obvious a thing should be so long unknown.

b Magia Naturæ lib. 17. cap. 10.

c Rem. 67.

Telescopick observations by *Galileo* and *Hugenius*.

105. But as soon as the fame of these dutch telescopes was spread abroad, *Galileo* immediately made others of the like sort, and soon after much better; by which he first discovered those celebrated appearances in the heavens; as the mountains and valleys in the moon, the spots in the sun, and thereby his rotation about himself; the satellits of Jupiter; the phases of Venus like those of the moon, and their various apparent magnitudes; the great difference between the apparent diameters of the planets and of fixt stars; and of these a much greater multitude than had ever been known. He also observed the phænomena of Saturn as well as he could with such short telescopes; but could not discover his true shape, nor any one after him for many years. For though they had lengthened their tubes very much, yet they improved their power and efficacy but very little. As to my self, I undertook this business with a better prospect of success; and after I had acquired a thorough knowledge of the laws of refraction, and had made my own object-glasses and telescopes above twenty feet long, with these I discovered the true forms

forms of saturn, never seen before; and the cause of them to be a ring surrounding his body; the like of which is not to be found in any other planet. I also discovered a satellit revolving about him in 16 days; all which I published 26 years ago in a treatise by it self [a]. By the encouragement of these discoveries Astronomers and Artificers went on to make longer telescopes; the best of which are made by *Campani* at *Rome*. By these, about ten years after, *Cassini* had the good fortune to discover two more satellits besides mine. He also observed some spots in Jupiter and Mars, by which he determined the times of their rotations about themselves.

a Systema Saturnium. *Haga* 1659.

Telescopes improved by *Hugenius*.

106. These are the present improvements of this noble instrument; and this is a summary of those things in the heavens, which it has revealed to the inhabitants of the earth. And who is so stupid as not to perceive how great and excellent they are? and who does not understand, that has any philosophy, what a light they afford to the contemplation of nature? Certainly we may congratulate the happiness of the present age, for so great acquisitions of knowledge. Those famous and excellent men, *Copernicus*, *Regiomantanus*, *Braheus*, so lately excluded from it, what an immense treasure would they have given for it? And those ancient philosophers, *Pythagoras*, *Democritus*, *Anaxagoras*, *Philolaus*, *Plato*, *Hipparchus*; would they not have travelled over all the countries in the world for the sake of knowing such secrets of nature, and of enjoying such sights as these? and perhaps we may shortly expect a discovery of many more and quite new ones, by practising a late invention of mine for using a long telescopick glass, without that excessive trouble in managing those long and weighty tubes, and indeed without any tubes at all; it being as easy now to observe with an object-glass of 100 or 200 feet focal distance, as it was before with a tube of ten feet [b]. Especially since many persons have now undertaken to cultivate the art of grinding and polishing such large glasses; which I also after a long intermission have resumed again with good success.

b Art. 892.

Their magnifying power not yet demonstrated.

107. But now for the causes and properties of this artificial eye, which are not yet so happily explained. The chief thing still wanting and not yet demonstrated is this. Having the form and position of the lenses, to find the manner and measure by which they magnify objects. For *Kepler* has not done it, though highly commendable for his new discoveries in his dioptricks. Nor had *Des Cartes* better success; for in truth he is quite mistaken in his method of demonstrating the effects of telescopes. This indeed is hardly credible of so great a man and so conversant in these things; yet it was proper to be mentioned, to save people the trouble of endeavouring to understand what can never be made sense of. And though many others have since been labouring at the same problem, which is the chief of all, yet none of them have been able to solve it.

Additions to the history of telescopes.

108. So far *Hugenius*. *Borellus* in his book *de Telescopii inventore* [c] has printed the affidavits of several inhabitants of the city of *Middleburgh*, made before the Consuls in the year 1655; wherein the son of *Zacharias Jansen* attests, that his father, a spectacle-maker of that city, made telescopes in the year 1590, as he had often been informed; and presented one of them to Prince *Maurice* and another to the Arch-Duke *Albert*; and that *Metius* and *Drebel* in the year 1620 came to *Middleburgh* and bought of these telescopes; which at first were but 16 inches long. Another attests upon his own knowledge that his neighbour *Hans Lipperhey*, a spectacle-maker in the same city, made telescopes before the year 1605; two others say before the year 1609 or 1610. Likewise *Antonius de Rheita* in his *Oculus Enoch & Eliæ* published at *Antwerp* in 1645, fixes *Lipperhey*'s invention in the year 1609, and that he casually hit upon it, by happening to look through a concave and a convex spectacle glass, both at once, while he held them in each hand; that he himself upon *Kepler*'s thought, made the first inverting telescope with a convex eye-glass; which takes in much more at a view than the other; that he also invented and perfected the Binocular telescope [d], which shews things much more bright and vivid and seemingly larger too than a single telescope. But *Fontana* in his *Novæ observationes cœlestium & terrestrium rerum*, printed at *Naples* in 1646, says that he invented the inverting astronomical telescope in the year 1608, three years before *Kepler*'s dioptricks were printed. But his friend *Zupus* only testifies that this author shewed him one of them in 1614.

c pag. 29.

d Art. 974.

Galileo a second inventor of telescopes. pag. 2.

109. *Galileo* has given us the following account of his part of the invention of telescopes in his *Nuncius Sidereus* published by him in March 1610. "Near ten months ago it was reported that a certain Dutchman had made a perspective, through which very distant objects appeared distinct as if they were near; several experiments were reported of this wonderful effect, which some believed and others denyed. But having had it confirmed to me a few days after, by a letter from the noble *James Badovere* at *Paris*, I applyed my self to consider the reason of it, and by what means I might contrive a like instrument, which I attained to soon after by the doctrine of refractions. And first I prepared a leaden tube in whose extremities I fitted two spectacle-glasses, both of them plane on one side, and on the other side, one of them spherically convex, and the other concave. Then applying

my eye to the concave, I saw objects appear pretty large and pretty near me; they appeared three times nearer, and nine times larger [in surface] than to the naked eye. And soon after I made another which represented objects above 60 times larger; and at last having spared no labour nor expence, I made an instrument so excellent as to shew things almost a thousand times larger and above thirty times nearer than to the naked eye. Whoever would be satisfied of the truth of the observations I am going to relate, must have an instrument that shall make things appear 20 times nearer at least. Which he may be sure of by this experiment. Let him make two squares or two circles, one 400 times larger than the other, as it will be if its diameter be made 20 times longer; then having fixt them both upon a very distant wall, let him view them both at once, that is the lesser with the perspective and the larger with his other eye, which may be done conveniently enough with both eyes open at the same time; and then the two papers will appear equal in bigness if the instrument magnifies as was expected.

110. He adds in his *Saggiatore*, that he was at *Venice* when he heard of the effects of Prince *Maurice*'s instrument, but nothing of its construction; that the first night after his return to *Padua* he solved the problem, and made his instrument the next day, and soon after presented it to the *Doge* of *Venice*; who to do him honour for his grand invention, gave him the Ducal Letters, which settled him for life in his Lectureship at *Padua*, and doubled his Salary; which then became treble of what any of his predecessors had enjoyed before. So far *Galileo*; whose best telescope so famous for its grand discoveries, may be reckoned about a yard long, as appears by art. 364.

Frier Bacon's pretence to this invention examined.

111. Hitherto we have made it appear that telescopes were certainly in use soon after the beginning of the seventeenth century. Let us now consider what pretences there are for a greater antiquity of this invention. Mr. *William Molyneux* has told us that Frier *Bacon* who dyed in 1292, *did understand all sorts of optical glasses perfectly well; and knew likewise the way of combining them, so as to compose some such instrument as our telescope*[a]; and his son, the Honourable *Samuel Molyneux*, has declared his opinion rather more expresly, *that the invention of telescopes, in its first original, was certainly put in practice by an Englishman, Frier* Bacon; *although its first application to astronomical purposes may be justly attributed to* Galileo[b]. To examine this opinion of these learned and judicious persons, which has since been followed by some others, it is necessary to transcribe a whole chapter of our author's upon vision by refractions, which immediately follows after his discourse upon vision by reflections[c].

a Rem. 81.

b Art. 781. Chap. 2.

c Opus Majus Lond. 733. pag. 357.

112. *De visione fracta majora sunt. Nam de facili patet per canones supradictos, quod maxima possunt apparere minima, & e contra; & longe distantia videbuntur propinquissime, & e converso. Nam possumus sic figurare perspicua, & taliter ea ordinare respectu nostri visus & rerum, quod frangentur radii & flectentur quorsumcunque voluerimus; ut sub quocunque angulo voluerimus, videbimus rem prope vel longe: & sic ex incredibili distantia legeremus literas minutissimas; & pulveres ac arenas numeraremus propter magnitudinem anguli sub quo videremus; & maxima corpora de prope vix videremus propter parvitatem anguli sub quo videremus. Nam distantia non facit ad hujusmodi visiones nisi per accidens, sed quantitas anguli. Et sic posset puer apparere gigas, & unus homo videri mons, & in quacunque quantitate; secundum quod possemus hominem videre sub angulo tanto sicut montem & prope ut volumus. Et sic parvus exercitus videretur maximus, & longe positus appareret prope, & e contra. Sic etiam faceremus solem & lunam & stellas descendere secundum apparentiam hic inferius; & similiter super capita inimicorum apparere; & multa consimilia; ut animus mortalis ignorans veritatem non posset sustinere.*

What problems in vision he thought practicable by refractions.

113. In English thus. Greater things than these may be performed by refracted vision. For it is easy to understand by the canons abovementioned, that the greatest things may appear exceeding small, and on the contrary; also that the most remote objects may appear just at hand, and on the contrary. For we can give such figures to transparent bodies, and dispose them in such order with respect to the eye and the objects, that the rays shall be refracted and bent towards any place we please; so that we shall see the object near at hand or at a distance, under any angle we please. And thus from an incredible distance we may read the smallest letters, and may number the smallest particles of dust and sand, by reason of the greatness of the angle under which we may see them; and on the contrary we may not be able to see the greatest bodies just by us, by reason of the smalness of the angles under which they may appear. For distance does not affect this kind of vision, excepting by accident, but the quantity of the angle. And thus a boy may appear to be a giant, and a man as big as a mountain, forasmuch as we may see the man under as great an angle as the mountain and as near as we please. And thus a small army may appear a very great one, and though very far off yet very near us, and on the contrary. Thus also the sun, moon and stars may be made to descend hither in appearance, and to appear over the heads of our enemies;

and

and many things of the like sort, which would astonish unskilful persons.

And what by reflections.

114. In the foregoing chapter upon vision by reflections, he has also told us, that by several speculums properly disposed, a single soldier may be multiplyed so as to appear like an army; and one army like many armies, for terrifying infidels and enemies; that speculums may be raised up on high against opposite armies and cities, so as to discover all the enemies hidden transactions; and that this may be done from any distance whatever. Because by the Book upon speculums, one and the same thing may be seen by reflection from as many speculums as we please, if they be disposed in proper situations; and therefore some may be placed nearer and some farther off; so that the object may be seen from any distance whatever, *ut videremus rem quantum a longe vellemus*.

They are much alike.

115. I have added this extract to the other passage, to shew that his thoughts about the manner of executing these designs by reflections and by refractions are much alike; as is evident from the passage last mentioned compared with this, *figurare perspicua & taliter ea ordinare respectu nostri visus & rerum, quod frangentur & flectentur radii quorsumcunque voluerimus*, to dispose the glasses in such order with respect to the eye and objects, *that the rays shall be refracted and bent towards any place we please*.

His solution of them impracticable.

116. It seems then as if he did not think of performing these problems by a single portable instrument like a telescope; but by fixing up several glasses in proper places at large intervals from one another; which would certainly prove ineffectual. Because no surfaces can be figured and polished so perfectly, as to reflect and refract light to very great distances, without great aberrations, or straglings of the rays, from the places or points intended; and these aberrations would be increased by the interposition of more glasses; to say nothing of the loss of light at every surface, and of colours generated by refractions from such great distances; so that the object must appear at last so faint, so deformed and confused as to excite no determinate idea.

He relates a fact impracticable by speculums.

117. What he mentions of *Julius Cæsar* [a], that he raised up speculums to a great height upon the coast of *France*, to discover the disposition of the cities and camps in *England*, when he was going to invade it, is therefore impracticable; and probably a fiction, if there be not a mistake in the interpretation of the word *specula* for glasses instead of a *watch-tower*. The same is to be understood of the story mentioned by *Porta* [b], that *Ptolomy* by speculums could discern ships at the distance of six hundred miles; which could not possibly be done by our best telescopes. Mr. *Waller* imagines he had his intelligence by *speculis* or watch-towers, placed at several intermediate distances; where several signs might be given and received successively from the first to the last [c]. Dr. *Hook* has given us some ingenious thoughts upon discoursing by signs at a great distance in the same book [d].

a Ibid. p. 357.

b Magia Natur. lib. 17. cap. 11.

c Philos. Exp. and Obs. by Hook &c. p. 360.

d Ibid. p. 142.

He was not qualified to invent a telescope by theory.

118. But to return to our author, considering the false notions he had from the ancients about distinct and confused vision; the false principle he maintains, that the apparent magnitude of an object is as the angle subtended at the eye by its image, and reciprocally as the distance of the image too; and lastly the false conclusions he has drawn, and must always draw, from these principles; as I have shewn in his attempt upon making spectacles [e]; it was certainly next to impossible for him to have invented by theory a much nicer and more complex instrument, I mean a telescope of any sort. Indeed Mr. *Molyneux* says it is manifest he knew what a concave and a convex glass was [f]. But this does not appear from the passage there cited, nor from any other that I have yet met with. Mr. *Molyneux* was misled by the inaccuracy of the author's language. *Si vero corpora non sint plana*, per *quæ visus videt, sed sphærica, tunc est magna diversitas; nam vel concavitas corporis est versus oculum vel convexitas* &c. instead of this he should have said *si vero corporum superficies non sint plana*, per *quas visus videt, sed sphærica* &c. For in this whole chapter he treats of nothing else but the appearance of an object through a single concave or convex surface of a large body or medium, the object being placed within it and not beyond it; nor do I find that he has ever considered the appearance of an object placed at a distance beyond a refracting body of any shape. Indeed he mentions the refractions of the sun's rays through a sphere, to shew how they burn things [g]; but not a word how an object appears through it.

e Rem. 87.

f Rem. 82.

g Opus Majus pag. 67. & 397.

nor by experiments for want of lenses.

119. Besides, I have shewn above, that he never had handled a convex spectacle-glass when he wrote a passage in this same treatise [h]; and without a variety of lenses he could try but few experiments, and could never have hit upon a proper combination of them for a telescope. Indeed Dr. *Jebb* the editor of the late edition of the *Opus Majus*, in his dedication to Dr. *Mead*, produces a passage from a manuscript [i] to shew that this author actually applyed telescopes to astronomical purposes. *Sed longe magis quam hæc oporteret homines haberi, qui bene, immo optime, scirent perspectivam & instrumenta ejus*, —— *quia instrumenta astronomiæ non vadunt nisi per visionem secundum leges istius scientiæ*. To this it may be answered, that the ancients had some occasion for perspective in plain instruments, before the invention of telescopick ones. But as this passage stands alone, it is not easy to know the intent of it; however had there been any more to

h Rem. 88.

i MS. Cot. Tib. C. V. fol. 6.

the

the like purpose, no doubt this gentleman, so much versed in this author's works, would have found them out and obliged us with them.

His imagination carried him too far.

120. In short this author speaks only hypothetically, saying that glasses may be figured and objects may be magnified so and so; but never asserts one single trial or observation upon the sun or moon, (or any thing else,) though he mentions them both. On the other hand he conceives some effects of telescopes that cannot possibly be performed by them.

How he came by these notions.

121. If it be asked how he came by these notions; I answer from the common doctrine of refractions in his Canons, and from common appearances by refraction and reflection; especially from concave speculums, whose effects were well known to him, both by the accounts of them in ancient authors, and by his own experience. And this I take to be a sufficient ground for a man of good sense and fancy to produce all that he has said. I conclude then that the time of the invention of telescopes was not earlier than the beginning of the seventeenth century.

History of the invention of microscopes. *Hugenii* Dioptr. p. 221.

122. I will add the history of the invention of microscopes written by *Hugenius*. "It is probable says he, that the use of simple microscopes, for magnifying objects with a single globule or a little lens, was known soon after the invention of telescopes; but the contrivance of compound microscopes, being less obvious, seems to have been about ten years later. It does not appear that these microscopes were made in the year 1618; because *Syrturus*, who published a book that year about the origine and construction of telescopes, would hardly have been silent upon so remarkable an invention, if it had been then known. *Fontana* indeed lays claim to it from the year 1618, in his book of Observations published in 1646; but the testimony of *Syrsalis* there printed, goes no higher than the year 1625. But that my countryman *Drebelius* made these compound microscopes at *London* in the year 1621, I have often been informed by several eye-witnesses, and that he was then reckoned the first inventor of them. For nothing hinders but both these persons may have hit upon the same thing, by trying various compositions of glasses, though quite ignorant of geometry aud of the causes of these effects.

Another demonstration of telescopes, Fig 21, 22. a Art. 120. 123.

123. So far *Hugenius*, who demonstrates the effects of telescopes in this manner. The glasses L, E being placed as usual[a], in their axis EL produced forwards, take LN equal to Lq, the focal distance of the object-glass LM. Then any ray as PNM which passes through N, and falls upon the object-glass LM, will be refracted through it into a line MA parallel to the axis LE; and falling upon the eye-glass AE, it will be refracted to the principal focus O; or else will diverge from it, if this glass be concave. Therefore in both telescopes, the object PQ appears under the angle AOE, and to the naked eye at N, under the angle PNQ; and so the apparent magnitude is to the true, as the angle AOE to PNQ or LNM, that is, since LM equals AE, as the focal distance LN, to the focal distance EO*. * Art. 60.

Fig. 23.

124. If two more equal eye-glasses BF, CG be subjoined to the astronomical telescope, as in art. 121, and O be the common focus of the glasses AE, BF; the ray AOB will again be refracted into a line BC parallel to the axis; and consequently will be refracted through the last eye-glass to its principal focus D; where the eye being placed, will see the object upright, and magnified just as much as before; because GD being equal to FO, the angle CDG is equal to BOF or AOE.

Colours appear in telescopes. b Phil. Transf. N°. 1.

125. This excellent composition of glasses was invented by somebody at *Rome*, probably by *Campani* b. It is usual and useful to put a plate with a round aperture in it, at the place of the first image at q or of the second at κ, to circumscribe the visible prospect, and to cut off the colours from the edges of it; though they are not more apparent in this telescope with three eye-glasses than with one; but rather less. For though there be more refractions, yet the colours generated by the refractions at the two first glasses, made both the same way, are somewhat corrected and contracted by the refractions at the two last; being both made the contrary way to the former[c]. For the edge of a lens has the like effect as the edge of a prism. c Art. 171.

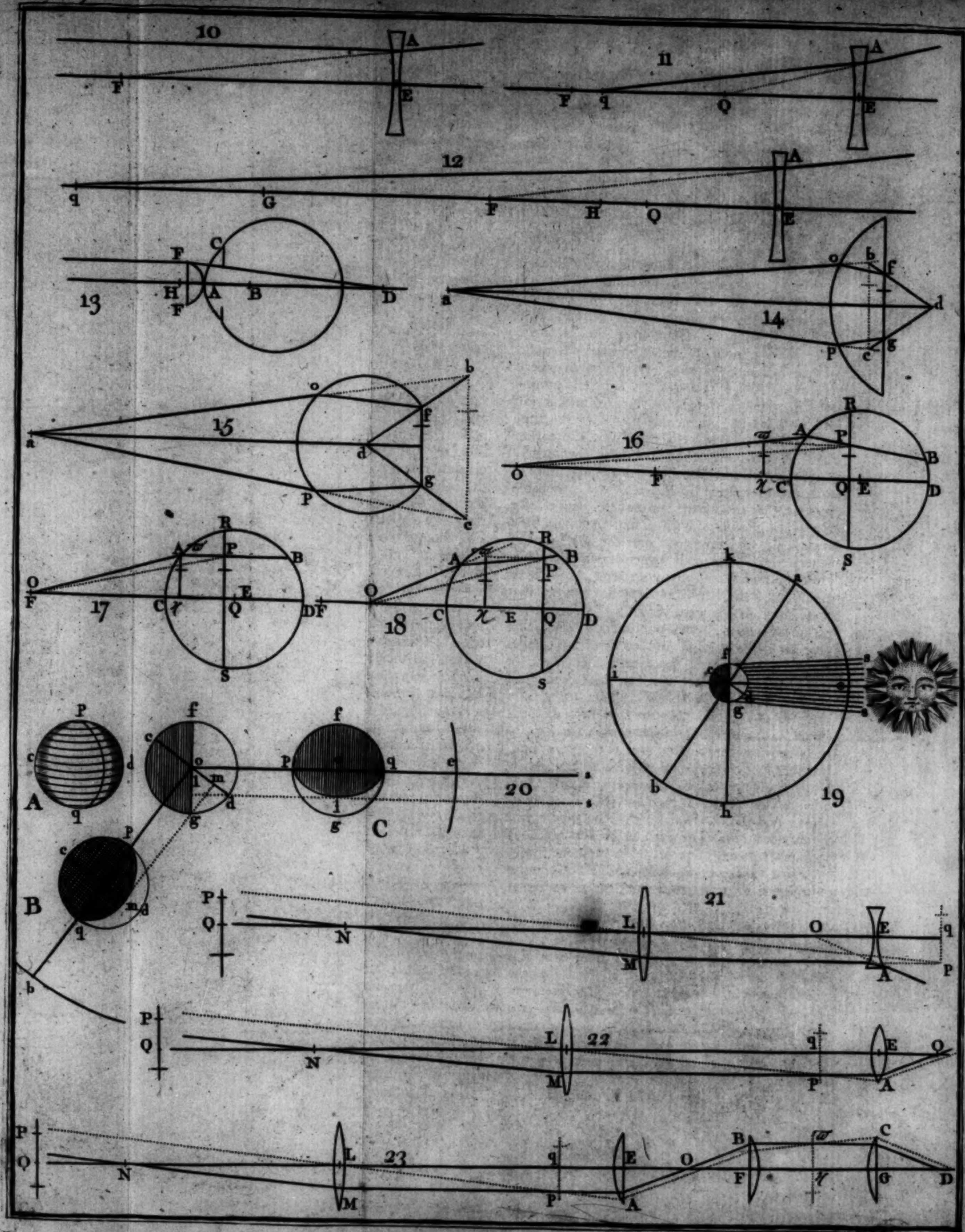
A more general demonstration of telescopes. Fig. 24, 25, 26, 27. d Art. 128.

126. Hitherto we have supposed the interval LE between the two convex glasses to be equal to the sum of their focal distances. Now let this interval be bigger or less, as is requisite for defective eyes[d], and let EF be the focal distance of the eye-glass, and Lq that of the object-glass; I say the apparent magnitude will be to the true as LF to FE, that is as the interval of the glasses diminished by the focal distance of the eye-glass, to the focal distance of the eye-glass. For the axes of all the pencils which pass through L, as PLA, will be refracted by the eye-glass to a focus G, where the eye being placed will see the whole object PQ, though the aperture of the pupil and of the object-glass be never so small; and the object PQ will appear under the angle AGE. But L being a focus of incident rays upon the eye-glass, we have $LF : LE :: LE : LG$*, and disjointly $LF : FE :: (LE : EG ::$ the angle EGA, to the angle ELA* or PLQ*) as the apparent magnitude, to the true. * Art. 239. * Art. 60. * Art. 43.

127. Hence according as the interval of the glasses is greater or less than the sum of their focal distances, the apparent magnitude is to the true, in a greater or less proportion than that of the focal distances.

128.

Pag. 22 of Remarks Plate 2.



To fit a telescope for observing eclipses and spots in the sun. Fig. 26, 27.

128. When the telescope is lengthened, the place of the image or focus q is farther from the eye-glass than its focal distance; and therefore this glass will make the emergent rays converge and form a second image or picture $\pi\kappa$ of the object PQ, upon any white surface, placed at the distance $q\kappa$, which is a third proportional after qF and qE *. And the apparent magnitude of this picture, seen by the naked eye from a distance equal to κG, is the same as if it could be seen distinctly from G through the telescope; the angles $\pi G\kappa$, and AGE being equal; and consequently is somewhat greater than when the length of the telescope is adjusted for distinct vision. The same demonstrations are applicable to *Galileo*'s telescope.

* Art. 239.

Brightness of the sun's image.

129. This is the way used by some astronomers, in observing eclipses of the sun and spots upon his body; which may also be observed in looking through the telescope, by applying a plane smoaked glass to the eye, or by smoaking the eye-glass it self. In the other way the image will appear brighter if the end of the telescope be put through a hole in the window-shutter of a dark room. And then if the round image upon the paper be equal to the aperture of the object-glass, it will appear as bright as if the paper was illustrated by the direct light of the sun; supposing no rays were intercepted by reflections from the surfaces of the glasses. And therefore as much as the image is made larger than this, by shortening the telescope a little, it will be so much the fainter, and not quite so distinct. But in this matter experience is the best guide.

Telescopes made with three glasses.

130. In the next book I have shewn the construction of telescopes with three glasses, but those made with two or four are the best, for shewing more of the object at one view, and less coloured about its circumference, as *Hugenius* has observed [a]. An object may be seen upright through two lenses, but not distinct nor much magnified and but little of it at one view; and besides, the distance between the glasses must be much greater than ordinary.

a Dioptric. prop. 54.

Upon Art. 125.

History of reflecting telescopes.

131. After the discovery of the true law of refraction, according to the given ratio of the sines [b], *Des Cartes* and other mathematicians soon found that all the rays of a large pencil could not possibly be collected to a distinct point by any object-glass composed of spherical surfaces [c]; and that the aberrations of the rays from that point were increased with the breadth of the glass. And this they took to be the cause of the apparent indistinctness of an object seen in a telescope, when the focal distance of the eye-glass is too much shortened. For as a shorter eye-glass increases the area of the picture upon the retina [d], it is necessary to inlarge the area of the object-glass for the reception of more light [e]; and this inlargement increases the aberrations of the rays in its focus, and consequently in the picture upon the retina.

b Rem. 4.

c Art. 81.

d Art. 120.

e Art. 129.

Hyperbolick and elliptical lenses why introduced. Fig. 28.

132. These aberrations, caused by the spherical surfaces of glasses, were then thought the only impediment to the perfection of telescopes; and this engaged the mathematicians in determining what figure a glass must have to refract all the rays of a pencil to a given point; and among others they found, that glasses figured according to the surfaces described by conick sections, turned about their axes, would have that effect [f]. For example if F and G be the focuses of two opposite hyperbola's, whose axis CD is to FG in the ratio of the sines of incidence and refraction, suppose of 2 to 3; and if one of these hyperbola's ACB be turned about its axis GCD, and a portion of the solid so generated be cut off by a plane AB perpendicular to the axis; then all the rays which fall perpendicular upon this plane will be refracted by the convexity ACB to the outward focus F.

f *Cartesii* Dioptr. Cap. 8. *Newtoni* Princip. Phil. Lib. 1. pr. 97. *Huygens* de la Lumiere p. 101. *Dechales* Tom. 3. pag. 681.

They do not answer the purpose.

133. This discovery presently engaged all the mathematicians and mechanicks to contrive engines for grinding and polishing glasses according to the shapes of these conick surfaces; and amongst the rest Sir *Isaac Newton* himself, in the beginning of the year 1666. But having a curiosity at the same time to try the celebrated phænomena of the colours generated by refraction of the sun's rays through a triangular prism, by which he happily discovered their causes, he presently left off his glass works [g]. "For I saw, says he, that the perfection of telescopes was hitherto limited, not so much for want of glasses truly figured according to the prescription of optick authors, (which all men have hitherto imagined,) as because light it self is a heterogeneous mixture of differently refrangible rays [h]. So that were a glass so exactly figured, as to collect any one sort of rays into one point, it could not collect those also into the same point, which having the same incidence upon the same medium, are disposed to suffer a different degree of refraction. This, says he, made me take reflections into consideration; and finding them regular, so that the angle of reflection of all sorts of rays was equal to their angle of incidence, I understood that by their mediation optick instruments might be brought to any degree of perfection imaginable; provided a reflecting substance could be found which would polish as finely as glass, and reflect as much light as glass transmits, and the art of communicating to it a parabolick figure could also be obtained.

g Phil. Transf. N°. 80.

h Art. 171, &c.

Newton's reflecting telescope when invented.

134. Amidst these thoughts, he tells us, he was forced from *Cambridge* by the intervening Plague in 1666, and proceeded no farther till above two years after. When having thought of a

a tender way of polishing, proper for metal, by degrees he perfected an instrument 6 inches long, which magnified between 30 and 40 times, and which he first described in the Phil. Transf. No. 80, and afterwards in his Opticks, page 91. 8°. And this is indisputably the greatest improvement that telescopes have ever received since their first invention.

Mr. Gregorie's was invented before it.

135. Nevertheless it must be acknowledged that Mr. *James Gregorie* of *Aberdeen* was the first inventor of a reflecting telescope. But his construction is quite different from Sir *Isaac Newton*'s, and not near so advantageous, as Sir *Isaac* himself has shewn in the Phil. Transf. No. 83. Mr *Gregorie* describes this telescope at the end of his *Optica promota* published in 1663; and was led into the invention of it, not by the consideration of the different refrangibility of rays, which was not then known, but by an inconvenience he foresaw would follow from an hyperbolick object-glass. For he observes that if it be sufficiently broad, to receive light enough into a telescope that shall magnify very much, it must of consequence be very thick; and then the clearest glass would hinder too much of the light from being transmitted. He might also have added another inconvenience, that though it will collect a pencil of rays coming parallel to its axis into a single point, yet it cannot collect the rays of an oblique pencil so accurately as a glass consisting of spherical surfaces will do; as has been found by experience [a]; and therefore spherical lenses upon this and some other accounts are fitter for optical uses than those of any other figure [b].

a Dechales Tom. 3. p. 686.

b Newtoni Princip. lib. 1. prop. 98. Schol.

Mr. Hadley brought them both into use.

136. These reflecting telescopes were first brought to perfection in practice about the year 1719, by the great ingenuity and industry of Mr. *John Hadley* [c]; Sir *Isaac Newton*'s first, and Mr. *Gregorie*'s soon after; which in small lengths has an excellent effect and is exceedingly commodious. The following description of it differs from the author's chiefly in this; that he directs his larger reflecting concave to be made of a parabolick figure, and his lesser of an elliptical one, instead of the spherical surfaces now used; which are the only figures that can be polished without insuperable difficulties.

c See Art. 582.

Mr. Gregorie's reflecting telescope described. Fig. 29.

137. It is proposed to make a reflecting telescope with two concave metals and a convex eye-glass and to shew its effects. Let the given focal distances of the lesser and the larger concave and of the convex eye-glass, be equal respectively to the lines t, T, q; and in a given line $ctqCl$, designed for their common axis, take in one and the same direction, $ct=t$, $tq=T$, $qC=\frac{t\times t}{T}$ and $ql=q$; and place the eye-glass at l, the lesser concave at c, and the larger at C; so that their concavities may respect each other; and let the incident rays, as QA, QB, be reflected from the larger to the lesser concave, and from thence to the larger again, where let them pass through a moderate hole made in the middle of it at C, and then be refracted through the eye-glass kl to the eye at o; I say a remote object will appear distinct and upright and magnified in the ratio of $T\times T$ to $t\times q$; that is, of the square of the focal distance of the larger concave, to the rectangle under the focal distances of the lesser concave and of the eye-glass.

138. For a pencil of rays QA, QB coming parallel to the common axis, will be reflected from the larger concave ACB to its principal focus T; where crossing one another and falling upon the lesser concave acb, they will be reflected from it to the point q. For since the focal distance $TC=T=tq$ by construction; by taking away the common part Tq, we have $tT=qC=\frac{t\times t}{T}$ by construction; that is, we have tT, tc, tq continual proportionals, as they should be [d]; and since pl is the focal distance of the eye-glass kl, the rays that flow from q will emerge from it in parallel lines, and therefore will produce a distinct appearance of the remote point Q from which they came.

d Art. 207.

139. Let ST be the image of the object PQ formed by reflection from the large concave; and it will be terminated by the line PES, drawn through E, the center of this concave, parallel to the rays PA, PA that flow from P *. Again the rays that flow from this image ST, will be reflected from the lesser concave and form a second image pq; which will be terminated by the line Sep drawn through the center e of this concave [e]; and the rays that diverge from p will emerge from the eye-glass kl in the lines ko parallel to the line pl, drawn through the center of the eye-glass [f]. Therefore the object PQ will appear upright, because the rays ko lye on the same side of the common axis Qlo as the point P from which they came.

Fig. 30.

* Art. 115.

e Art. 115.

f Art. 46.

140. In the second image pq take a line qs equal to the first image TS; and if the image pq was equal to qs, the object would appear through the eye-glass under an angle equal to qls*; which is to the angle PEQ or SET, under which it appears to the naked eye at E, as TE or TC to ql*; and so the object would be magnified in the same ratio as in Sir *Isaac Newton*'s telescope. But since the triangles epq, eST are similar; and since we had tq to te (as te to tT*, and disjointly as eq to eT, that is,) as pq to ST or qs; it appears that pq is bigger than qs, and also the visual angle kol or plq bigger than qls, in the said ratio of tq to te. And so the object being farther magnified in this ratio of tq to te or, by construction, of TC to tc, is magnified in the whole in the compound ratio of TC to tc, and of TC to ql, that

* Art. 107. 111.

* Art. 60.

* Rem. 138.

that is in the ratio of TC square, to the rectangle under tc and ql.

141. The magnifying power may also be demonstrated in Mr. *Huygens*'s manner [a]; that is, by considering a ray of an oblique pencil to go parallel to the axis between the lesser concave and the eye-glass, and by determining the ratio of the angles in which it will cut the axis.

a Rem. 123.

To adapt it to a near object.

142. For viewing near objects the little concave must be removed a little from the large one by the contrivance mentioned in Art. 124. Because while a remote object approaches, its image TS will also approach towards t; and while tT is diminished, its reciprocal tq will be increased [b].

b Rem. 138.

And to short-sighted eyes.

143. Therefore to fit this telescope for a short-sighted person, since the eye-glass is usually fixt, the little speculum must be moved somewhat nearer to the large one. For then the interval tT will also be diminished and its reciprocal tq will be increased; and so the rays will fall upon the eye-glass diverging from a nearer point than its focal distance; and consequently will emerge from it diverging upon the eye.

A more general rule for the magnifying power.

144. By a farther contraction of the interval between the concaves, the image pq may be projected through the hole in the large concave, to any given place behind it; and by removing the eye-glass to the same distance from the image as before, the vision would become distinct again; and the object would be more magnified than before, as much as the ratio of tq to te or tc is made bigger than the ratio of TC to tc; as appears by the demonstration [*]. But by enlarging the image pq, it becomes more obscure and imperfect, as shall be shewn hereafter, and consequently the appearance of the object less bright and distinct. Besides, as the image becomes larger, the less of it, and of the object, can be seen at one view through a given eye-glass.

* Rem. 140.

The visible area is as the breadth of the eye-glass.

145. All things being fixt in their places, the diameter of an object taken in at one view, is proportionable to the breadth of the eye glass, if the hole in the large concave does not limit it. For the angle of reflection pce, at the middle point of the lesser concave, being equal to the angle of incidence ecS; it appears, that while pq and kl are increased or diminished in any ratio, the image ST and the object PQ will also be increased or diminished in the same ratio.

It is inlarged by two eye-glasses.

146. Now if an eye-glass of a given focal distance and convexity, be made very broad, it will become too thick; and so the rays will fall too obliquely upon one or both its surfaces near the margin of it; and this obliquity will cause too many of them to be reflected, and the rest that are transmitted, to be too much refracted, in comparison to those pencils that pass through the middle of the said lens [c]. Therefore to increase the visible area of the object, it is necessary to project the image pq two or three inches beyond the hole in the large concave, and to intercept the rays that are tending towards it, with a thinner and broader convex glass fg put close to the backside of this concave; which glass will cause the rays to converge quicker than before, and to form an image vx nearer to it, and smaller, than pq; both being terminated by a line pvg drawn through the center of this glass [d]. And then the rays of each pencil diverging from this new image vx, must be received by another convex eye-glass bi, that shall make them emerge towards the eye in parallel lines. A meniscus glass, whose convex side is placed towards the converging pencils fvb, is fittest for this purpose; because the rays will pass through its edges less obliquely, than through a glass of any other shape.

c Art. 73.

Fig. 31. 32.

d Art. 55.

This telescope compared with a common one.

147. Having the places and focal distances of these eye-glasses, I could easily give a rule for finding the magnifying power of the telescope [e]; but the measures of such small distances being liable to errors, it is better to find it by experiment: either in *Galileo*'s way of viewing two unequal circles, one with the naked eye and the other through the telescope; or by comparing this telescope with a dioptrick one, whose magnifying power is known or easier to be found [f]. One of these telescopes 16 inches long, is reckoned to magnify as much as a dioptrick one of 15 or 16 feet.

e See Rem. 156.

f Art. 126.

The eye-stops.

148. To prevent collateral rays, that pass by the sides of the smaller concave, through the hole in the larger, and those also which are reflected from the imperfect margins of them both, from entering into the eye; it is necessary to place a thin plate with a proper hole in it to circumscribe the image at x, and also another very small hole at o, where all the pencils cross one another immediately before they enter the eye. The breadth of this latter hole must be no bigger than that of the principal pencil at o, and the places of them both must be exactly adjusted; otherwise the telescope can have no good effect.

Some problems solved. Fig. 34.

149. If the focal distance of the lesser concave be a given line t, and it be required to place it so, that the rays shall be reflected from the given focus T to a given point q; bisect Tq in m, and to mT erect a perpendicular Tn equal to the line t; and joining mn, towards T set off mt equal to mn; and t will be the point where the focus of the lesser concave must be placed. For let a semicircle, described with the center m and semidiameter mn or mt, cut the axis again in z; and we have qz equal to Tt, and consequently Tz equal to tq; we have also Tn a middle proportional between the segments tT, Tz of the diameter tz [*], that is, the focal distance t or tc is a middle proportional between tT and tq, and therefore the

* Eucl. VI. 13.

rays that flow from T, will be reflected from the lesser concave to the given point q *.

* Art. 207.

150. Hence also, if it be required to find the focal distance of the lesser concave, whose focus being placed at a given point t, shall cause the rays to be reflected from a given point T to a given point q; bisect Tq in m, and with the center m and semidiameter mt describe a circle, cutting an indefinite perpendicular erected from T, in the point n; and you have Tn equal to the focal distance required.

151. The larger concave, and the convex eye-glass, and the interval Tq between the two images of a remote object being given; if it be required to find the focal distance and place of the lesser concave, that shall cause the telescope to magnify the object in any proposed ratio; since this given ratio is compounded of the given ratio of TC to ql and of tq to tc*, this latter ratio is also given; for which putting n to 1, take tT to Tq as 1 to $nn-1$; and you have tT; then take tc to tT as n to 1; and you have the position and magnitude of tc. For since the unknown lines tT, tc, tq are continual proportionals in the given ratio of 1 to n, we have tT to tq as 1 to nn*, and disjointly, tT to Tq as 1 to $nn-1$.

* Rem. 144.

* Eucl. VI. 20. cor. 2.

The little concave speculum may be changed for a convex one. Fig. 34.

152. Telescopes of this kind are sometimes made with a little convex speculum instead of the concave one. If their focal distances be equal, and the vertex of the convex de, be placed at e, where the center of the concave was, the telescope will magnify in the same ratio as before; but will shew the object inverted; unless it be set upright by three convex eye-glasses, as in a dioptrick telescope. For a pencil of rays converging from the large concave towards its focus T, being intercepted by the little convex de, will be reflected by it to the same point q as before by the little concave bc. For the point t being the principal focus of both these little speculums, we have tT, te (or tc) and tq continual proportionals as before[a]. Through any point S of the first image ST and through the center e of the little concave, draw Sep terminating the image pq formed by this concave[b]; in like manner through c the center of the little convex de, and through the same point S, draw cSr terminating the image qr formed by this convex. These images qp, qr lye on contrary sides of the axis, and therefore the object appears in contrary positions. But these images are equal, and of consequence the object appears equally magnified. For we have $tq : te :: te : tT :: tq \mp te : te \mp tT$, that is $:: eq : eT :: cq : cT$. And the triangles peq, TeS being similar, and also qcr, Tcs, we have $pq : ST$ $(:: eq : eT :: cq : cT ::)$ $qr : ST$; therefore pq is equal to qr.

a Art. 207.

b Art. 215.

Upon Art. 127.

To compute the magnifying power of a double microscope. Fig. 185.

153. Hence we may compute the magnifying power of a double microscope in this manner. When the object appears distinct, measure the distances LQ and LE, and also Eq the focal distance of the eye-glass. Then by substracting Eq from EL we have Lq, and also the quotient of Lq divided by LQ. Again by dividing the measure of the least distance from which we commonly view minute objects, that is 6 or 8 inches, by the measure of the focal distance Eq, we have another quotient; which being multiplyed by the former quotient, gives the number of times, by which the diameter of the object is magnified: as I said in the present article. For the triangles pLq, PLQ being similar, the object PQ is contained in its image pq, as often as LQ is contained in Lq. But the rule being more general than this demonstration, which supposes the rays of each pencil to emerge parallel from the eye-glass, or the image pq to fall upon its principal focus, I will add another demonstration of it.

Another demonstration of double microscopes. Fig. 35.

154. Let the image pq fall at any distance from the eye-glass AE, and let EF be its focal distance; and with the center E and semidiameter EF, describe an arch FG cutting the axis PLA of any oblique pencil, in G. Draw GE, and AO parallel to it; and the ray PLA will be refracted into the line AO*. Draw PR parallel to AO or GE, and supposing the naked eye placed at any point Ω in the axis LQR produced, join $P\Omega$. Now since the angle PLQ or FLG* is very small, the arch FG may be taken for a stright line perpendicular to the axis LE; and therefore the figures $LPQR$, $LGFE$ are similar[c]. Consequently we have $QR : QL :: FE : FL$. Whence $QR = \frac{QL \times FE}{FL}$. But the apparent magnitude of the object seen at O, is to its apparent magnitude seen at Ω, as the angle AOE or PRQ, to the angle $P\Omega Q$; that is, as $Q\Omega$ to QR* or $\frac{QL \times FE}{FL}$; that is, as $Q\Omega \times FL$ to $QL \times FE$, or as $\frac{Q\Omega}{FE} \times \frac{FL}{LQ}$ to 1.

* Art. 51.

* Art. 43.

c Art. 104.

* Art. 60.

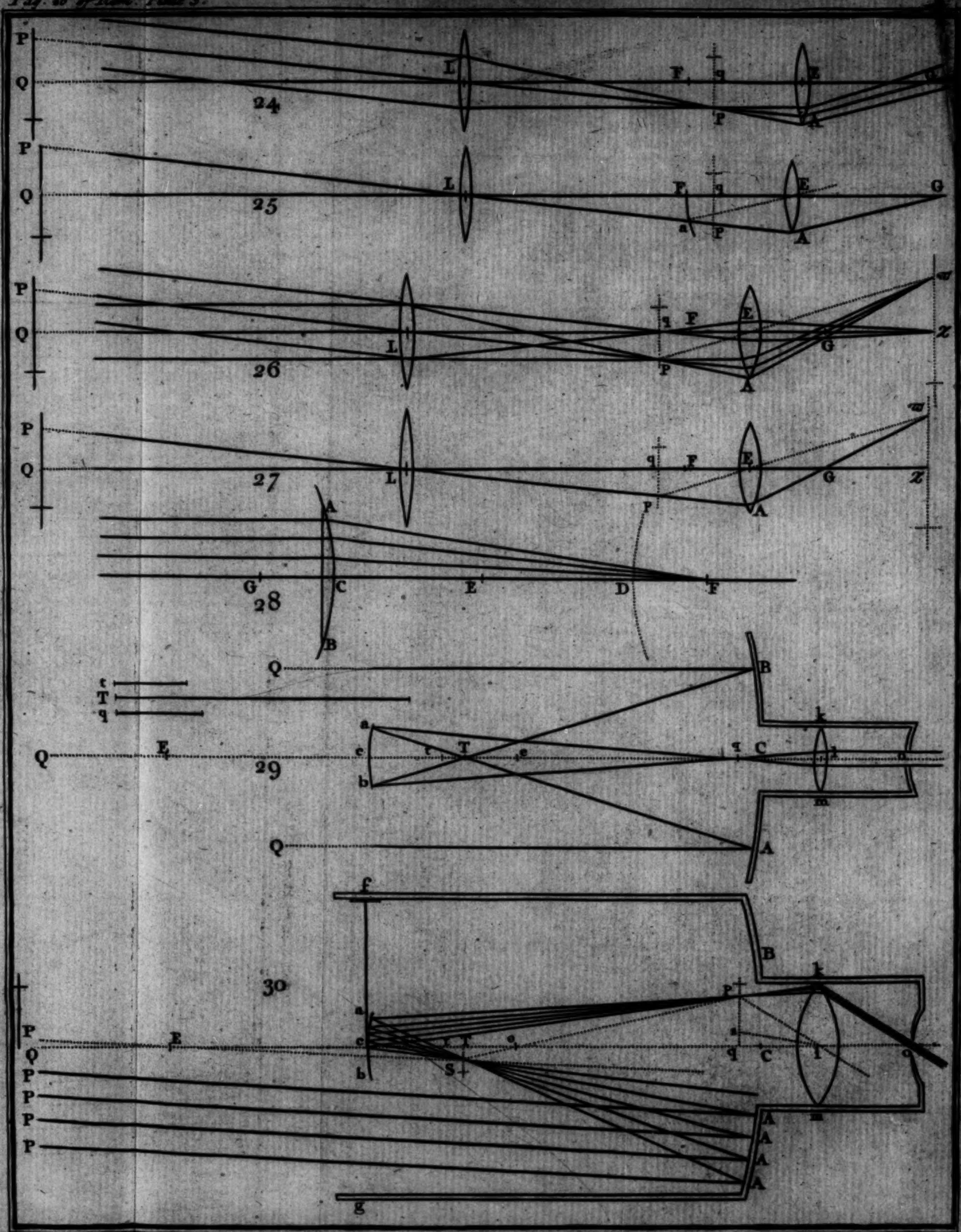
Visible area considered.

155. Hence it appears that the apparent magnitude of the object may be increased, either by bringing it nearer to the object-glass, and consequently by inlarging its image, or by viewing the same image through a smaller eye glass. But in this proceeding there are two limitations. The first is, that the aperture of the object-glass must be increased to admit more light[d], which will increase the imperfections of the image[e]; and the second is, that the visible area of the object will be diminished, either by increasing the image or by viewing it through a smaller and narrower

d Art. 129.

e Art. 81.

Pag. 26 of Rem.ts Plate 3.



er eye-glass. The first limitation is considered in the next book, and the second may be removed as follows.

A microscope with two eye-glasses. Fig. 36. * Art. 55.

156. When it is required to see much of an object at one view, it is usual to interpose a pretty broad convex glass AE, between the object-glass L and the image pq formed by it. For this glass AE will contract the image pq into a shorter one $\varpi\kappa$, terminated by the line pE * ; and then the rays that diverge from this image $\varpi\kappa$, may all be received upon a narrower eye-glass ae, and be refracted, to the eye at o, either into parallel or in diverging lines, as ao. When all these glasses are fixt at proper intervals to be found by experience, distinct vision may be procured by a gradual alteration of the distance $L\mathcal{Q}$. Then let all the distances $L\mathcal{Q}$, LE, Ee be measured, and also the focal distances EF and ef of the two eye-glasses, by Art. 63; and by saying as LF to LE so LE to Ll, we have the line Ll; and by taking Lf from it, we have fl; and the apparent magnitude of the object seen through the microscope, will be to its apparent magnitude seen by the naked eye from the distance $\mathcal{Q}\Omega$, as $\frac{\mathcal{Q}\Omega}{\mathcal{Q}L} \times \frac{FL}{FE} \times \frac{fl}{fe}$ to 1.

Fig. 37.

157. For with the centers E, e and semidiameters EF, ef describe the arches FG, fg; and let the axis $PLGA$ of an oblique pencil, cut FG in G; and joining GE, the ray LA will first be refracted into the line Al parallel to GE * ; and consequently since the triangles LGE, LAl are equiangular, we have LF to LE (or LG to LA *) as LE to Ll. Let the ray Al cut the arch fg in g, and the eye-glass ea in a, and it will be refracted into a line ao parallel to ge *: and so the eye being placed at o will see the object $P\mathcal{Q}$ under the angle aoe. But this angle aoe or feg is to the angle flg as fl to fe * ; and again this angle flg or FEG is to FLG as FL to FE * ; and lastly this angle FLG or $PL\mathcal{Q}$ is to $P\Omega\mathcal{Q}$ as $\mathcal{Q}\Omega$ to $\mathcal{Q}L$ *. And by compounding these ratios, the angle aoe is to $P\Omega\mathcal{Q}$ as $\mathcal{Q}\Omega \times FL \times fl$ to $\mathcal{Q}L \times FE \times fe$, or as $\frac{\mathcal{Q}\Omega}{\mathcal{Q}L} \times \frac{FL}{FE} \times \frac{fl}{fe}$ to 1.

* Art. 51. * Art. 204. * Art. 51. * Art. 60. * Art. 60. * Art. 60.

158. This middle glass is only useful for seeing more of the object at one view. For the more glasses are used, the more light is lost by reflections at their surfaces; and a single eye-glass will magnify more, and more distinctly, than two of them.

Upon Chapter 5. Concerning our ideas acquired by sight.

Upon Art. 132.

159. I must here make my acknowledgements, once for all, to my highly honoured friend the learned and judicious Dr. *Jurin*, for obliging me with some of his curious remarks and dissertations, written at my request upon the subjects of Art. 132, 135, 137, 160; which I have specified in the margin, as follows.

Dr. *Jurin's* solution of Mr. *Molyneux's* problem.

160. It is a thing well known that when several ideas have been habitually joined and connected together, if one or two of these be accidentally presented to the mind, the rest of the class which have been usually so connected, are immediately excited in us. In hot weather the sight of sun-shine is instantly accompanied with the ideas of heat and uneasiness, and the view of a grove of trees does as immediately produce those of shade and refreshment. In winter the sight of the sun, or of a fire in the room we come into, is as readily followed by the ideas of warmth and pleasure.

161. But this connexion or association of ideas is no where more constant, and at the same time no where less attended to by the generality of mankind, than in the different ideas of sight and touch. When a solid body is presented to our view, the idea first and immediately excited in us, is no other than that of a various complication of light and shade, which instantly raises in our mind the other subsequent ideas belonging to the touch, of solidity, convexity, or angularity, which have been used to accompany such a sensation of sight, or to speak in the language of the acute and judicious author of the *Theory of Vision*, the visible idea excites in us those tangible ideas which have been used to go along with it. And this is so instantly and imperceptibly done, that we can hardly avoid esteeming that to be a bare sensation of our sight, which is indeed the act of our memory and of our judgement, the greater part of mankind thinking they see a globe to be convex and a cube to be angular, when really they only judge them to be so. The truth of this doctrine has been so well demonstrated by Mr. *Locke* and the abovementioned ingenious writer, and is so clearly confirmed in this very chapter, particularly by the curious observations of Mr. *Chesselden*, as to put the matter out of all doubt. But I must confess, I do not see that the famous question of Mr. *Molyneux*, as it is by him stated, concerning the globe and cube set before a man born blind and now made to see, is of any use towards establishing this doctrine: nor could I ever bring my self to think that he had rightly solved his own problem. I am not afraid to say so, though Mr. *Locke* himself has been pleased to declare, that

he agrees with that thinking gentleman. For notwithstanding this piece of complaisance, Mr. *Locke*'s determination appears to me to be expressed in such a manner, and with such limitations not to be found in the problem proposed by Mr. *Molyneux*, as intirely convince me that the judgment of this great and clear reasoner was widely different from that of his friend. In order to make this appear more plainly, I shall here set down Mr. *Molyneux*'s problem, with his determination of it, and that of Mr. *Locke*, in their own words.

Essay on Human Understanding. Book 2. Chap. 9. Sect. 8.

162. "*Suppose a man born blind, and now adult, and taught by his touch to distinguish between a cube and a sphere of the same metal, and nighly of the same bigness, so as to tell when he felt one and the other, which is the cube, which the sphere. Suppose then the cube and sphere placed on a table, and the blind man to be made to see. Quære, whether by his sight, before he touched them, he could now distinguish and tell, which is the globe, which the cube.* To which the acute and judicious Proposer answers: *Not. For though he has obtained the experience of, how a globe, how a cube affects his touch; yet he has not yet obtained the experience, that what affects his touch so or so, must affect his sight so or so; or that a protuberant angle in the cube that pressed his hand unequally, shall appear to his eye as it does in the cube.* I agree with this thinking gentleman, whom I am proud to call my friend, in his answer to this his problem; and am of opinion, that the blind man, at first sight, would not be able with certainty to say which was the globe, which the cube, whilst he only saw them; though he could unerringly name them by his touch, and certainly distinguish them by the difference of their figures felt. So far Mr. *Locke*.

163. Here it is to be observed that by Mr. *Molyneux*, the blind man now made to see, is debarred of no other help towards distinguishing which is the globe and which the cube, except only that of touching those bodies. He is left at full liberty to make use of his sight as he pleases, to look at them again and again, and to view their several sides by walking round them; as likewise to use his memory and his reason, if they can any way assist him. These last are so far from being excluded by Mr. *Molyneux*, that the use of them is manifestly implied by the words of the problem and his determination of it. For the blind man is supposed to be *now adult*, and to have *obtained the experience of how a globe and how a cube affects his touch*; though the author is of opinion that this experience will be of no use to him on the present occasion.

164. But does Mr. *Locke* allow the blind man all these advantages? No. As soon as he is made to see, he requires him to pronounce, *with certainty at first sight*, which is the globe, which the cube; without giving him leave to take a second view, much less to recollect himself and to reason upon what he sees. The question therefore determined in the negative by Mr. *Locke* is widely different from that proposed by Mr. *Molyneux*; and we have no reason to think that so accurate a writer would have made such a material alteration in the conditions of the problem, if, without doing so, he could have subscribed to his friend's determination.

165. Setting aside therefore the authority of Mr. *Locke*, or rather taking it in to my own assistance, I proceed to prove against Mr. *Molyneux*, that the blind man, now brought to sight, shall be able to distinguish and tell which is the globe, and which the cube, before he touches them.

166. But here, in order to prevent mistakes, I must observe, that the question is not, whether this man, upon seeing the two bodies, shall of himself know them to be a globe and cube: this I take to be utterly impossible for him to discover, without touching them. All that Mr. *Molyneux* queries, is, whether he can distinguish and tell which is the globe, which the cube; which question manifestly implies the two following conditions.

First. That the blind man shall by sight perceive the globe as one thing, distinct from the cube and all other bodies; and shall likewise perceive the cube as one thing distinct from the globe and all other bodies.

Secondly. That he shall be told, the two bodies he sees are one a globe and the other a cube; without which information it is to no purpose to ask him which is the globe and which the cube.

This therefore being done, I will suppose him to take a careful and repeated view of the two bodies in open sky-light, and by walking round the table on which they are placed, to see and observe them in all their different situations; after which he will be able to reason in this manner.

167. The two bodies before me, I am told, are one a globe and the other a cube.

They are therefore of a different figure one from the other.

The sensations I receive from them, are likewise different.

I learn therefore from the little experience I have already had of this new sense of seeing, that it is differently affected by diversity of bodies. And my reason tells me it ought to be so. For, if two bodies are alike in figure, I must needs be affected in like manner by them both: and on the contrary, when I am differently affected by two or more bodies, I must conclude that this diversity of sensations is occasioned by the diversity of figure of those bodies.

168. Farther, I find my sense to be differently

ly affected by one of these bodies, when I view its several sides, and observe its several parts.

These parts therefore are in themselves different from one another, for the reason just now mentioned, and consequently the body composed of them is not all over alike.

The other body, in what manner, or on what side soever I view it, always gives me the same sensation, and is therefore all over alike.

Now I remember, when a globe and cube were given me to handle, before I received my sense of seeing, that by reasoning then in the same manner upon what occurred to my touch, as I now do upon what is presented to my sight, I discovered not only that a globe and cube were bodies of a different make; but that a globe was a body alike all over, and that a cube was not so, but composed of parts greatly differing from one another.

This body therefore, which my sight informs me is alike all over, is undoubtedly the globe, and the other is the cube.

169. Thus, I think the blind man will unerringly distinguish between the two bodies, and that by the use of this single principle, that his senses were not given to deceive him; but that the different sensations, which several bodies raise in him, are caused by the difference of those bodies; without which our senses would not only be fallacious, but utterly useless.

170. I cannot omit taking notice that, upon talking about this problem not long since, with my highly esteemed friend the author of this treatise, I had the satisfaction to be informed by him, that the greatest blind philosopher that ever was, whom I need not call Dr. *Sanderson*, is of the same opinion with me, and reasons upon the problem much in the same manner. And I must observe that this learned author himself, by using these words, *by sight alone*, which seem to exclude the use of reason, as much as those of Mr. *Locke*, *at first sight*, appears not to be intirely of the same sentiment with Mr. *Molyneux*.

Upon Art. 135.

Dr. *Jurin's* remark upon the association of ideas.

171. Though it be beside the present purpose of the author, yet it may not be wholly improper to observe here, that when the ideas of sight excite in us those tangible ideas, that have been used to go along with them, this is not owing to any peculiar virtue or property in the ideas of sight to exhibit other ideas, but only to the general faculty of our memory; which upon any idea being presented to the mind, does readily suggest to us such other ideas, whether of the same or of any different sense, as have usually accompanied it.

172. That this is true of the ideas of hearing is evident from the use of language; the various sounds of which do readily excite in us such ideas, whether visible, or tangible, or of any other sense, with which those sounds have usually been connected. And any person who is ever so little attentive to what passes in his own mind, will find the same to hold in every one of our senses.

173. As I go home in the dark, I find by my touch several different objects in my way. I feel but a part of them, and immediately the visible ideas of the whole are excited. I know the one to be a post, the second a man, and the third a house.

174. I hear a rattling noise behind me, and presently the visible idea of a coach and horses arises in my mind, and with it the corresponding tangible ideas likewise, and I get out of the way to avoid the danger.

175. I pass by two several places, where I meet with very different scents, which raise in me the visible ideas, here of woodbinds growing in a court yard, and there of a dunghil.

176. I come home and go into a room, where I feel for a table with several sorts of fruit upon it. I tast one, and immediately know it to be a bergamot pear; another I find to be a golden pippin, the visible idea of each being instantly suggested by their taste.

177. If these instances are not thought sufficient, or not sufficiently clear, the imagination of every thinking person will readily supply him with innumerable others, to prove that the ideas received by any one of our senses do readily excite such other ideas, either of the same sense or of any other, as have habitually been associated with them. So that, if on this account we are to suppose, with a late ingenious writer, that the ideas of sight constitute a visual language, because they as readily suggest to us the corresponding ideas of touch, as the terms of a language excite the ideas answering to them, I see not but we may for the same reason allow of a tangible, an audible, an olfactory, and a gustatory language as well as a visual: though it must be owned that, as sight is without comparison the most comprehensive of all our senses, the visual language will be abundantly more copious than any of the rest.

Upon Art. 137.

178. A person is said to squint, when the axes of both his eyes are not directed to the same object. This defect, I think, is by the generality of physicians supposed to proceed from the want of a due correspondence in the muscles of the eyes, which not acting in a proper concert with one another, as in persons free from this blemish, are not able to point both eyes at one object. But the famous Mr. *de la Hire* a who has since been followed by a very great and learned Professor, whose sentiments are more generally embra-

Dr. *Jurin's* dissertation upon squinting.

a Mem. de Math. & Phys. Ann. 1694. Accid. de la vue. art. 10.

embraced by those of the faculty throughout *Europe*, than perhaps those of any physician since the time of *Galen*, is of a contrary opinion; and imagines this deformity not to arise from any depraved habit, or want of consent in the muscles of the eyes, but from a defect in the eye it self, which he explains after this manner.

The opinion of *de la Hire* and others.

179. He supposes that, in the generality of mankind, that part of the retina, which is seated in and about the axis of the eye, is of a more delicate sense and perception, than what the rest of that coat is endowed with; and therefore that we direct both axes to the same object, not only for the sake of direct vision, whereby the image of the object may be more distinctly and perfectly painted upon the retina; but likewise and indeed chiefly, in order to receive the picture upon that part of the retina, which can best and most accurately perceive it. But in persons who squint, he conceives the most sensible part of the retina of one eye, not to be placed in the axis, but at some distance from it on the one side, or on the other; and that therefore in the eye so unusually framed, not the axis, but this more sensible part of the retina is turned toward the object, on which the axis of the other eye is fixed; and consequently both axes are not directed to the same point.

Confuted by experiments.

180. If this opinion be true, that the eye is thus distorted for the sake of seeing more distinctly with it; then, if the other eye be shut, and the distorted eye alone be used to look at an object, it must still be as much distorted as before, for the same reason. The contrary of which is true in fact, as may easily be experienced.

181. Desire such a person to close his other eye, and to look at you with that which is usually distorted. He will immediately turn the axis of it directly towards you. Bid him open the undistorted eye, and look at you with both eyes. You will find the axis of this last now pointed at you, and the other eye turned away from you and drawn close to the nose, or perhaps to the upper eye-lid, as I have sometimes observed it. Make as many tryals as you please, you will always find the same event.

182. Likewise, if a squinting person uses a glass to read with, or to look at any minute object, with the eye usually distorted, you may easily observe the axis of that eye to be pointed directly at the object.

Squinting persons see objects distinctly with one eye only.

183. From these tryals it plainly appears, that the eye is thus distorted not for the sake of seeing better with it, but rather to avoid seeing at all with that eye, as much as possible. For its pupil being generally drawn close to the nose, cannot receive the image of the object, towards which the other eye is directed, but in a very oblique and indistinct manner; and consequently can be no more affected with it, than the eyes of other persons are by such objects as lye at a considerable distance laterally from that which we look at. So that in reality a squinting person sees the object before him distinctly with one eye only; namely that whose axis is pointed directly at the object. Consequently we may conclude, that this defect is not caused by any such præternatural make of the eye, as those two learned gentlemen have supposed.

Other opinions confuted.

184. Nor is it occasioned by any defect in the muscles of the distorted eye. For when the other is shut, this eye is moved by the action of its muscles, in all possible directions, as freely as that of any other person,

185. Neither is it owing to the want of correspondence in the muscles of both eyes, such as to hinder them from being both moved the same way at the same time. For when both eyes are open, and the undistorted eye is moved either upwards or downwards, or to the right or left, the other always accompanies it, and is turned the same way at the same instant of time.

The disposition of the axes of common eyes.

186. But to understand particularly wherein this defect consists, it will be necessary to consider the disposition and situation of the eyes, in persons who are not affected with such a blemish.

For seeing remote objects.

187. When we look directly forwards at a distant object, the pupil of each eye lies in the middle of the aperture formed by the eye lids, so that the distance between the two pupils consists of the breadth of the nose and half the breadth of the aperture of each eye: and howsoever obliquely we turn our eyes, this very same distance between the two pupils is always preserved.

For seeing near objects.

188. In looking at near objects the distance between the pupils is something less; but still this same lesser distance is always preserved in all oblique directions of the eyes, as well as when we look straight forwards. By which means the axes of both eyes are directed towards the same object, in both these cases of looking at distant aud near objects.

The disposition of the axes of squinting eyes.

189. But in those who squint, when the pupil of the undistorted eye is seated in the middle of the aperture, as in looking directly forwards, the pupil of the other eye is drawn close to the nose, and consequently the distance between the two pupils is considerably less than in other persons; and this lesser distance between the two pupils continues the same in all oblique directions of the eyes; so that the two axes are never pointed at the same object, though the muscles do so far act in concert with each other, as to move both eyes the same way at the same instant of time,

One probable cause of squinting.

190. This vitious habit may easily be contracted by a child, if he is often laid into his cradle in such a position, as to be able to see either the light

light, or any other remarkable object, with one eye only.

The common methods of cure why ineffectual.

191. And when by this means he is brought to squint, and is afterwards conformed and settled in the practice of doing so, I apprehend, it will be in vain to attempt a cure by his wearing tubes, or shels with small holes in them to look through. Do what you will of this kind, he will continue to see through them distinctly with one eye only, and will still distort the other.

The true method of cure.

192. The true method of cure I take to be this. When the child is arrived at such an age as to be capable of observing directions, place him directly before you, and let him close the undistorted eye and look at you with the other. When you find the axis of this eye fixt directly upon you, bid him endeavour to keep it in that situation, and open his other eye. You will now immediately see the distorted eye turn away from you towards his nose, and the axis of the other will be pointed at you. But with patience and repeated tryals he will by degrees be able to keep his distorted eye fixt upon you, at least for some little time, after the other is opened. And when you have brought him to continue the axes of both eyes fixt upon you, as you stand directly before him, it will be time to change his posture, and to set him first a little to one side of you, and then to the other, and so to practise the same thing: and when in all these situations he can perfectly and readily turn the axes of both eyes towards you, the cure is effected. An adult person may practise all this by a glass, without any director; though not so easily as with one. But the older he is, the more patience will be necessary.

An instance of it.

193. About twenty years ago I attempted a cure after this manner, upon a young gentleman about nine years of age, with promising hopes of success; but was interrupted by his falling ill of the small pox, of which he dyed.

Another opinion confuted.

194. I must not forget to take notice that Mr. *de la Hire*, in art 62, supposes squinting to proceed sometimes from another cause, namely the oblique situation of the crystalline humor in one of the eyes: but this is easily refuted by the experiments abovementioned, in the same manner as his former opinion. So far Dr. *Jurin*.

A remarkable instance of double vision.

195. After this 137th article was printed, I received an account of another remarkable instance of double vision, from my highly honoured friend *Martin Folkes* Esquire. Who tells me, he was informed by Dr. *Hepburn* of *Lynn*, that the late Reverend Mr. *Forster* of *Clenchwharton* in that neighbourhood, having been blind for some years with a *gutta serena*, was restored to sight by salivation; and that upon his first beginning to see, all objects appeared to him double; but afterwards, the two appearances approaching by degrees, he came at last to see single; and as distinctly as he did before he was blind.

Opinions upon the cause of double vision.

196. Another instance or two of this kind may be seen in Dr. *Briggs's Nova Visionis Theoria* pag. 25; wherein he proposes a theory, or rather an hypothesis, to account for the single and double appearances of an object, by means of equal degrees of tension of the fibres, of both the optick nerves, continued from the brain to corresponding parts of the two retinas; in which parts the two pictures of an object usually fall. So that the isochronous vibrations of these corresponding fibres agitated by the rays, may stir up a single sensation in the mind; in like manner as unisons in musick are hardly distinguished from one sound. But when the two pictures of an object fall upon parts of the retina, where the tensions of the fibres are different; their discording vibrations may stir up two distinct sensations in the mind; as in musical concords and discords. But for a fuller account of this hypothesis I chuse to refer the reader to the author himself; because there are several hints and observations in that little treatise, and in his *Anatomy of the Eye*, which may be useful and entertaining to the curious. Sir *Isaac Newton*'s sentiments upon this subject of single and double vision, may be seen in the 15th Query at the end of his Opticks. *Kepler* has observed a, that the supposed union of the nerves, any where between the two retinas and the brain, is repugnant to the double appearance of an object. Because, if the fibres of the nerves were there united, he reckons we should always see an object single, but never double. See also *Dechales*'s Cursus Math. Tom. 3. pag. 410.

a Dioptr. prop. 62.

Upon Art. 138.

The design.

197. In the present article, to avoid being tedious and troublesome to readers unprejudiced by received opinions, I gave but a summary view of the result of my thoughts and experiments upon apparent distance. But as this subject has never been rightly settled, and as it is higly necessary thereunto, that the principles of it should be clearly explained, and firmly established; for farther satisfaction it may not be amiss, to be somewhat more explicit. I observed that, by abundance of experiments made with glasses of all sorts, I found that an object always appeared to approach while its apparent magnitude increased, either by moving the glass, the eye or the object, backwards or forwards; and that it always appeared to recede while its apparent magnitude decreased; just as in vision with the naked eye. A few of the easiest and plainest experiments will be abundantly sufficient to establish this truth.

I. Exper. with a concave lens at rest.

198. Let any concave lens be fixt between your eye and any remote objects; and while your eye is drawn back from the lens, (the other being

being shut,) the apparent magnitudes of the objects will continually decrease, and their apparent distances from you will continually increase; and the contrary will appear while your eye approaches to the lens.

II. Exper. with a concave lens in motion.

199. Or if your eye be fixt, and the concave lens be gradually removed from it; the apparent magnitudes of the remote objects will continually decrease, and their apparent distances from you will continually increase; even though the lens were carried half way towards the objects: and the contrary will appear while the lens is brought back to your eye.

III. Exper. with a convex speculum.

200. Or if a convex looking-glass be used instead of the concave lens, the apparent magnitudes and distances of remote objects, lying laterally behind you, or of your own face, will also vary reciprocally; as they did in both the foregoing experiments.

IV. Exper. with a convex lens at rest.

201. Now let any convex lens be fixt between your eye and any remote objects; and when your eye is first put close to the lens, and then drawn back from it, the apparent magnitudes of the objects will continually increase, and their apparent distances from you will continually decrease; so long as the objects appear upright. And even after they are inverted, if the eye be drawn farther back, their apparent magnitudes will continually decrease, and their apparent distances from you will continually increase.

V. Exper. with a convex lens in motion.

202. Or if your eye be fixt, and the convex lens be applied close to it, and then be gradually removed from it; the apparent magnitudes and distances of remote objects will correspond reciprocally, and in the same order, as they did before; even though the lens were carried half way towards the objects.

VI. Exper. with a concave speculum.

203. Or if a concave looking-glass of a large sphere, be used instead of this convex lens; the apparent magnitudes and distances of remote objects, lying laterally behind you, or of your own face, will always vary reciprocally with respect to each other. The reason of these variations of apparent magnitude, may be seen in Art. 106 and 110; and the corresponding variations of apparent distance are the facts, for which we are to assign a reason, that shall hold good in all cases.

VII. Exper. with a plane speculum.

204. But first it is observable that while the apparent magnitude varies quicker or slower, the apparent distance does also vary quicker or slower; as will be evident by moving the eye or the glass, quicker or slower, or by using glasses of lesser or larger spheres. And even the slow variations of apparent magnitude and distance, are also observable in a plane looking-glass; especially by attending to the minute parts of objects, as they seem to depart from you; otherwise the imagination supplies a magnitude, which the sense does not perceive. But if the plane of the looking-glass be not perfect, it will frequently distort objects at a great distance, and gently increase their apparent magnitudes, and thereby deceive you.

Hence apparent distance is reciprocally as apparent magnitude.

205. It is also observable in all these experiments, when the eye and the glass are close together; that the apparent magnitudes and distances of all objects, are the same as to the naked eye; and when the eye and the glass are separated, that the apparent distance varies reciprocally in the same proportion as the apparent magnitude varies; that is, when one becomes double or treble, the other becomes half or one third respectively; as near as the sense can distinguish: as any one will find by comparing the appearances of the same objects, seen at one view through the lens, and by the sides of it with the naked eye.

Uses of this general principle.

206. It is true that the sense alone cannot accurately determine such ratios of apparent distances, or even of magnitudes, as should be expressed by larger numbers; and therefore a general rule derived from the simplest ratios and experiments, is the more useful and necessary to conduct our inquiries in more complicated cases; and to examine how near the appearances of things, and the causes assigned for them, do agree in Quantity with each other. For as this is the surest and the best means of distinguishing true causes from false ones, so the general neglect of it has been the chief occasion of all the errors in philosophy.

The certainty of its foundation.

207. The apparent distances of things seen clearly by the naked eye, are unalterable by the power of imagination; and therefore being determinate in themselves, they have determinate ratios to each other, and determinate causes: and those that look into glasses will be sensible of the same thing. And that all people agree in their judgments of the measure of apparent distance in glasses, will appear by this experiment. I well remember, when several persons were trying to read a gazette at a great distance, through one of Mr. *Gregory*'s reflecting telescopes, that I asked them one by one, how near they thought it appeared through the telescope; and whether as near as my face appeared to their naked eye, when I placed it before them, by the side of the visual rays coming from the gazette; and removed it backwards and forwards, as they directed, till they judged the two objects appeared equidistant, by the side of each other. And by marking the several stations to which they reduced me, I found their differences were very little, even in this gross way of tryal; though the objects were of different kinds, and the spectators of different ages; some of them being children. Therefore since determinate judgments have adequate causes, the judgments may be measured with the same certainty as you can measure the causes.

208.

The reason of it.

208. This constant and regular connection between the quantities of apparent magnitude and apparent distance, being proved to be matter of fact, let us now consider how it comes to be so. In describing the experiments above I have supposed the objects to be pretty remote; not because they will not succeed when the objects are near, but because they are somewhat simpler and plainer; and also because the eye can comprehend at one view a larger system of remote objects in all manner of oblique and direct situations to the visual rays. Which shews that their distances are suggested to the mind by the same cause, acting in the same manner in all these cases. Now since any one will acknowledge, at first sight of these objects through a concave lens, that they all appear smaller, exactly in the same manner as if he saw them with his naked eye from a greater distance; what can be plainer than that this smaller appearance suggests to him the usual idea of that greater distance, which has been constantly annext to it by experience from his infancy [a]? And the like may be said of the greater and nearer appearances of objects seen through a convex lens or in a concave looking-glass. Intending to prove this general solution of these Phænomena in a more particular manner in a remark upon Art. 148, at present I will proceed to consider the received principles and opinions of optick writers upon this subject.

a Art. 135. and Rem. 171.

Dr. Barrow's difficult case of apparent distance solved.

209. The famous difficulty which so much distressed the learned Dr. *Barrow*, was to account for the apparent distance of an object seen as in our 4th, 5th and 6th experiments [b]; that is when the visual rays fall upon the eye converging towards an image behind it. This difficulty arose naturally from a principle universally received in opticks, viz. that an object seen by reflections and refractions, appears always in the place of its image from which the rays diverge upon the eye. Which principle, though it happens to agree with experience in two or three common cases, is in general quite disagreeable to it, and to reason too; though not so manifestly as in that uncommon case which the Doctor thus describes.

b Rem. 201.

His own description of it.

210. *Hæc sunt, quæ circa partem opticæ præcipue mathematicam dicenda mihi suggessit meditatio. Circa reliquas, (quæ φυσικώτεραι sunt, adeoque sæpiuscule pro certis principiis plausibiles conjecturas venditare necessum habent) nihil fere quicquam admodum verisimile succurrit, a pervulgatis (ab iis, inquam, quæ* Keplerus, Scheinerus, Cartesius, *& post illos alii tradiderunt) alienum aut diversum. Atqui tacere malo, quam toties oblatam crambem reponere. Proinde receptui cano; nec ita tamen ut prorsus discedam anteaquam improbam quandam difficultatem (pro sinceritate quam & vobis & veritati debeo minime dissimulandam) in medium protulero, quæ doctrinæ nostræ, hactenus inculcatæ, se objicit adversam, ab ea saltem nullam admittit solutionem. Illa, breviter, talis est:* lenti vel speculo cavo EBF *exponatur punctum visibile* A, *ita distans ut radii ex* A *manantes, ex inflexione versus axem* AB *cogantur. Sitque radiationis limes (seu puncti* A *imago, qualem supra passim statuimus) punctum* Z. *Inter hoc autem & inflectentis verticem* B *uspiam positus concipiatur oculus. Quæri jam potest ubi loci debeat punctum* A *apparere? Retrorsum ad punctum* Z *videri non fert natura (cum omnis impressio sensum afficiens proveniat a partibus* A*) ac experientia reclamat. Nostris autem e placitis consequi videtur, ipsum ad partes anticas apparens, ab intervallo longissime dissito, (quod & maximum sensibile quodvis Intervallum quodammodo exsuperet) apparere. Cum enim quo radiis minus divergentibus attingitur objectum, eo (seclusis utique prænotionibus & præjudiciis) longius abesse sentiatur; & quod parallelos ad oculum radios projicit, remotissime positum æstimetur. Exigere ratio videtur, ut quod convergentibus radiis apprehenditur, adhuc magis, si fieri posset, quoad apparentiam elongetur. Quin & circa casum hunc generatim inquiri possit, quidnam omnino sit, quod apparentem puncti* A *locum determinet, faciatque quod constanti ratione nunc propius, nunc remotius appareat? Cui itidem dubio, nihil quicquam ex hactenus dictorum* analogia, *responderi posse videtur, nisi debere punctum* A *perpetuo longissime semotum videri. Verum experientia secus attestatur, illud pro diversa oculi inter puncta* B, Z, *positione varie distans, nunquam fere (si unquam) longinquius ipso* A *libere spectato, subinde vero multo propinquius apparere; quinimo, quo oculum appellentes radii magis convergunt eo speciem objecti propius accedere. Nempe, si puncto* B *admoveatur* oculus, *suo (ad lentem) fere nativo in loco conspicitur punctum* A *(vel æque distans, ad* speculum;) *ad* O *reductus oculus ejusce speciem appropinquantem cernit; ad* P *adhuc vicinius ipsum existimat; ac ita sensim, donec alicubi tandem, velut ad* Q, *constituto oculo objectum summe propinquum apparens, in meram confusionem incipiat evanescere. Quæ sane cuncta rationibus atque decretis nostris repugnare videtur, aut cum iis saltem parum amice conspirant. Neque nostram tantum sententiam pulsat hoc experimentum; at ex æquo cæteras quas norim omnes, veterem imprimis ac vulgatam, nostræ præ reliquis affinem, ita convellere videtur, ut ejus vi coactus doctissimus* A. Tacquetus *isti principio (cui pene soli totam inædificaverat* Catoptricam *suam) ceu infido ac inconstanti renunciarit, adeoque suam ipse doctrinam labefactarit; id tamen, opinor, minime facturus, si rem totam inspexisset penitius, atque difficultatis fundum attigisset. Apud me vero non ita pollet hæc, nec eousque præpollebit ulla difficultas, ut ab iis, quæ manifeste rationi consentanea video, discedam; præsertim quum, ut hic accidit, ejusmodi difficultas in singularis cujuspiam casûs disparitate funde-*

Fig. 38, 39.

E

fundetur. Nimirum in præsente casu peculiare quædam, natura subtilitati involutum, delitescit, ægre fortassis, nisi perfectius explorato videndi modo, detegendum. Circa quod nil, fateor, hactenus excogitare potui, quod adblandiretur animo meo, nedum plane satisfaceret. Vobis itaque nodum hunc, utinam feliciore conatu, resolvendum committo.

Translated. 211. I beg leave to borrow the following translation from an ingenious writer. "I have here delivered what my thoughts have suggested to me, concerning that part of opticks which is more properly mathematical. As for the other parts of that science (which being rather physical, do consequently abound with plausible conjectures instead of certain principles) there has in them scarce any thing occurred to my observation, different from what has been already said by *Kepler*, *Scheinerus*, *Descartes*, and others. And methinks, I had better say nothing at all, than repeat that which has been so often said by others. I think it therefore high time to take my leave of this subject: but before I quit it for good and all, the fair and ingenuous dealing that I owe both to you and to truth, obligeth me to acquaint you with a certain untoward difficulty, which seems directly opposite to the doctrine I have been hitherto inculcating, at least, admits of no solution from it. In short it is this. Before the double convex glass or concave speculum *EBF*, let the point *A* be placed, at such a distance that the rays proceeding from *A*, after refraction or reflection, be brought to unite somewhere in the axis *AB*. And suppose the point of union (i. e. the image of the point *A*, as has been already set forth) to be *Z*; between which and *B*, the vertex of the glass or speculum, conceive the eye to be any where placed. The question now is, where the point *A* ought to appear? Experience shews that it doth not appear behind at the point *Z*, and it were contrary to nature that it should; since all the impression which affects the sense comes from towards *A*. But from our tenets it should seem to follow, that it would appear before the eye at a vast distance off, so great as should in some sort surpass all sensible distance. For since, if we exclude all anticipations and prejudices, every object appears by so much the farther off, by how much the rays it sends to the eye are less diverging; and that object is thought to be most remote, from which parallel rays proceed unto the eye; reason would make one think, that object should appear, at yet a greater distance, which is seen by converging rays. Moreover it may in general be asked concerning this case, what it is that determines the apparent place of the point *A*, and maketh it to appear after a constant manner, sometimes nearer, at other times farther off? To which doubt, I see nothing that can be answered, agreeable to the principles we have laid down, except only that the point *A* ought always to appear extremely remote. But on the contrary, we are assured by experience that the point *A* appears variously distant, according to the different situations of the eye between the points *B* and *Z*. And that it doth almost never (if at all) seem farther off, than it would if it were beheld by the naked eye, but on the contrary, it doth sometimes appear much nearer. Nay, it is even certain, that by how much the rays falling on the eye do more converge, by so much the nearer does the object seem to approach. For the eye being placed close to the point *B*, the object *A* appears nearly in its own natural place, if the point *B* is taken in the glass; or at the same distance, if in the speculum. The eye being brought back to *O*, the object seems to draw near; and being come to *P*, it beholds it still nearer. And so on by little and little, till at length the eye being placed somewhere, suppose at *Q*, the object appearing extremely near, begins to vanish into mere confusion. All which doth seem repugnant to our principles, at least, not rightly to agree with them. Nor is our tenet alone struck at by this experiment, but likewise all others that ever came to my knowledge are, every whit as much, endangered by it. The ancient one especially (which is most commonly received, and comes nearest to mine) seems to be so effectually overthrown thereby, that the most learned *Tacquet* has been forced to reject that principle, as false and uncertain, on which alone he had built almost his whole catoptricks, and consequently by taking away the foundation, hath himself pulled down the superstructure he had raised upon it. Which, nevertheless, I do not believe he would have done, had he but considered the whole matter more throughly, and examined the difficulty to the bottom. But as for me, neither this, nor any other difficulty shall have so great an influence on me, as to make me renounce that which I know to be manifestly agreeable to reason: especially when, as it here falls out, the difficulty is founded in the peculiar nature of a certain odd and particular case. For in the present case something peculiar lies hid, which being involved in the subtilty of nature, will, perhaps, hardly be discovered till such time, as the manner of vision is more perfectly made known. Concerning which, I must own, I have hitherto been able to find out nothing that has the least shew of probability, not to mention certainty. I shall, therefore, leave this knot to be untied by you, wishing you may have better success in it than I have had."

Fig. 38, 39.

His principle and the old one compared. 212. Dr. *Barrow*'s manner of proposing and describing this difficult case, so opposite to his own theory, which he had cultivated throughout his Lectures, is indeed a noble declaration of his regard

regard to truth, and highly worthy of a great mind. The old principle that he mentions, which *Euclid*, *Albazen*, *Tacquet*, and almost all optick writers had hitherto followed, is this. Any given visible point of an object, appears at the intersection of the reflected or the refracted visual ray produced, and of a line drawn through the visible point perpendicular to the reflecting or refracting surface, whether plane or spherical. This intersection always coincides with our image of the visible point in a plane looking glass, and also in reflections and refractions at plane and spherical surfaces, provided the angle of incidence be very small as it generally is. But whatever be the magnitude of the angle of incidence Dr. *Barrow's* principle is, that the visible point appears by reflections or refractions at a certain point, from which those rays of a slender pencil diverge, in falling on the eye, that will enter the pupil. Which point in strictness of geometry, is always some point of a caustick formed by reflections or refractions of all the rays that flow from the visible point [a]: and therefore differs widely from the place of our image, when the eye lyes wide of the axis of the caustick, that is when the angle of incidence is large, but nearly coincides with it when the eye is near the axis, as in the present experiments.

[a] Art. 154.

And both proved to be insufficient by experiments with lenses. Fig. 40, 41.

213. In the mathematical part of his Lectures this learned Author has made many useful improvements; but in the physical part he seems to have failed in his principal design; which was to determine the apparent place or distance of an object universally and more accurately than by the anciently received principle. Now the reason of this failure and consequently of the falsity of both these principles, appears to me very plain by the following experiments. Holding any concave lens very close to your eye, repeat the Doctor's experiment with his lens or speculum, and you will presently find that the concave eye-glass will not sensibly alter the apparent distance of the object. This will be evident by slipping the eye-glass sideways off and on your eye alternately, while fixt at any given distance from the convex object-glass; and by comparing the appearances. Now if the focal distance Bb of this concave eye-glass B, be less than the focal distance of the convex object-glass A; the rays which flow from a point Q of a remote object, and after refractions through the object-glass A, go converging towards a point q, being intercepted and refracted through the concave eye-glass B, will fall upon the eye diverging from a point k; provided the interval of the glasses A, B be small enough for the principal focus b of the concave eye-glass B, to fall between it and the image q [*]. It is plain then that the object appears at one and the same distance, whether seen by rays diverging from the last image k, or converging towards the first image q when the eye-glass is taken away; and consequently that the apparent distance of the object has no dependance upon the places of its images. For these may be still varied at pleasure, by applying different glasses, either concave or convex, to the eye in the same place B; and yet the apparent distance will always be the same as if they were taken quite away. And by drawing your eye and eye-glass back towards the image q; while the principal focus b approaches to q and passes over it, the second image k will run out to an infinite distance, and then will return back the contrary way, from an infinite distance behind the eye [c]; and yet all this while the apparent distance will vary exactly in the same proportion as in Dr. *Barrow's* manner of tryal, when the rays fell converging, upon his naked eye, towards a fixt image q. Which plainly shews that his principle of divergency of rays, has nothing at all to do with apparent distance, to a single eye at least.

[*] Art. 48.

[c] Art. 48.

And with a looking-glass.

214. And the like experiments, tryed upon objects seen in a common looking-glass, will shew that concave or convex glasses, held close to the eye, do not alter the apparent distances of the objects; though the last image, or place from which the rays diverge upon the eye, or towards which they converge, after passing through the different eye-glasses, is varied thereby at pleasure. By which it is also evident to me, that no author, that I have seen, has given the true reason of the apparent distance of an object seen in a common looking-glass. See Art. 146. and Rem. upon it.

The origine of this false principle.

215. What misled them was this. Finding by experience that the apparent place of an object seen in a plane looking-glass, is always as far behind the glass, as the real place of the object is before it; finding also, by the known law of reflections, that the rays must diverge upon the eye, as if they had come from that place [d]; from this coincidence of facts, (in this and in a like case, of seeing objects that lye under water [e], which are the most obvious of all others,) they concluded too hastily, that the divergency of the visual rays from the place of the image of an object, was the cause of its appearing in that place; and of consequence that the same cause must hold good in vision by reflections and refractions at spherical surfaces. But this principle being false even in that simplest case of a looking-glass, as we shewed above, it is no wonder that so many difficulties have arisen from it in more complex cases; as in those observed by Dr. *Barrow*, *Gregory*, *Tacquet*, *Molyneux* and all the best writers.

[d] Art. 25.

[e] Art. 146.

Not true in vision with the naked eye.

216. But the fundamental error is still unobserved. Those that use spectacles and concave glasses to help the defects of their eyes, perceive objects through them very distinctly; and so

so do others that have no defects, provided the convexities and concavities of the glasses be not too great. Now when these glasses are put close to their eyes, all objects appear to them nearly of the same magnitudes and at the same distances, as they do to the naked eye[a]. But now the rays of the pencils do not diverge upon the eye from the place of the object, but as if they had come straight through the concave from a place much nearer, or straight through the convex from a place much remoter, than that of the object, or even converged towards a place lying behind the eye. And yet the object appears in its usual place. And consequently the divergency of the rays upon the naked eye from this real place, cannot be the cause of its appearing there. Nor indeed are we certain that it does appear there, but only thereabouts when the place is near to us. But when we look at very remote objects, they evidently do not appear, even to the naked eye, in the real places and positions which we know they possess; but sometimes nearer, sometimes remoter; some towards the one hand and some towards the other. See instances of this in Art. 138. pag. 51 line 11; and in Art. 158 to 163. and in Art. 169, 170. and Rem. upon it. Thus by arguing analytically from observations and experiments, it is evident to me, that the divergency of rays from an object in any place, is not the cause that suggests its apparent place, even to the naked eye.

a Art. 117.

Otherwise confuted in Dr. *Berkeley*'s new theory. sect. 13.

217. Upon this occasion I cannot omit taking notice, that this received principle was first rejected (upon proof of its insufficiency by arguments *à priori*) by the most ingenious and learned Author of the late *Essay towards a New Theory of Vision*; concerning the manner wherein we perceive by sight, the distance, magnitude and situation of objects: a Work highly entertaining and useful to all that are duly qualified to consider it. And herein, had this worthy gentleman, whom I greatly esteem, thought proper to have joined some experiments with glasses, or even geometrical inferences from glasses, with the other helps which his most eminent qualifications afforded him for this subject; I cannot but think, that he might have been furnished with several reasons for rejecting another principle or two, which he substituted instead of that old one; and which, in pursuance of my design of settling this science upon a firm foundation, I cannot avoid considering in the following remarks.

218. Having done with the divergency of rays, which is not perceived by sense; let us now consider the degrees of apparent confusion, that frequently result from different divergencies. For the confusion being certainly perceived, may be reasonably thought to have an influence upon the mind in its judgments of distance: and indeed one can hardly help thinking so, upon perusing the following argument of the ingenious writer abovementioned.

His new principle; that apparent confusedness suggests distance. ibid. sect. 21.

219. "An object placed at a certain distance from the eye, to which the breadth of the pupil bears a considerable proportion, being made to approach, is seen more confusedly: and the nearer it is brought, the more confused appearance it makes. And this being found constantly to be so, there ariseth in the mind an habitual connexion between the several degrees of confusion and distance; the greater confusion still implying the lesser distance, and the lesser confusion, the greater distance of the object.

And is the occasion of those judgments attributed to diverging rays. ibid. sect. 22.

220. "This confused appearance of the object doth therefore seem to be the medium, whereby the mind judgeth of distance in those cases, wherein the most approved writers of opticks will have it judge by the different divergency, with which the rays, flowing from the radiating point, fall on the pupil No man, I believe, will pretend to see or feel those imaginary angles, that the rays are supposed to form according to their various inclinations on his eye. But he cannot choose seeing whether the object appears more or less confused. It is therefore a manifest consequence from what has been demonstrated[a], that instead of the greater, or lesser divergency of the rays, the mind makes use of the greater or lesser confusedness of the appearance, thereby to determine the apparent place of an object".

a Ibid. sect. 13. See Rem. 217.

And is shewn to agree with Dr. *Barrow*'s difficult case. ibid. sect. 31. Fig. 38, 39.

221. "Let us now see how Dr. *Barrow*'s experiment agrees with our tenets. The eye the nearer it is placed to the point *B* in the foregoing figures, the more distinct is the appearance of the object; but as it recedes to *O*, the appearance grows more confused; and at *P* it sees the object yet more confused; and so on till the eye being brought back to *Z* sees the object in the greatest confusion of all. Wherefore by Rem. 219 the object should seem to approach the eye gradually, as it recedes from the point *B*, that is at *O* it should (in consequence of the principle I have laid down in the aforesaid Remark) seem nearer than it did at *B*, and at *P* nearer than at *O*, and at *Q* nearer than at *P*; and so on, till it quite vanishes at *Z*. Which is the very matter of fact, as any one that pleases may easily satisfy himself by experiment."

Ibid. sect. 36.

222. Dr. *Berkeley* farther observes; that though the confusion of near objects, seen by the naked eye, results from too great a divergency of the rays, and in Dr. *Barrow*'s experiment, from a convergency of them; yet that equal degrees of confusion, either way produced, will have the same effect upon the mind. For, says he, "the eye, or (to speak truly) the mind perceiving only the confusion it self, without ever considering the cause from which it proceeds, doth constantly annex the same degree of distance

stance to the same degree of confusion. Whether that confusion be occasioned by converging, or by diverging rays, it matters not. Whence it follows, that the eye viewing the object Z through the glass QS (which by refraction causeth the rays ZQ, ZS, &c. to converge) should judge it to be at such a nearness, at which if it were placed, it would radiate on the eye with rays diverging to that degree, as would produce the same confusion, which is now produced by converging rays, i. e. would cover a portion of the retina equal to DC in figure 42. But then this must be understood (to use Dr. *Barrow*'s phrase) *seclusis praenotionibus & prajudiciis*, in case we abstract from all other circumstances of vision, such as the figure, size, faintness, &c. of the visible objects; all which do ordinarily concur to form our idea of distance; the mind having by frequent experience observed their several sorts or degrees, to be connected with various distances.' So far Dr. *Berkeley*.

Fig. 42.

Fig. 43.

But it disagrees with Dr. *Barrow*'s experiment in the quantity of apparent distance.

223. Now supposing his principle to be true; namely that apparent distance (or the idea of distance) is suggested by apparent confusion, I agree with him perfectly in this his conclusion, viz. " that the eye viewing the object through the glass, should judge it to be at such a nearness, at which if it were placed, it would radiate upon the eye with rays diverging to that degree, as would produce the same confusion, which is now produced by converging rays [a]." This is a just consequence from the principle; but it shews, that an object seen but the least confusedly in glasses, ought always to appear within a foot or two of the eye at the farthest. Because most people perceive by the naked eye, little or no confusion in objects, placed at those distances, or even much smaller. But in Dr. *Barrow*'s experiment, objects appear confused at all degrees of apparent distance, not exceeding their apparent distance to the naked eye. For the object being placed any where beyond the focal distance of the glass, will appear confused though the eye should touch the glass; because the rays fall converging upon the eye; and if the object be removed from the glass gradually, or which comes to the same, if the eye and the glass close together, be gradually drawn from the object, the confusion will increase [b] with the apparent distance; which is always the same very nearly as it would have been to the naked eye thus drawn back [c]. And by using glasses of different convexities put close to the eye, the apparent confusion may be altered at pleasure, without altering the real and apparent distance. It follows then, from the disagreement of these appearances with the conclusion abovementioned, that the principle it self is insufficient; and consequently that the agreement between greater nearness and confusedness, when the eye is drawn back from the glass, is only accidental with respect to confusedness; and necessarily owing to the apparent magnitude increasing with the confusedness [d].

a Rem. 222.

b Art. 48.

c Art. 117.

d Rem. 208.

And with the same experiment tryed with a concave eye-glass. Fig. 40.

224. Again, let a concave eye-glass, of a much shorter focal distance than the convex lens used in the last experiment, be held close to your eye, and also to this lens, and then the object will appear confused through both, by reason of too great a divergency of the rays in falling upon the eye. But while these glasses are gradually separated, the confusion will first decrease to nothing, and then increase again, while the apparent distance of the object is decreasing continually. The reason of these variations of confusion, will appear from the motion of the second image k in Rem. 213; and shall be farther explained in the next remark but one.

[A digression upon the use of perspectives without tubes.]

225. And by the way, this experiment shews that pretty long perspectives may be used without tubes; it being sufficient to hold the head of your cane with the eye-glass in one hand, and any other part of the cane with the object-glass in the other hand; and then to slide this glass to a proper distance, soon found by tryal, for shewing the objects most distinctly. Thus you may use the glasses of a perspective as long as your arm, as I frequently do when I carry them about with me. But to procure a brighter appearance of objects, it is better to have a broader object-glass than those that are commonly included in tubes.

And with other experiments tryed with a convex eye-glass. e Rem. 198.

226. But to return. Let the order of the glasses be now inverted; that is, hold a convex lens close to your eye, whose focal distance is somewhat greater than that of the concave lens or the convex speculum, used in the three first experiments [e]; and by repeating them over again with this convex eye-glass, you will find the same variations of apparent distance and magnitude as without the eye-glass [f]. But now the apparent confusion and distance will always increase both together, or decrease both together. Which is contrary to the principle of confusion. The reason of these variations of confusion is this. Let the concave object-glass be at A, the remote object at Q, its image formed by this concave at q, from which the rays diverge in falling upon the convex eye-glass B; and let the second image formed by this glass be at k, from which the rays diverge, or to which they converge in falling upon the eye at B. Now since the focal distance Bb of the convex eye-glass B, is supposed to be longer than Aq; it follows, when the glasses are near to each other, that the image q will be nearer to the eye glass B, than its principal focus b; and then the rays will diverge upon the eye as from the image k, and the vision will be distinct, if k be not too near the eye. But while the glasses are drawn asunder, and the interval qb decreases to nothing

f Art. 117.

Fig. 44, 45.

Fig. 45.

thing and then becomes negative, the distance *Bk* will increase to infinity and then become negative and decrease on the other side of the eye [a]; and consequently as the rays fall more and more converging upon the eye, the confusion will increase; while the apparent distance decreases [b], as it did without the eye-glass [c].

a Art. 48.

b Rem. 198.

c Art. 117.

Objection answered.

227. As to excluding prænotions and prejudices; that is, as the author expresses it, in case we abstract from all other circumstances of vision, such as the figure, size, faintness &c. of the visible objects [d]; I answer, that when a given fixt object is viewed successively through different eye-glasses put close to the eye, fixt at any given distance from the fixt object-glass; the figure, size and faintness of the visible object are not sensibly altered by the different eye-glasses; and yet the confusedness, though very sensibly altered, or reduced to distinctness, produces no sensible alteration of the apparent distance of the object in any of my tryals.

d Rem. 222.

Therefore apparent confusion does not suggest distance.

228. It appears to me by these experiments and many more that I have tryed, that confusedness of appearance does not suggest nearness or any distance at all. Perhaps the reason may be, that most children and young people do not perceive any confusedness in near objects; because they commonly pore upon their letters in learning to read and write; their eyes, like other parts of their bodies, being then more pliant than afterwards. And therefore though a confused appearance of near objects, may gradually come upon them as they grow older; yet they rather avoid it than attend to it as a sign of nearness; because they did not want it, having constantly been used to other signs of nearness sufficient for the purpose.

Nor does the straining of the eye.

229. But we are told that a straining of the eye, to prevent confusedness, is a sensation that supplies the place of it in suggesting nearness [e]. I answer that this sensation being somewhat painful might hinder young people from poring. But they do pore, and therefore seem to have no sensation of straining; nor any need of it, to suggest nearness, though it afterwards comes upon them. For as they grow older they strain their eyes to avoid confusedness, or to restore the usual distinctness and largeness, together with a perception of the minuter parts of things; and are not these usual perceptions sufficient to suggest the usual idea of nearness? When this straining is no longer effectual, they are forced to use spectacles, by which the old associated ideas, of distinctness, largeness and nearness, are again revived in their minds. Small objects seen through spectacles, by those that want their help, appear much distincter than to the naked eye; and therefore should appear much remoter, according to the principle of confusedness; but I am told that they appear rather nearer, as they should do by theory; because they appear rather larger, by reason of a small distance of the spectacles from the eyes.

e Ibid. sect. 27.

Nor does faintness of appearance, perceived through pin-holes.

230. Lastly for farther satisfaction, that apparent distance does not depend upon apparent confusedness, nor upon the straining of the eye, nor upon degrees of brightness and faintness of appearance, let the following experiments be tryed. To prevent confusedness and the straining of the eye, make a fine pin-hole in a card, or a paper, and having applyed it close to your eye, repeat any of the experiments abovementioned; and you will find that the objects will always appear distinct, even when the rays converge towards the eye; and yet the degrees of apparent distance will continue the same as before; and will be somewhat more determinate, because the apparent magnitudes are now more distinctly defined by distincter out-lines [f]. It is best to look at objects without doors, because they radiate stronger than those within doors, and appear more evident through a small hole. Now by slipping the hole to one side of the pupil, and by restoring it alternately, while the eye, the object and glasses are fixt; you cannot be certain of a sensible alteration of their apparent distances; though their distinctness and brightness be greatly altered. And the same holds good in using the pin-hole without any glasses. Because, when it is close to the eye, it does not sensibly alter the apparent magnitudes of objects placed at any distances whatever; and yet it causes very near ones to appear distinct.

f Art. 117.

Or through smaller apertures of telescopes.

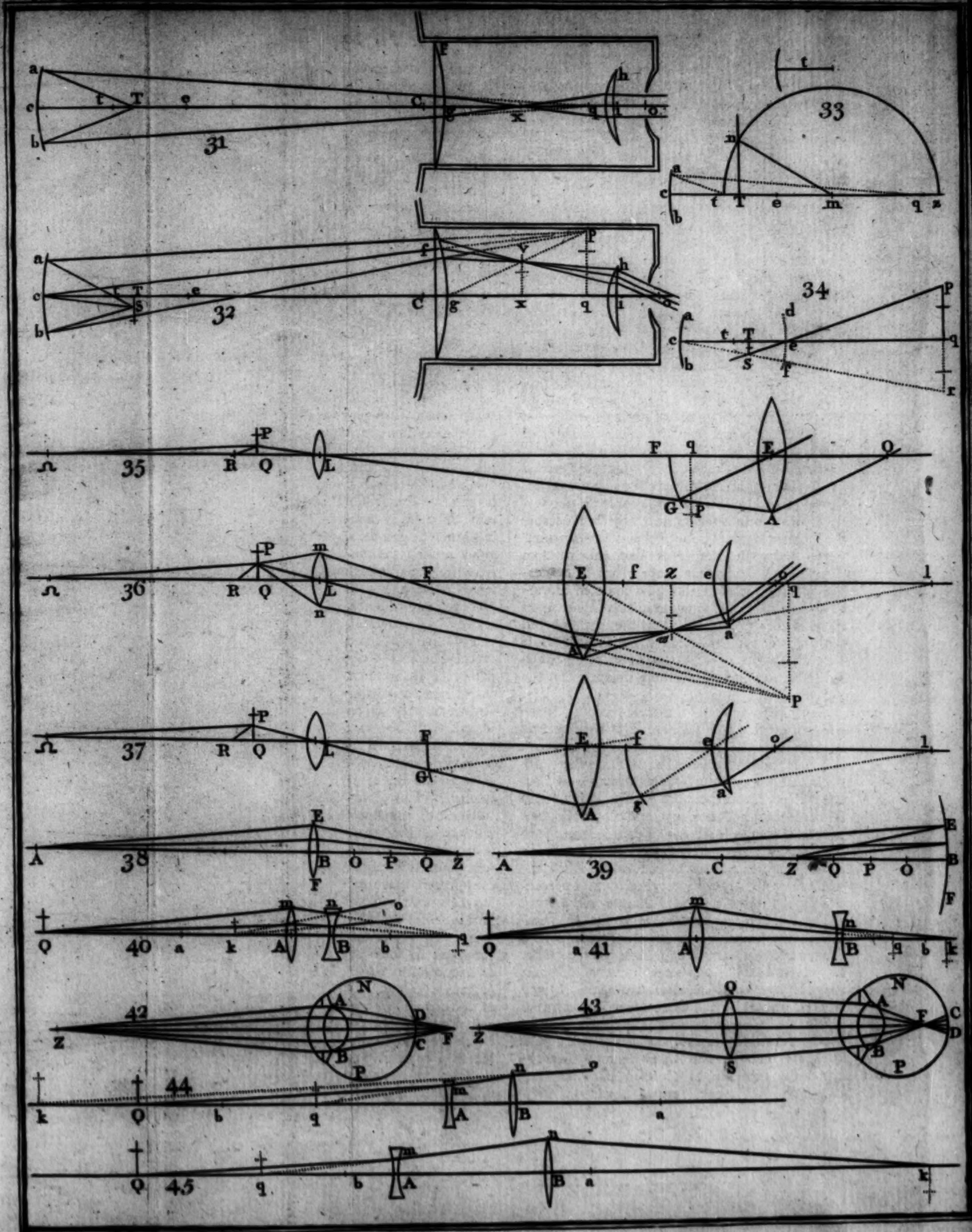
231 In like manner if the aperture of the object-glass of a telescope be varied, by covering it successively with circles of paper, having holes in their centers of very different sizes; it is well known that the apparent brightness of any given object, will be varied in proportion to those apertures. But its apparent distance will not be varied thereby, so long as the same eye-glass is retained. But if it be changed for other eye-glasses of shorter focal distances, the apparent distance of the object will now be diminished; notwithstanding that its faintness is increased [g]. This may be accurately tryed in the manner described in Rem. 207. Therefore the increase of the apparent magnitude is the cause of the greater nearness notwithstanding the faintness; because nothing else is sensibly altered.

g Art. 348.

General conclusion.

232. I conclude then by induction of particulars in all these experiments (to say nothing of many more) that, whatever be the variations of the divergency and convergency of rays, or of distinctness and confusedness, or of brightness and faintness, the apparent distance of a given system of known objects seen with glasses, in the field or in any spacious place where the fancy has room enough to work in [h], is suggested to us either principally or solely by its apparent magni-

h See Rem. on Art. 151.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magnitude. And since our judgments of appearances in glasses, are indisputably derived from our experience in vision with the naked eye; it follows that here also the same general conclusion takes place. And therefore this conclusion, thus discovered by analysis, may be assumed as a principle in synthesis, in order to explain the phænomena of distance in vision of all sorts; according to the best method of philosophising described by our great Philosopher in the following words.

The method of philosophising. Newt. Opt. Query 31. pag. 380.

233. "As in mathematicks, so in natural philosophy, the investigation of difficult things by the method of analysis, ought ever to precede the method of composition. This analysis consists in making experiments and observations, and in drawing general conclusions from them by induction, and admitting of no objections against the conclusions, but such as are taken from experiments, or other certain truths. For hypotheses are not to be regarded in experimental philosophy. And although the arguing from experiments and observations by induction be no demonstration of general conclusions; yet it is the best way of arguing which the nature of things admits of, and may be looked upon as so much the stronger, by how much the induction is more general. And if no exception occurs from phænomena, the conclusion may be pronounced generally. But if at any time afterwards any exception shall occur from experiments, it may then begin to be pronounced with such exceptions as occur. By this way of analysis we may proceed from compounds to ingredients, and from motions to the forces producing them; and in general, from effects to their causes, and from particular causes to more general ones, till the argument end in the most general. This is the method of analysis: and the synthesis consists in assuming the causes discovered and established, as principles, and by them explaining the phænomena proceeding from them, and proving the explanations.' So far Sir *Isaac Newton*.

Exceptions when proper to be taken notice of.

234. In the 139th article and those that follow, I proceeded synthetically upon the principle of apparent magnitude; thereby to determine the apparent distances of objects seen by reflections and refractions in all cases. And in the following remarks I intend to take notice of such exceptions to the general determination, as have hitherto occurred to me; and also to shew the reason why they must be exceptions. But at present I will proceed a little farther with the extent of this principle.

Maxims in painting and perspective.

235. To shew that objects do not always appear to the naked eye in the real places from which the rays diverge, but frequently in other places, suggested to the imagination by their apparent magnitudes, I instanced in perspective and painting [a]: in which these maxims are universally followed. First, to diminish the dimensions of the figures of given objects in proportion as the objects themselves are remoter from the eye; so that the magnitudes of the figures may always discover the distances of the objects. And secondly, to make the contours that bound the figures more faint or more sensible according as the objects are more or less removed from the eye; and lastly to leave out the minuter parts of small figures, especially about their contours, and to sketch out the rest more slightly and indeterminately, in proportion as the objects are more remote. Because while an object recedes from the eye, the apparent magnitude of the whole and of its several parts are continually decreasing; and the contour that bounds it, being but a physical line, soon appears faint and confounded with the colours of contiguous objects, and then disappears, and next to that, the minuter exteriour parts; till at last nothing remains but the appearance of the grosser parts of a confused, indeterminate figure. And what is all this but chiefly the decrease of apparent magnitude of the whole and of its parts?

[a] Art. 138.

Description of a perspective draught. Fig. 46.

236. But to be somewhat more particular, let the figure *ABCD* contain a perspective draught of the inside of a long gallery, seen from one end of it, when the axis of the eye is directed in a line parallel to the length of the room. And let this axis cut the plane of the draught in *o*; and by the known rules of perspective [b], the representations of all lines, which in the room are parallel to the axis of vision, or in this instance to the length of the room, must be drawn converging towards the said point *o*; thence called the center of the draught. Such are the lines *Aa*, *Bb*, *Cc*, *Dd*, which represent the four intersections of the two sides of the room with the floor and cieling. It is manifest then, that the representations of all equal lines, which in the room are perpendicular to these parallels, and are intercepted between them, or between any other parallels to them, are continually diminished as they fall nearer and nearer to the center *o*. Which is the same thing as to say, that all the linear dimensions of the representations of any row of equal objects, ranged in the room upon any line parallel to the axis of vision, either on the floor below it, or in the cieling above it, or on either side of it, are continually diminished as they fall nearer to the center *o*, and consequently as they belong to remoter objects. See Art. 156. FIG. 266.

[b] See Art. 156.

The effects of it upon the eye.

237. These are the proportions of the parts or figures of the draught; and in looking upon it, we find, that the diminished parts, placed in all positions round about the center *o*, have a like effect upon the mind in suggesting distance from the eye. So that this effect is not owing to any particular position of the smaller parts, placed

ced above or below or on either side of the center *o*, but to their magnitudes only. Which is still more evident in the perspectives of small rooms, churches and all short prospects; in which all the larger parts and figures, though drawn almost with equal strength, do yet suggest unequal distances by their unequal magnitudes. And it is acknowledged by all the Artists, that I have consulted, that the bare out-lines of a draught justly executed, will sufficiently discover the distances of the original objects, especially when viewed with one eye placed in the point of view. Because the bare shapes of figures are frequently sufficient to discover what objects are represented by them. And herein the imagination is helped by looking through a short tube, or through ones hand coiled round like a tube, to hide the appearance of the collateral parts of the paper, not put into perspective.

The lights and shadows of a draught.

238. As to the disposition of the lights in a draught, they are frequently made as strong in and about its center as any where else; and sometimes stronger. Because a side light, thrown equally upon the original objects, is reckoned the best; and because a strong light is the more necessary for distinguishing the smaller figures near the center of the draughts. The shadows indeed of remote objects are generally omitted, being originally too weak to affect the eye.

Aereal perspective what.

239. These are the principal maxims and effects of Linear perspective; which in painting is improved by Aereal perspective, or the art of imitating the apparent degradation of the colours of natural bodies, in proportion as the bodies are gradually removed from the eye. This degradation is partly caused by the transmission of the rays of some colours through the air and vapours, in greater plenty than the rays of other colours; but chiefly by their mixture with the rays of an azure colour, reflected from very long quantities of intermediate air and vapours.

Its effects give place to those of linear perspectives.

240. The aerial perspective of a remote country is entirely preserved when we view it through a concave lens. And here the linear perspective, from which the rays diverge upon the eye, is accurately proportioned in the image of the country, formed by the lens upon a plane passing through its principal focus at right angles to its axis. Now when the eye is applyed close to the lens, this imaginary perspective, or indeed the similar perspective upon the retina, excites the same idea of the country as is usually excited by an equal [a] perspective of it formed upon the retina of the naked eye. But when the lens is fixt and the eye is drawn back from it, and consequently from the imaginary perspective at its focus, the similar perspective upon the retina is now diminished[b], and excites an idea of a greater distance of the country, though the aereal perspective of colours continues the same as before. But in looking through plane glasses very lightly tinged with the smoke of a candle or with any faint and transparent colours, I do not remember that either the apparent magnitudes or the apparent distances of remote objects were sensibly altered; though the natural degradation of their own colours was quite disturbed by the tincture of the colour of the glass. By putting these things together, it seems evident to me, that the linear perspective of a painted landskip has a more powerful effect in suggesting distances, than the aereal perspective of colours.

a Art. 117.

b Art. 106.

Objection answered.

241. To this it may possibly be objected, that certain large and remote objects, as mountains or cities, seen very clearly through an air more pure than usual, appear sensibly nearer than they generally do when the air is grosser[c]. I have been assured of two or three instances of this kind by very good judges in these matters; particularly by Dr. *Berkeley* himself; who has told me, that when he travelled in *Italy* and *Sicily*, where the air is generally much clearer than here in *England*, he was frequently surprised with appearances of this sort: viz. that a distant city, or the like object, that appeared very lively and strong, appeared also much nearer to him, than he knew it to be, by more certain ways of estimating its distance. But in my opinion these phænomena are not at all disagreeable to the usual relation between apparent magnitude and distance. For our idea of smaller or greater magnitude is not a simple idea of a smaller or greater uniform surface; but includes also an idea of a smaller or greater number and variety of distinct parts of the known object, not imagined, but actually perceived by the eye; together with various degrees of their evidency. Now since the minute parts of a known object, which are usually obscured by a grosser air, become more evidently revealed in a purer; the object ought to appear somewhat nearer through the purer than through the common grosser air: and almost at the same nearness, at which it would appear, if the spectator approached so far towards it, till he could see the same number of minute parts as clearly in a grosser air, as he saw them in the purer from a greater distance: I say almost at the same nearness; because the greater apparent magnitude of the whole object, seen from the nearer station, if it be known and familiar to us, will also contribute to lessen its apparent distance: not to mention the view of a smaller extent of the country interposed.

c See *Hook's* Exper. and Obser. pub. by *Derham* 8o. p. 143.

Idea of magnitude not simple but compound.

The answer confirmed by telescopes.

242. Now though this explanation does not totally exclude all concurring suggestions of nearness by the aereal perspective abovementioned, (as I do not intend it should,) yet I think it is farther confirmed by its close agreement with the principal, if not the only, cause that suggests

suggests the nearness of objects seen through telescopes, (especially such as are adapted for observations by day-light[a];) which cause does not consist so entirely in the greater apparent magnitude of the whole object, (which frequently cannot be wholly seen at one view, through a fixt telescope,) as it consists also in a clear and distinct perception of a much greater number and variety of minuter parts, than can possibly be perceived by the naked eye.

a Art. 356.

Why objects appear smaller through telescopes than we are apt to imagine.

243. Hence (by the bye) it comes to pass that persons unexperienced in looking through telescopes, are often disappointed in their expectations of their performance. For example, when the face of a man, sitting at 100 yards distance, is proposed to be viewed through a telescope that magnifies 100 times in diameter; they expect to see a gigantick sort of a face at least as broad as the full moon: (which I am satisfied was Frier *Bacon's* notion of this matter, and shews that he never looked through a telescope[b]:) and indeed it is natural enough, considering they are apt to think, that the object will appear in the same place and at the same distance, as it does to the naked eye; and consequently like a surface 100 times broader than that of the real face. But upon tryal it appears about 100 times nearer; and then this nearness of appearance becomes the only cause of their admiration. For since the face appears in the telescope of the usual size, and with the usual distinctness and variety of features and minuter parts, as it would do to the naked eye about 100 times nearer to it[c], they are not surprized with any uncommon circumstance in this appearance, excepting that of the nearness it self. Another thing is, that in looking at the face with the naked eye at 100 yards distance, we are apt to conceive it larger, in proportion to the distance, than if it was nearer. For the general air of a known person suggests to our memory a general notion of several features, which upon examination are not perceived by the eye: as we are plainly sensible in looking at strangers or any unknown objects, which require a more particular perception of their minuter parts. So that this prænotion contributes also to frustrate the observer's expectation of the performance of the telescope, though very compleat and admirable in discovering the minuter parts of things. And this is the just and true sense of the definition of apparent magnitude in Art. 98 and 104.

b Rem. 111.

c Art. 143.

An observation upon the relievo in painting.

244. *Leonardo da Vinci* has made a curious observation not improper to be mentioned in this place[d]: that a painting, though conducted with the greatest art, and finished to the last perfection, both with regard to its contours, its lights, its shadows and its colours, can never shew a relievo equal to that of the natural objects; unless these be viewed at a distance and with a single eye. Which he thus demonstrates. If an object *C* be viewed by a single eye at *A*, all objects in the space behind it, included as it were in a shadow *ECF* cast by a candle at *A*, are invisible to the eye at *A*; but when the other eye at *B* is opened, part of these objects become visible to it; those only being hid from both eyes that are included as it were in the double shadow *CD*, cast by two lights at *A* and *B* and terminated in *D*; the angular space *EDG* beyond *D* being always visible to both eyes. And the hidden space *CD* is so much the shorter, as the object *C* is smaller and nearer to the eyes. Thus he observes that the object *C* seen with both eyes becomes as it were transparent, according to the usual definition of a transparent thing; namely, that which hides nothing beyond it. But this cannot happen when an object, whose breadth is bigger than that of the pupil, is viewed by a single eye. The truth of this observation is therefore evident; because a painted figure intercepts all the space behind its apparent place; so as to preclude the eyes from the sight of every part of the imaginary ground behind it.

d Treatise of painting p. 178. *Lond.* 1721.

Fig. 47.

Some advantages in vision with both eyes.

245. Hence we have one help to distinguish the place of a near object more accurately with both eyes than with one; in as much as we see it more detached from other objects beyond it; and more of its own surface, especially if it be roundish. And therefore supposing we judged of its distance by nothing else but by its apparent magnitude, our judgments must be somewhat different with one eye and with both; for this other reason also, that with both eyes we see objects much clearer and stronger as well as larger. And upon second thoughts[e] I will not dispute whether the feeling of the turn of our eyes, in directing their axes successively from a remoter object to a nearer, or on the contrary, may not also contribute to correct our judgment of its distance: certainly, as soon as this position of the eyes is obtained, our perception of the object becomes much clearer and stronger than before. For before that, by attending nicely we may always perceive two fainter appearances of the object approaching each other swiftly and transversely, till they jump together into one clear image. And therefore it is no wonder that persons upon losing one eye, are subject to mistakes in small distances, as in pouring out liquors from one vessel to another, or in snuffing a candle, or in the like actions. But by degrees they become more expert, as I am well informed by an experienced person; and the Hon. Mr. *Boyle* takes notice of the same thing[d]; and adds, that by an experiment purposely made, he found several times, that with his two eyes together he saw an object in another place, than he could do by either of them alone. Instances of this kind may be seen in Art. 137. But when the objects are pretty large

e See Art. 138. p. 51. line 25.

d 7th obs. upon vitiated sight.

large and pretty remote in comparison to the interval between the eyes; all these smaller helps become ineffectual, except that of the greater vivacity or strength of appearance.

In what case greater faintness suggests greater distance and how.

246. Lastly, when the most remote objects of unknown magnitudes are seen in the horizon one above another; as mountains above mountains, houses above houses, and the like; no doubt it is the greater blueness, or faintness, and the greater height together, which suggest the idea of a greater distance of the bluer and higher objects; because we are sensible of the same thing in painting, and because we have no other means of information in both these cases. Nor do I deny, that some degrees of faintness, still smaller than these, by obscuring the contours and some of the smaller parts of an object, placed at a less distance than that of the horizon, may cause it to appear somewhat smaller, and consequently almost as much remoter, as it would appear to the eye removed so much farther from it in a clearer air, till it shall appear of the same size and with the same number and variety of minute parts as it did before in the grosser air [a].

a Rem. 241.

The principal point established.

247. But, when we take a view of a scene or landskip of known objects, not so remote as to be sensibly obscured by the air, nor so near at hand or so small as to appear sensibly different to one eye or to both [b]; but situated at any moderate distances whatever, at which we usually perceive them and their distances, most distinctly, clearly, accurately and in the utmost perfection; what I chiefly contend for is this; that their apparent distances from the eye, as well as from one another, are principally, if not solely, suggested to us by their apparent magnitudes: which I think has been sufficiently proved.

b Rem. 245.

The conclusion of this dissertation.

248. To conclude, it is the business of opticians first to find out and establish the most general cause that suggests the distances of things, when seen in the utmost perfection; then, in order to account for the like appearances when the vision is less perfect, we must consider the extent and limitations of the regular operations of that general cause upon our minds; and also the impediments to its operations, such as the littleness and the faintness of objects too remote, the confusedness of objects too near, and the like. So that in order to the solution of any given phænomenon of distance, the effects of the general cause, if found inadequate to the given appearance, may be equated or corrected by the effects of the particular impediments, when proved to be such in the given case. Just as the place and distance of a planet must first be computed from the law of its mean motion; and then be corrected by proper equations that measure its irregularities. This is the only just method of proceeding in all physical enquiries; and therefore till the law of the principal cause, answering to this mean motion, be well adjusted, it will be in vain to apply the equations.

Upon Art. 142.

249. Pag. 53. line 2. *Fourthly, when a ray PO coming directly to the eye, makes an angle POQ equal to AOC or $\varpi O x$:* which problem may be solved as follows. In the axis of any convex lens whose center is E, take EG and EH severally equal to twice its given focal distance, and let the distance EQ, of the object from the lens, be bigger than EH; and taking OG to GE as EH to HQ, and placing it from the lens, the eye being placed at O will see the object Q, through the lens, in the same place as if the lens was taken quite away. Likewise, if the place of the eye be given, the place of the object may be had by the same proportion: which may be demonstrated from Art. 139, or from the first term of the series in Art. 247, by putting the value of the apparent distance $O\Pi = OP$.

Fig. 203. Fig. 48.

Upon Art. 146.

250. line 1. *Therefore in these two cases the object appears in the place of its image.* This will be farther evident by producing $P\varpi$ till it cuts the refracting plane CA at right angles in D. For supposing rays as PA to flow from P, since $D\varpi$ is to DP, in the given ratio of the sine of incidence to the sine of refraction [b]; it follows that while the focus P is moved along the object PQ, the corresponding focus ϖ will describe an image ϖx, parallel and equal to PQ [*].

Fig. 209. b Art. 223. * Art. 244.

251. In the other case of reflections from the plane CAD; since $D\varpi$ is equal to DP, all the rays that flow from P will diverge from ϖ after reflections [c]. And consequently while the focus P is moved along the object PQ, the focus ϖ will describe an image ϖx parallel and equal to PQ.

Fig. 210. c Art. 201.

252. I touched upon some other cases in Art. 142, in which a small object is equal to its image and appears in its place, when it touches any thin lens, or any reflecting or refracting surface; and also when it is placed in the center of a concave speculum; or in the center of a single spherical surface of a dense medium.

253. I think there is but one case more of this kind, in single bodies, viz. when the distance EQ of an object PQ, from the center of a convex lens or of a sphere or of a single convex surface of a dense medium, is equal to the sum of its two focal distances. For then the opposite distance Eq of its image, is also equal to that sum [d]; and consequently the object PQ is equal to its image pq [*]; and should appear in its place, by the general rule mentioned at the end of the present article; viz. that Ox, the apparent distance of the object, is to Oq, the distance of its image,

Fig. 49, 50. d Art. 236. * Art. 247.

image, as the object PQ or $\varpi\kappa$ is to its image pq: as is evident by the similar triangles, $O\varpi\kappa$, Opq.

254. By which it is very plain that the object cannot appear in the place of its image, (at least to a single eye,) but when they are equal to each other.

Upon Art. 148.

A fuller explanation of the apparent motions of a fixt object seen in a fixt glass. [a] Art. 139.

255. Since the phænomena of apparent motions in glasses, were deduced from the definition that I gave of apparent distance [a], by no other mediums than those of geometry; it may possibly be more satisfactory to explain them a little farther, by means of a closer comparison of them with the like phænomena to the naked eye. And hereby we shall better understand what insuperable difficulties and contradictions will arise, in attempting to explain them by the received principle of the divergency of rays from the place of the image.

Fig. 51, 52, 53. * Art. 55. & 245. [b] Art. 43.

256. Other things remaining as they were, join OP and through the center of the lens draw PE, cutting the visual ray OA (produced) in p; then the line pq, perpendicular to the axis, will be the image of the object PQ*; and when the eye is put close to the lens, the angles pOq, POQ will be very nearly equal to each other [b].

Fig. 51. [c] Art. 60. [d] Art. 106. 108.

257. *Case* 1. Hence while the eye is receding from the lens to any point O, if the distance Oq be less than OQ, the angle pOq will decrease quicker, or in a greater proportion, than the angle POQ does [c]; and consequently the apparent magnitude of the fixt object PQ, seen in the lens, will also decrease quicker than it would do to the naked eye if the lens was taken away [d]. But supposing at the same time, that the object PQ were moved from the naked eye into the successive parallel places $\varpi\kappa$, so quick as to subtend a decreasing visual angle $\varpi O\kappa$ constantly equal to the angle pOq, formed by the refracted rays; its apparent magnitude to the naked eye, would then become constantly equal to its apparent magnitude in the lens: and therefore it would appear to recede from the naked eye, with the same velocity, with which it will appear to recede in the lens, while it stands fixt in the place PQ.

Demonstration.

258. For the two pictures upon the retina being both distinct and constantly equal and similar to each other; and the sense being not at all affected by the refractions of the rays, but only by the magnitude of the picture thence resulting; the mind must needs make the same judgment upon the sensations of this decreasing picture, as it usually and constantly makes upon the like sensations of a like decreasing picture of an object moved forwards from the naked eye.

Fig. 52. [e] Art. 60. [f] Art. 106. 108.

259. *Case* 2. Now let the distance Oq be greater than OQ; and while the eye is receding from the lens to any point O; the angle pOq will decrease slower, or in a lesser proportion, than the angle POQ does [e]. And consequently the apparent magnitude of the object PQ, seen in the lens will also decrease slower than it would do to the naked eye, if the lens was taken away [f]. But supposing at the same time, that the object PQ were moved towards the naked eye into the successive parallel places $\varpi\kappa$, so quick as to subtend a decreasing visual angle $\varpi O\kappa$ constantly equal to the angle pOq, formed by the refracted rays; its apparent magnitude to the naked eye, would then become constantly equal to its apparent magnitude in the lens: and therefore it would appear to approach towards the naked eye with the same velocity, with which it will appear to approach in the lens, while it stands fixt in the place PQ*. * Rem. 258.

Dr. Barrow's difficult case demonstrated. Fig. 53. * Rem. 258.

260. *Case* 3. Lastly, if the fixt image pq be behind the eye, moving backwards from the lens to any point O; the angle AOE or pOq will now increase, while the angle POQ is decreasing: and of consequence the apparent magnitude of the object, seen in the lens, will increase, while its apparent magnitude to the naked eye would decrease, if the lens were taken away. But if the object be moved towards the naked eye into the successive parallel places $\varpi\kappa$, so quick as to subtend an increasing visual angle $\varpi O\kappa$ constantly equal to pOq or AOE, its apparent magnitude to the naked eye would become constantly equal to its apparent magnitude in the lens; and therefore it would appear to approach towards the naked eye with the same velocity, with which it will appear to approach in the lens while it stands fixt in the place PQ*: especially if the object be viewed through a pin-hole to procure distinct vision.

Fig. 54. * Art. 139.

261. When the eye is put close to a single refracting surface AC, let the refracted visual ray DCp cut a line $P\varpi$, drawn parallel to the axis, in r; and the object PQ will appear in the place of the perpendicular rs*. And while the eye is receding from C to any point O, the apparent distance of the fixt object PQ may be shewn, as above, to vary in the same proportion, as it would vary to the naked eye, if the equal object rs were moved from the place rs, with such a velocity as always to subtend the variable visual angle pOq or AOC.

Fig. 55. * Rem. 250.

262. Hence while the eye recedes from a refracting plane surface AC, the object PQ will appear immoveably fixt in the place of its image pq; because it coincides with the place and magnitude of the perpendicular rs*.

Demonstration.

263. For since we are constantly accustomed to certain known degrees of increase of the apparent magnitude of a fixt object, while we are approaching towards it; and to the like degrees of decrease while we are receding from it; it is necessary that the degrees of apparent mag-

nitude perceived in glasses, as well as by the naked eye, should vary quicker than these, in order to excite an idea of the departure or approach of an object.

Fig. 56, 57.

264. When the eye is put close to a reflecting surface AC, let the reflected visual ray DCp cut the line $P\varpi$ drawn parallel to the axis, in r; and the object PQ will appear in the place of the perpendicular rs*. And while the eye is receding from C to any point O, the variations of apparent distance may be explained as above; by conceiving the motion of the equal object rs, seen by the naked eye, to begin from the place rs.

* Art. 139.

265. In reflections at any single surface it might easily be shewn that the distance Cs is equal to CQ; and in refractions at a single surface, that Cs is to CQ in the ratio of the sines that measure the refractions [a].

a Rem. 250.

Fig. 58.

266. Hence also while the eye is receding from a plane looking-glass, the object PQ will appear immoveably fixt in the place of its image pq, because it coincides with the place and magnitude of the perpendicular rs; and for the reason just mentioned in Rem. 263.

The phænomena agree with this explanation.

267. These and many more conclusions of the like sort, deduced from the definition of apparent distance in glasses [b], I find are agreeable to the phænomena, as well when the image is behind the eye as when it is before it. But the image is here introduced, not as a medium that affects the sense, but the understanding only; in order to determine, by geometry, the variations of the visual angle subtended by it. When the image is behind the eye the phænomena of distance are allowed, by Dr. *Barrow*, Mr. *James Gregory** and the best writers, to be inexplicable, upon the received principle of the divergency of rays from it. But still they contend that the object appears in the place of its image when before the eye; which indeed, by a coincidence of the effects of different causes, is agreeable to reason and to the phænomena in two or three obvious cases abovementioned [c], but in general it is equally opposite to both, as I shewed above [d], and will now shew it again in a different manner.

b Art. 139.

* Optica promota prop. 29.

c Rem. 250. &c.

d Rem. 213. &c.

But are inexplicable by the divergency of rays.

268. For, setting aside the sensations of different degrees of apparent magnitude, why does not the object appear always in the place of its image, when bigger or less than it self, as well as when equal? And upon moving the eye backwards or forwards, why does the object appear at rest in the latter case, and in motion in the former; sometimes backwards and sometimes forwards; with certain degrees of apparent velocity in certain cases? Allowing that the object appears in the place of its fixt image to the eye fixt in any one given place, it ought also to appear there to the eye in any other given place; that is, it ought to appear fixt in that place, while the eye is in motion from place to place. I think nothing can be plainer than this conclusion, but still we may consider it in this other light. Supposing a real object, equal and similar to the image pq of the object PQ, to be put into the place pq; it would certainly appear at rest to the naked eye approaching towards it or receding from it; and its picture upon the retina being constantly equal and similar to the picture of the object PQ seen in the glass; this object PQ ought likewise to appear at rest. Which being directly contrary to the general phænomena of vision in all sorts of glasses, might be sufficient to shew the unreasonableness of the received hypothesis; viz. that an object appears always in the place of its image even before the eye.

Fig. 51 to 57

Apparent lateral motions explained.

269. But besides this, there is another plain consequence from it, as opposite to experience as the former; to wit, whenever an object appears in the place of its fixt image, it ought to appear fixt, not only to the eye moving backwards or forwards, but sideways also. Nevertheless in most cases these apparent lateral motions are very great, though that of the eye be very small; and I find both by theory and experience that they are reducible to the following rules.

Fig. 52, 56.

270. When the fixt image q lyes between the eye at O and the apparent place x of the point Q; the apparent lateral motion of the point Q (like a real motion of the point x carried by the angular motion of the line Oqx about the fixt point q,) will tend the contrary way, from the axis of the glass, to the eye's lateral motion from it. Because the visual rays that fall successively upon the eye in different places, do all diverge from the fixt point q very nearly; and because we refer the appearance of the point Q to the place x beyond the image q.

Fig. 55, 58.

271. Consequently when the points q and x coincide, there can be no apparent lateral motion of the point Q; which agrees with common experience, of objects seen by reflection from a plane looking-glass, if it be wrought perfectly plane, or from stagnating water; and also of objects placed under water and seen by refractions thro' its suface; provided in this latter case, wherein the image is imperfect [a], that the lateral motion of the eye be not very great.

a Rem. 273.

Fig. 52, 57.

272. But when the image q lyes beyond the apparent place x, the apparent lateral motion of the object Q, agreeable to the real motion of the point x, will tend the same way from the axis as the eye's lateral motion does; as likewise, when the image q is infinitely remote or behind the eye. This is evident for the like reasons as in the first case, Rem. 270.

Fig. 53.

273. I have here supposed all the rays flowing from the point Q, to belong to the fixt point q, after reflections or refractions, either accu-

accurately or very nearly; because when the apertures of the glasses or the lateral motions of the eye are not too great, the aberrations of the outward rays from q will be found too small and therefore insufficient to account for these large quantities of apparent lateral motion; even upon Dr. *Barrow*'s hypothesis, that the point Q appears in the successive points of a portion of a caustick adjoining to q. See Art. 154.

Upon Art. 151.

Geometrical constructions of the apparent distance magnitude and position of an object seen in any glass.

274. Though the manner, in which I have explained this theory, is the easiest that I could think of for determining any particular quantity of apparent distance, and also for demonstrating the reason of its increase or decrease in general; yet a more accurate and comprehensive idea of it in all cases may be easier had by inspection of the figures, which I am going to describe

Glass and object fixt, while the eye is moved. Fig. 59 to 67.

275. *Case* 1. Let an object be fixt at any point Q of the axis QC of any given lens or speculum fixt at C, whose focal distance is CF; and let the image of the object Q be at the point q, to be found by Art. 207 or 236. To the axis CQ erect a perpendicular CK equal to CQ, and join Kq; and while the eye at O is moved along the axis CO, let it carry a line OD always perpendicular to the axis, and terminated at D by the fixt line Kq produced; I say, this perpendicular OD will be continually equal to the successive apparent distances of the fixt object Q from the moving eye at O: and when OD tends upwards, the object will appear upright, otherwise inverted.

276. The figures represent all the different positions of the line Kq to the axis of the glass, occasioned by the different positions of the fixt object Q and its image q with respect to the principal focus F.

Glass and eye fixt, while the object is moved. Fig. 68 to 76.

277. *Case* 2. Let the eye be fixt at any point O of the axis OC of any given lens or speculum fixt at C, whose focal distance is Cf; and let the image of the eye O be at the point o, to be found by Art. 207 or 236. To the axis CO erect a perpendicular $C\Omega$ equal to CO, and join Ωo; and while the object Q is moved along the axis QC, let it carry a line QD always perpendicular to the axis, and terminated at D by the fixt line Ωo produced; I say, this perpendicular QD will be continually equal to the successive apparent distances of the moving object Q from the eye fixt at O: and when QD tends upwards, the object will appear upright, otherwise inverted.

Eye and object fixt, while the lens is moved. Fig. 77, 78, 79.

278. *Case* 3. Let the eye be fixt at any point O, and the object at any other point Q of the axis OCQ of any given lens C, whose focal distance is the given line f. Upon the interval OQ, between the eye and object, build a square $QOK\Omega$; and let the side $K\Omega$, opposite to the interval QO, be an ordinate to the axis of a parabola $KD\Omega$, whose parameter belonging to its axis, is the given focal distance f; and whose legs tend towards OQ if the lens be concave, otherwise from it; and while the lens C is moved along its axis OQ, let it carry a line CD always perpendicular to the axis, and terminated at D by the first parabola $KD\Omega$; I say, this perpendicular OD will be continually equal to the successive apparent distances of the fixt object Q from the eye fixt at O: and when OD tends upwards, that is, towards $K\Omega$, the object will appear upright, otherwise inverted.

Limits of inversion determined. Fig. 80.

279. If the interval OQ between the eye and object, be bigger than four times the focal distance f of the convex lens; from each end of it take off Of and Qf severally equal to twice the focal distance f, leaving the part ff in the middle; and between this part ff and OQ, take a middle proportional GH, and place it also in the middle; I say, the parabola will pass through the points G, H; and therefore while the convex lens is passing over either of them, the apparent object will invert it self. See Art. 151.

Eye and object fixt, while the speculum is moved. Fig. 81, 82.

280. *Case* 4. Let the eye be fixt at any point O, and the object at any point Q of the axis QOC of any given speculum C, whose principal focus is F. Upon the interval OQ as a base, make a rectangular parallelogram $QOKL$, whose height OK is equal to the focal distance CF; and let the side KL, opposite to the interval QO, be an ordinate to the axis of a parabola DKL, whose parameter, to its axis, is the focal distance CF, and whose concavity shall contain the interval OQ; and while the speculum C is moved along its axis CF produced, I say, that a line drawn from F perpendicular to FC, and terminated at D by the fixt parabola DKL, will be continually equal to the successive apparent distances of the fixt object Q from the eye fixt at O: and if the parallelogram QK be placed upwards for a concave speculum and downwards for a convex one, the object will appear upright or inverted according as FD tends upwards or downwards.

281. And if the eye and object be conceived to change places, the apparent distance and position of the object, will continue the same as before.

Limits of inversion determined.

282. Bisect OQ in M, and a circle described upon the center M through the point K or L, will cut the axis in the points G and H through which the parabola will pass; and while the focus F is passing over G, the apparent object will invert it self.

Fig. 83, 84.

283. Hence when the interval OQ between the eye and object is contracted to nothing, that is, when the eye views it self in a moving speculum

lum, the lines MK, MG, MH become severally equal to the focal distance CF.

Speculum fixt, while the eye and object are moved. Fig. 85, 86.

284. *Case* 5. If the interval OQ between the eye and object be given, while both are in motion along the axis of any given speculum fixt at C; (as when one end of a stick is viewed in it while the other is kept close to the eye;) bisect the interval OQ in M, and the semidiameter CE, of the speculum, in F; and let $CELK$ be a rectangular parallelogram whose height CK is to MQ as MQ to CF; then let the given side KL, opposite to the semidiameter CE, be an ordinate to the axis of a parabola DKL, whose parameter to its axis, is CF, and whose concavity contains the semidiameter CE; I say, that while the stick OQ is moved along the axis CE produced, the perpendicular MD to the stick, always terminated at D by the fixt parabola, will be continually equal to the successive apparent distances of the object Q from the eye at O: and if the parallelogram $CKLE$ be placed upwards upon CE for a concave speculum, and downwards for a convex one, the object will appear upright or inverted according as MD tends upwards or downwards.

Limits of inversion determined. Fig. 87, 88.

285. Take FG and $FH = \pm\sqrt{CF^2 + MQ^2}$, and the parabola will cut the axis in the points G and H; and when M passes by G the apparent object will invert it self.

286. When the length of the stick OQ is contracted to nothing, that is, when the eye views it self in motion, the parabola will pass through C and E; and FV, the distance of its vertex from F, will be equal to its parameter FC or FE.

How the apparent magnitude varies in all these cases.

287. In all these Constructions the apparent magnitude of the object is always reciprocally as the moveable perpendicular that represents its apparent distance. The demonstrations of all the cases are very short, but having dwelt so long upon this subject, I will leave them to be deduced from Art. 139 or 247 or 389 by those that take pleasure in Geometrick Places; and will conclude with a few general cautions and directions for a farther examination of this theory by experiments.

General directions for examining this theory by experiments.

288. In every tryal the observer must take due notice, whether the positions of things about him, and about the glass, and about the object in view, allow room enough for his imagination to work in. For instance, standing at a distance from a person placed near the far end of a room, suppose that I view him through a concave glass. While I gradually withdraw the glass from touching my eye, he will appear smaller and smaller and to recede a little; but though he appears continually diminished, yet I cannot conceive him farther removed than to the end of the room, though I see part of it also diminished; because I see the rest of it more evidently with my naked eye by direct rays passing by the circumference of the glass. But if I view him in an open field, he will appear to recede continually farther and farther from the place in which I see him with my naked eye by the sides of the glass; provided I remove the glass in a line directed exactly towards him, or even a little above him, or to either side of him. But if the line of direction falls below him, he will appear in the glass between me and his apparent place seen by the naked eye; in which case, according to the usual order of apparent objects on the ground, I shall be apt to think him nearer to me than his true place is; because the view of the intermediate ground by my naked eye confines the extent of my imagination in the glass. In fig. 89, the man or object is PQ, the observer's eye is at O, the upper direction of the lens at A, is OAC; and the lower direction of the same lens at B is OBD, terminated by the ground CD at D on this side of the object PQ. Fig. 89.

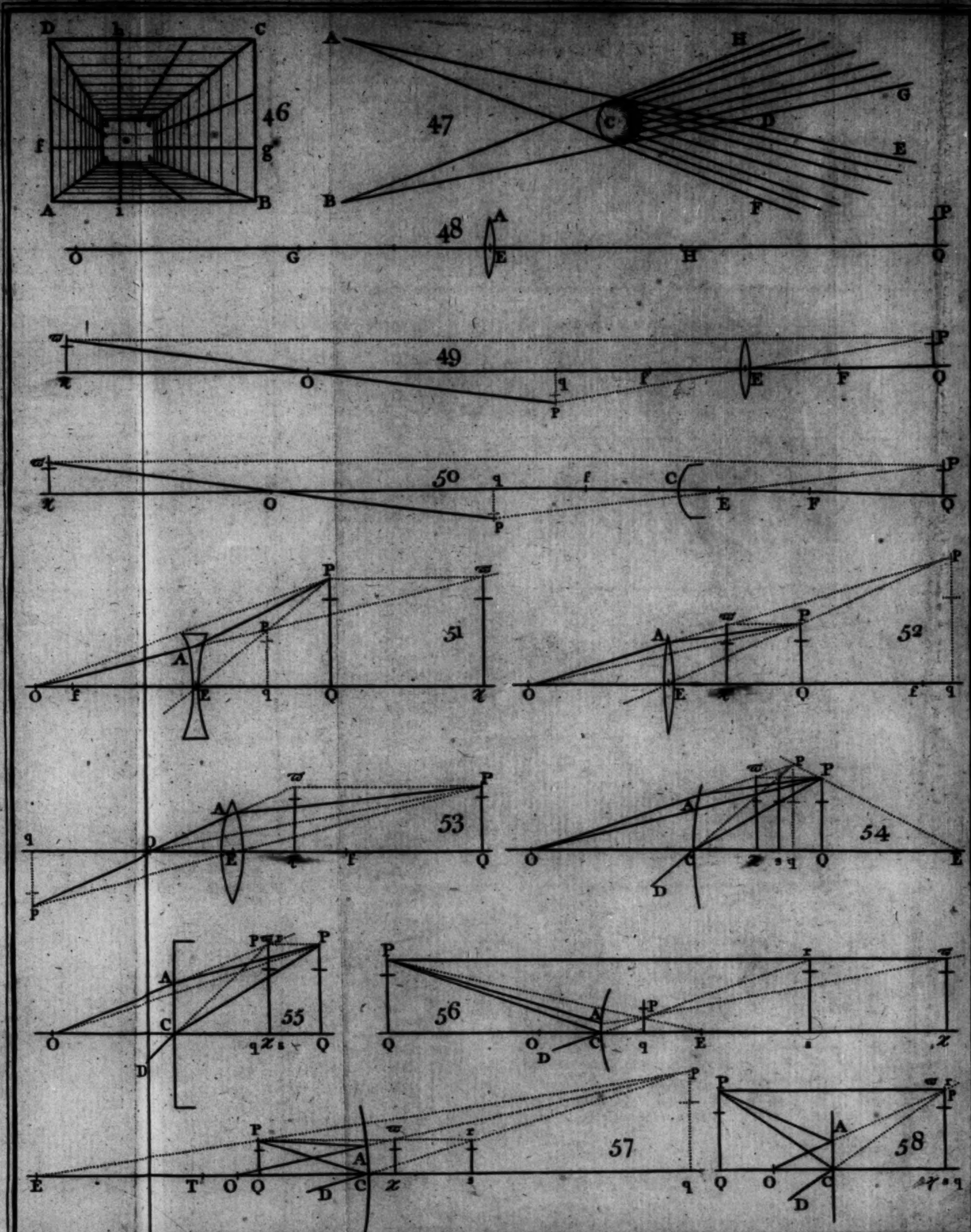
Apparent progressive motions stopt several ways.

289. In like manner if we conceive the line CD to represent the side of a long building, or of a street, instead of the ground; and the concave lens to be directed to any part of the houses nearer to the eye than the place of the object PQ, it will appear nearer than its true place does. Also if this figure be conceived to be inverted, so that CD may represent the cieling of a long gallery or of a cloister; the directing of the glass towards a nearer part of the cieling than that of the object upon it, will make it seem nearer than its true place. Which shews that the impediment to the imagination lyes equally in the nearness of the parts of those surfaces, that are seen by the naked eye; and not in their lower or higher situation above an horizontal line drawn through the eye. Which reason is farther confirmed, by fixing the same glass in a hole made in a broad thin board; or by looking into the wrong end of a small perspective, which will diminish objects, tho' it be close to the eye, and so it will intercept the direct view of all collateral objects from the naked eye, as well as the broad board; and then any diminished object will appear equally remote, whether the glass in the board or the perspective be directed exactly towards it, or below it, or above it, or towards either side of it.

The fields are the properest places for these experiments.

290 Another obstacle to the power of the imagination, not much different from the former, is the continual diminution of the quantity or number of objects, or of the parts of an object, seen through a concave lens, in drawing it farther and farther from the eye [a]; it being very difficult to conceive one part of a known object or system of objects to recede from the rest, that appear to the naked eye to be fixt in their places. The same may be said of the approach of objects seen

a Art. 113. 114.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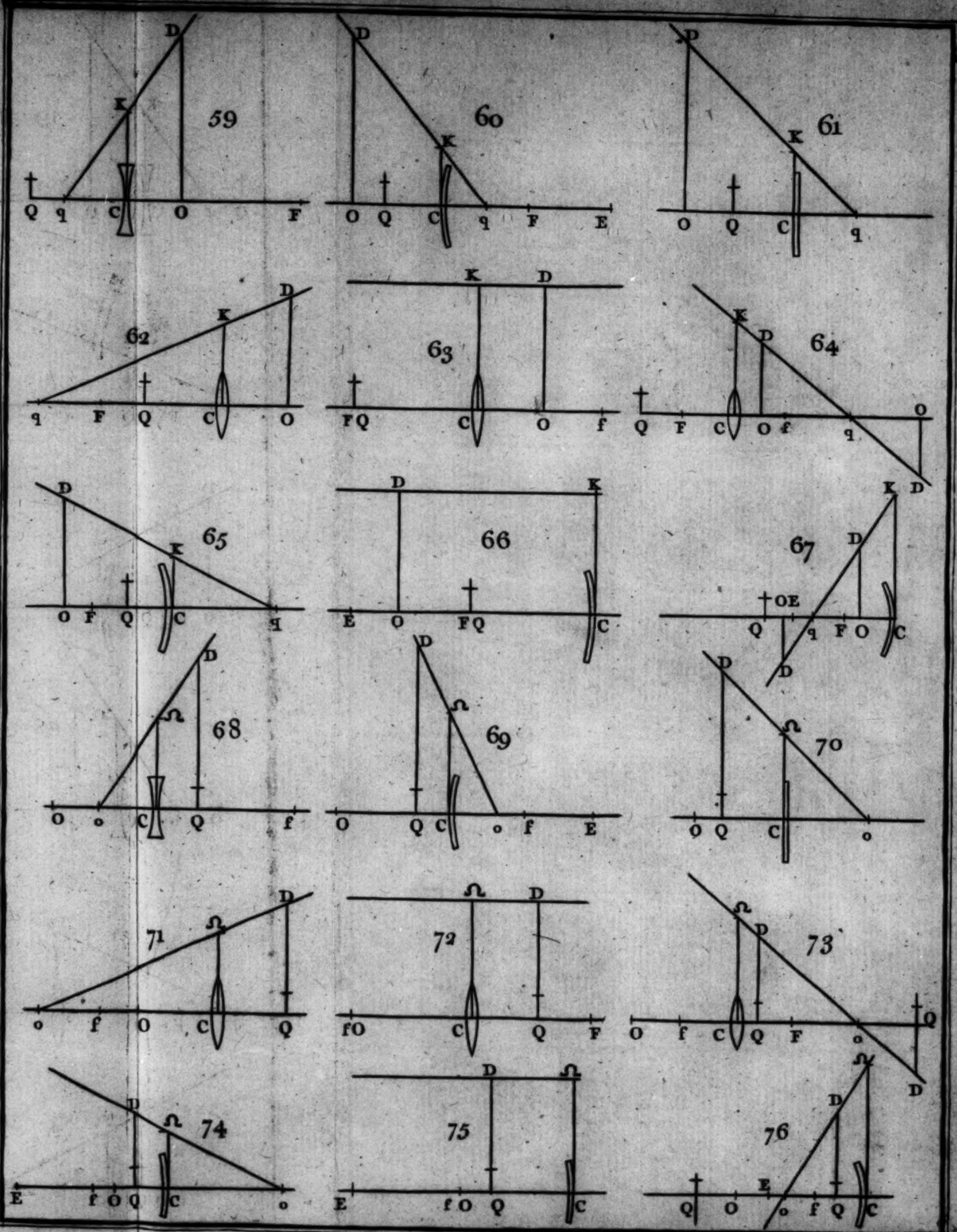
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seen through a convex lens drawn from the eye. And therefore upon several accounts, these experiments will succeed the best in the open fields, where the objects being remoter and subtending smaller angles at the eye, are more compleatly comprehended within the compass of the glass; where the imagination has more room to work in; and where the mind being less prepossessed with a knowledge of the distances and situations of objects than in a room, is more at liberty to perceive the difference between the ideas suggested by the glasses and by the naked eye.

Cases when an object should appear before a lens or speculum. Fig. 90, 91, 92.

291. I observed in Art. 144, that an object seen by refractions through any convex lens or by reflections from a concave speculum, ought to appear behind it, or at it, or before it, according as the real diameter of the object is bigger, equal to or less than the diameter of the part of the glass in which it appears. Hence it follows, that the object PQR placed at any distance CQ from the glass AC, will appear at the glass, when the eye is placed at its principal focus F *; and, that if the object be placed at a greater distance from the glass, than twice the focal distance CF or Cf, it will appear again at the glass when the eye is brought to another point O; to be found by bisecting the distance CQ in o, and by supposing the point o to be the focus of incident rays, and by finding their conjugate focus O after refractions or reflections. For then conceiving any ray OA to flow back from the eye at O, and, after its reflection or refraction at any point A and its passage through o, to fall upon the object PQR at R; the point R will be seen by a ray [a] returning back in the same lines RA, AO. And since oQ was made equal to OC, the leg QR of the right angled triangle oQR will be equal to the leg AC of the equal right angled triangle oCA; and therefore the part QR of the object PQR will appear in the same place by reflections or refractions, as if it were put into the place AC, and were viewed there by the naked eye at the same point O *.

* Art. 148.

a Art. 90.

* Art. 139.

292. Hence it is plain, if CQ be equal to $2Cf$, that the point o will coincide with the principal focus f; and so the conjugate focus O will be removed to an infinite distance; and if CQ be less than $2Cf$, and consequently Co be less than Cf, the point O will fall behind the glass, where the eye cannot come: and lastly if the object Q be very remote, the point O will come very near to F.

293. Now let q be the place of the image of the object PQR, to be found by Art. 236; then while the eye is receding from C to F the apparent object should follow it, till it touches the glass; and while the eye is receding from F to q, the apparent object should come through the glass, still following the eye till it touches it when they both come to q; and while the eye is receding farther back from q to O, the apparent object should return from q to C; and lastly while the eye is drawn farther back from O, the apparent object should again pass through the glass and go forwards behind it [c].

c Art. 47 and 139.

294. This should be the progress and regress of the apparent object by the theory, while the eye is continually moving from the glass. But by experiments with convex lenses, I find that it comes forwards from behind the glass no farther than till it touches it, when the eye is at F; and while the eye is receding from F to O, that it sticks all the while at the glass, without coming through it, and begins to return as soon as the eye has passed over the point O.

295. But when the object is placed not far from the center of a large concave speculum, though it does not appear to come out before the glass, while the eye is moving from the focus f towards the image q, yet when the eye is got past the image, the object will often appear before the glass; especially when viewed with both eyes. Which shews that the vigorous force of the united sensations of the two pictures falling upon corresponding places of the retina, together perhaps with the sensation of feeling the mutual turnings of the axes of the eyes, are helps to our apprehension of distance. Because we have not these helps when the eye, being placed between f and q, sees the object either faintly though distinctly through a pin-hole, or else confusedly with only one eye, or double with both eyes and confusedly too. Fig. 92.

296. Whether one might not contrive the experiment so advantagiously with a very broad lens as to make the object appear before it I dare not pronounce. But some experiments are so opposite to common experience, as to be hardly reconciled to this part of the theory. For example, when I place my fingers behind a round-bellyed decanter full of water, either touching its back surface or at a small distance behind it, they will appear much larger than the fingers of my other hand that touch the fore surface, and therefore by theory the former fingers should appear nearer to me than the latter. But this would be repugnant to constant experience both of seeing and feeling; since the fingers behind the decanter do not appear alone but joined to my hand and arm, part of which I know and perceive by direct vision to be behind the globe. Now whether the prejudice of knowing the place of any other object to be behind the globe or the convex lens, be not an impediment to its appearance before it, I must leave to be considered.

Observations upon an object placed in the focus of a convex lens. Fig. 93, 94.

297. If the object PQR be placed at the principal focus f of any convex lens ACB, its apparent distance Ox, from the eye at any point O, should be continually equal to the given focal

cal distance of the lens. This has been shewn before, but for the sake of the following application, I will prove it again. Let E be the center of the lens, and since the angle AOB, or $\pi O \rho$, under which the object PR appears, is always constituted under a couple of successive rays OA, OB parallel to the axes PE, QE of their respective pencils; it is continually equal to the given angle PER under the said axes. And since the apparent object $\pi \kappa \rho$ is always equal to the real object PQR, it follows that the moveable triangle $\pi O \rho$ is continually equal to the fixt triangle PER; and consequently, that the apparent distance $O\kappa$ should be continually equal to the focal distance Ef or EF. This agrees perfectly with experiments while the eye is receding from the lens as far as its focus F, but no farther; that is, the object appears to follow the eye till it touches the back side of the lens, and then it sticks there, though the eye recedes never so far back from F.

When it appears to stick at the lens it seems to increase.

298. In this experiment I have observed farther; that while the eye is moving any where between the lens and its focus F, as the object appears to accompany it always at the same distance $O\kappa$, constantly equal to EQ; so it appears constantly of the same magnitude. But while the eye is receding from F still farther back, and the apparent object seems to stick at the glass, it seems also to increase in magnitude; till the least circular spot upon it, whose diameter we may suppose to be PR, will appear to fill the whole aperture, whose diameter is AB, when EO, the eye's distance from the lens, is to the focal distance EQ, as the said AB is to PR; as I have found by accurate measures: which shews that the object was placed exactly in the focus f, or that the rays OA, OB were respectively parallel to the axes EP, EQ, of their pencils.

A surprising circumstance.

299. But the most surprising circumstance of the experiment is this, *viz.* that though the object be a piece of printed or written paper whose letters are but just legible at the distance EQ, either through the lens, or with the naked eye; yet they may be read through the lens with the same ease from any distances how great soever. But if the letters be rather too small to be read distinctly when the eye is close to the lens, or any where between it and its focus F; though they seem afterwards to increase prodigiously while the eye is receding from F, yet the difficulty in reading them still continues the very same as before; notwithstanding that they do not only appear magnified but also very distinct, the rays in each pencil being parallel as was proved by the experiment.

300. Now though these appearances seem odd at first sight, yet they follow naturally from the apparent sticking of the object at the lens, and from the definition of apparent magnitude. For since the subtense AB of the given visual angle AOB increases as its distance OC does, and since we refer the apparent object to the place of the subtense AB, we must needs conceive its real magnitude to increase. But since the picture of the letters upon the retina continues invariably the same wherever the eye is placed, by reason of the given visual angle AOB, whose given parts are also invariable; it must needs suggest to the mind the like invariable perception of the least sensible parts of the letters, to the eye in all places.

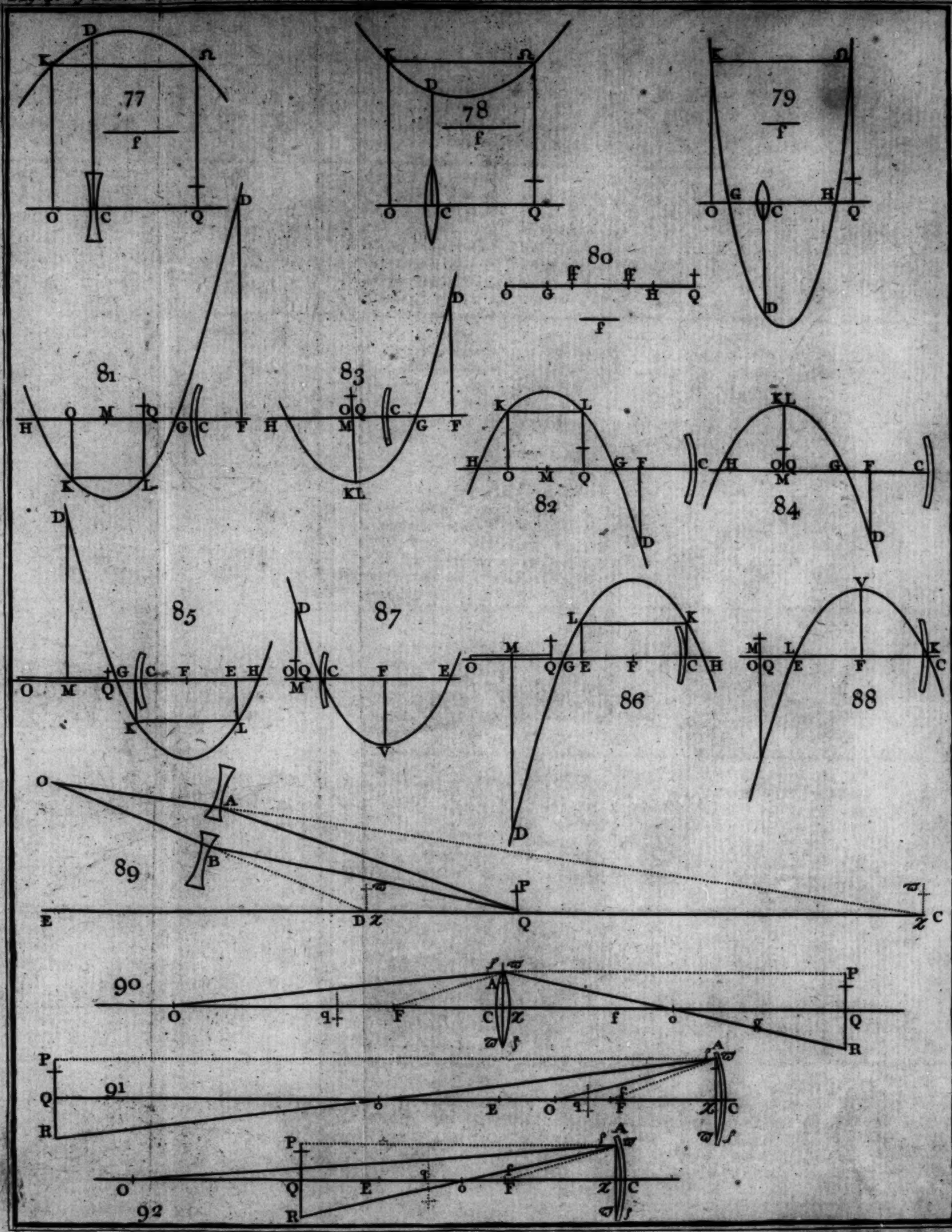
Hence two sorts of apparent magnitude.

301. It follows then from this experiment and from another that I am going to mention, that apparent magnitude should be distinguished into two sorts: the one, being proportionable to the magnitude of the visual angle or of the picture upon the retina, and consequently to the number and variety of the least sensible parts perceivable in an object, is unalterable either by the power of the imagination, or by any circumstances whatever that do not affect this picture. But the other, we have seen, is varied by the imagination in proportion to the distance at which we conceive the object to appear, whose picture upon the retina is invariable. For when the eye was at O and CO was double of CF, we imagined the real diameter of the object PQR, to be in the place AB, and to be double of what we thought it to be at the same place when the eye was at F.

A like appearance to that of the various magnitudes of the sun and moon. Fig. 93, 94.

302. I have dwelt the longer upon this experiment, in order thereby to illustrate and confirm the solution that I have given in Art. 164. &c. of that famous and perplexing phænomenon of the various apparent magnitudes of the sun, moon, and constellations, at various heights above the horizon. Let the object PQR, now intended to represent the moon, be a wafer stuck upon a card or white paper fixt upon a piece of a board; or it may be any round distinct spot, whose diameter is at least three or four times less than the diameter of the aperture of the lens AB. Then let this object and the lens be both fixt, upon a longer board MN, at an interval from each other exactly equal to the focal distance of the lens, to be found by Art. 63 or otherwise. And when the board MN is firmly fixt upon a level with the eye of the observer, let him recede for the space of four or five focal distances from the lens, till the wafer, seen just behind it, seems to fill its aperture or near it. Then let him imagine this apparent magnitude and distance of the wafer from his eye, to represent the apparent magnitude and distance of the horizontal moon; and while he is returning gradually towards the lens, the apparent magnitude of the wafer will seem to decrease in proportion to the decrease of its apparent distance from his eye; in like manner as the apparent magnitude of the moon seems also

Pag. 28 of Rem.ts Plate 7.



also to decrease in proportion to the decrease of her apparent distance, while she is ascending higher and higher, through several parts of the apparent concavity of the sky; which parts appear successively nearer and nearer to the eye while it is directed higher and higher above the horizon, as I have shewn in Art. 162, 163.

The two appearances compared.

303. Besides, as the moon either high or low, appears always at the same distance from us as those clouds or parts of the sky do that lye about her, so the wafer appears always at the same distance from the eye as the lens does with the frame about it, so long as the eye keeps farther from it than its principal focus. These two phænomena agree also in the only circumstance that creates the difficulty in solving the moon's appearances; namely, that each of the visual angles, under which the wafer and the moon do severally appear, continue invariable; not by means of any refractions of the moon's rays, in passing through that imaginary concave surface of the sky, answering to the refractions of the other rays in passing through the lens, but because the moon's real distance varies so little, during her ascent or descent, as to cause no sensible variation of apparent magnitude, in case her apparent distance did not vary. Nor can I find any one circumstance immediately perceiveable by sight, in which these two phænomena do not agree; in as much as the refractions of the rays at the lens do no more affect the eye, than if they had come straight through a plane glass from an increasing or decreasing wafer placed just behind it. But if any one should doubt whether the looking higher and higher at the moon, and constantly horizontal at the wafer, be a diversity of circumstances that can cause any difference in these appearances; he may easily remove this scruple by gradually elevating the
Fig. 94. far end of the board MN with the lens and wafer fixt to it, and by gradually moving his eye towards the lens at the same time. I must confess I have not tryed the experiment in this manner, not having the least suspicion that any diversity in the appearances can arise from this circumstance; but I find it succeeds very accurately with a broad 5 foot-object-glass of a telescope, while the eye is moving horizontally; and I think it will answer tolerably well with a glass of a shorter focal distance, or even with a common spectacle glass. It will also succeed well enough when the object is placed in the principal focus of a concave speculum, instead of a convex lens.

Other remarks on the theory of apparent distance. Fig. 76.

304. But to return to my remarks upon some other cases in this theory. When the interval beween my eye and a large concave speculum was given, and was greater than its radius; the apparent distance of any small bright object, first put close to the speculum, and then gradually drawn from it, seemed to decrease to nothing and then to increase, after the apparent inversion; according to the theory. But then I took care so to withdraw the object, that it always appeared to a single eye over against the same part of the speculum; otherwise when it was seen obliquely, and successively over against several parts, it seemed to move from me behind the speculum. Because the place of its image, from which the rays diverge upon the eye, at first recedes from behind the speculum; and thereby affects the eye by rays flowing obliquely and successively in different directions from different parts of the speculum. And when both eyes are open, one of them at least must be affected in this manner by oblique rays in different directions; and besides when the two images fall on corresponding places of the retina, the appearance must be stronger and remoter than to a single eye according to the experiment in Art. 977. The object here used was a small lighted candle fixt to the end of a long straight staff, which I drew back by the side of my eye, having sometimes a pin-hole before it.

305. Again, in viewing my own eye or face, in a large reflecting concave, through a pin-hole; Fig. 88.
while I recede from touching it to its principal focus, the apparent magnitude of my face decreased and its apparent distance increased; and the contrary happened in going still back from the focus to the center, according to the theory. But without the pin-hole, the decrease of apparent magnitude was scarce sensible in going from the glass to its focus.

306. As to the necessity of these and the like restrictions, I need only observe in general, that without the pin-hole our judgments of apparent magnitude are not so certain and determinate as with it, the appearances without it being frequently too much confused; and that our judgments of it are also more uncertain with both eyes than with one in many cases. Because in broad glasses we frequently see two appearances, which sometimes recede from, and sometimetimes accede towards each other, with transversely opposite motions; and when all things are fixt at rest and these appearances are but partially united, (as if they lapped over each other) we sometimes unwarily take them for a single appearance, much larger than it would be to a single eye; and sometimes the common middle part of both is mistaken for the whole, by not attending to the fainter and weaker extremities on each side of the middle; and then this middle part appears to be smaller than the whole would do to a single eye. Thus your face put near to a large concave speculum appears laterally larger to both eyes than to one; and being put near to a common looking-glass it appears laterally narrower. Also the wrist of your hand,

 held

held upright and close to your forehead, appears much thinner to both eyes than to one, or even to both eyes viewing it somewhat farther off. And the near approach of a broad image formed by a concave speculum may cause the like appearance. I mention these things as some general hints, by which the disagreement of the theory with experiments in a few particular cases may possibly be explained; and I leave them to be farther considered by those that have leisure and inclination.

307. Very remote objects, that appear contiguous and at equal distances from the naked eye, do generally appear contiguous and equidistant through glasses, though their real distances from the eye be never so unequal. The truth of this proposition is known to those that with a telescope have observed the appulses of the moon to the planets and fixt stars; and may be examined every day by viewing very remote mountains, towers or buildings extant one above another. For all objects so remote that their pictures are projected very nearly upon the same plane at the focus of a telescope, are equally magnified by it, that is, in the same given ratio [a]. And consequently whatever real distance is assumed to represent the same apparent distance of these objects from the naked eye, will be diminished by the telescope in the same given ratio in which the objects are equally magnified. For further illustration, let *KL* and *MN* be any two objects that appear to the naked eye contiguous to each other at one and the same apparent distance, represented by any given line *OQ*; and let certain parts *KL*, *MN* of these two objects appear under the two angles *KOL*, *MON* severally equal to any given angle *POQ*; and in the telescope let them appear under two other angles severally equal to any other given angle $\varpi O \varkappa$; then their common apparent distance in the telescope, suppose $O\varkappa$, will be to *OQ*, in the given ratio of the angle *POQ* to the angle $\varpi O\varkappa$, by Art. 141.

a Art. 120.

Fig. 95.

General conclusion.

308. Hence it is manifest that the apparent distance of an object seen in glasses, cannot be otherwise determined, than by assuming some certain measure of its apparent distance from the naked eye. But since we usually judge the apparent distances of known and familiar objects to be proportionable to their real distances; because we find they increase and decrease together to a degree of accuracy sufficient for common purposes; it follows, in these cases if we represent the apparent distance of an object from the naked eye by its real distance, that the apparent distance in the glasses will answer the measure of it determined by theory, as exactly as in vision with the naked eye. In other cases where we know there is a great disparity between the real and apparent distances of an object, we must assume a certain quantity of real distance for the measure of its apparent distance from the naked eye, and proceed with it in glasses as above: which is all the accuracy that can be expected.

Upon Art. 155.

Appearances through a drinking-glass full of liquor.

309. If this theory be examined, when the eye is placed within the caustick *vpx*, that is, between its cusp *p* and the center of the sphere or concave speculum; it will be necessary to apply a small pin-hole close to the eye, to procure distinct vision. And if it be examined by looking at a lighted candle or at any small, bright object, seen through a drinking-glass full of any clear liquor, the object will appear to the eye, at any place whatever, always upright properly speaking; and inverted only with respect to the right hand and left when the cusps *p*, *q* are before the eye. Because the refractions upwards, of the rays passing through the drinking-glass, follow the same laws as if they had passed through the edge of a prism held downwards [b]; (to which the conical figure of the glass may be compared;) but the refractions sideways follow the laws of refractions through a sphere; which last is manifest by the apparent contrary motions of the eye or the glass and of the object seen through it, though it appears upright.

b Art. 54.

Theory of apparent shapes postponed.

310. The chasm in the numeration of the figures between FIG. 253 and 266, was occasioned by reprinting this article and by changing the 19th plate for another. For at first I attempted in this place to explain the apparent shape and curvity of objects seen by reflections and refractions; but finding it could not be done with sufficient accuracy, without more geometry than what I proposed to use in the Popular Treatise, I chose to consider it more at large in the Remarks upon Art. 612. &c.

Upon Art. 160.

311. Since the printing of this article I was informed by the Honourable Mr. *North* that this apparent divergency of the rows of trees planted upon the rising ground at the end of his vista, is very manifest through a large perspective as well as to the naked eye.

Dr. *Jurin's* remarks upon fallacies in vision, &c.

312. "In shaving my self by a glass hung in a window, which looks into a garden, it has several times happened to me, that a fly has settled upon the window near my glass, and in creeping along has raised in me the idea of a large bird flying in the air, and sometimes that of a dog, or cat, running cross the garden, according as the situation of the fly, with regard to my eye, has projected its image either upon the air, or upon the ground. The reason of this

appearance

appearance I take to be this. My eyes were then fixt upon my own image reflected from the glass, and were so adapted, as to see that image distinctly in a place as much beyond the glass, as my face was then distant from it; and the fly being much nearer me than that image, and likewise being seen obliquely, must consequently appear very indistinct, as well as larger than it would have done, had my eyes been fixt upon it. And as I could not apprehend, that any creature answering to so large an image, could be so near me, immediately my fancy suggested to me that it was at a greater distance, and a much larger animal, and moving with a greater velocity. So true it is, that in the suddenest judgments we make, the idea of a greater distance raises in us the idea of an object of greater magnitude, and moving with a greater velocity; the apparent swiftness of its motion being magnified in the same proportion with the diameter of its bulk.

313. It has likewise happened to me, that upon using a sort of fence hung upon my head, to shade my eyes from the light of the candles in reading, a fly has settled upon the inside of that fence, within three or four inches of my eyes, and by creeping along the edge of it, has suddenly raised in me the appearance of a rat running along the floor. Which I account for in the same manner as in the other phænomenon; the book, on which my eyes were fixt, being considerably farther distant from them, than the edge of that fence; and consequently the picture of the fly upon the retina being larger and less distinct, and its apparent velocity greater than it would otherwise have been.

314. Children of three or four years of age, not having as yet learned how to allow for the lesser apparent magnitude of distant objects, take every thing seen at a distance to be less than it really is. If they see a man or a woman thirty or forty yards off, they take the one for a boy and the other for a girl. And though experience teaches them in a little time to correct this error in objects seen upon the level, or from a moderate height, yet if they happen to look upon them from any high building, they are apt to make the same mistakes as before.

315. Let a boy, who has never been upon any high building, go up to the top of the Monument, and look down into the street; the objects seen there, as men and horses, will appear so small as greatly to surprize him. But ten or twenty years after, if in the mean time he has used himself now and then to look down from that and other great heights, he will no longer find the same objects to appear so small. Because now, having had frequent experience of the lessening of the apparent magnitude by the increase of distance, he does by a quick and imperceptible judgment, conceive the objects to be of the same magnitude as if they were less remote: which judgment of the mind not being distinguished from the perception of the eye, he thinks he actually sees them larger than before. And if he were to view the same objects from such heights, as frequently as he sees them upon the same level with himself in the streets, I suppose, they would appear to him just of the same magnitude from the top of the Monument, as they do from a window one story high. For, in the street, a man at a hundred yards distance appears of the same height as another at ten, though the image of the former in the eye is but a tenth part of the length of the other; the smalness of the image being compensated by the knowledge of the greatness of the distance. And this allowance is instantaneously and imperceptibly made in such objects as are constantly and perpetually seen in any one situation.

316. But the famous Monsieur *Perrault* * seems to be greatly mistaken, when he applies this principle to the overthrowing the received opinion of all architects and sculptors since the time of *Phidias*; viz. that statues placed upon very high buildings ought to be made of a larger size, and to be heightened with bolder strokes than those which are to be seen at a nearer distance. He says the judgment of the eye is so perfect, and so accustomed to make the proper allowance for the distance of the object, that it appears of the same magnitude to us, whether at a greater, or a lesser distance. In things very familiar to us he is certainly right: but statues upon very high buildings are not so constantly and familiarly seen as men and horses in the streets; and therefore we cannot make so just an allowance for the distance in one case, as in the other. I believe very few spectators are able to make a near estimate of the length of the dragon upon *Bow* steeple, or of the height of the St. *Michael* at *Brussels*, except such as saw those figures before they were put up. Besides that all persons, except architects, are apt to take the height of a lofty building to be much less than it really is. Therefore I see no reason for that learned gentleman's ridiculing the celebrated story of the *Minerva* wrought by *Phidias*, which is said to have appeared exceedingly deformed, when seen near at hand in his work-house, but incomparably beautiful when placed at the height it was designed for. Something like this may be seen in pictures every day; many of which appear hard and course near hand, but perfectly soft and natural when viewed at a proper distance. The paintings in the cupola of St. *Paul's*, if they were to be seen from the distance of a few feet, would have a much less agreeable effect than when

* Ordonn. des 5 col. d'Architect. P. II. Ch. 7.

when they are viewed from the pavement.' So far Dr. *Jurin*.

Fallacy in the view of *Stilton* Church.

317. Travelling northwards in the great road to *Stilton* in *Huntingtonshire*, when I came within the distance of a mile or two from it, I observed by chance that the inns in the road and the church, on the left hand of them, appeared so far asunder, that I remember I asked my servant whether the church before us belonged to *Stilton* or to some other town, which he could not answer. We then judged them to be near a mile asunder. But soon after, as I came pretty near to the inns, happening again to look at the church, I was greatly surprised to find it so near them. This raised my curiosity to know the real distance between them; and upon pacing it over, I judged it to be not above 200 yards, or near an eighth part of a mile. About a month after, when I came back through the same village, I found the same deception in that apparent distance as strong upon me as before, notwithstanding that now I knew what it really was. I was therefore the more curious to observe the situation of the village and of the fields about it, in order to find out the cause of this fallacy.

318. The open field through which the road passes on the south side of the town, is almost level for a mile or two; and from this distance there appears to be a gentle rising of the fields on each side of the town for about half a mile or a mile; and the town appears in the horizon (nothing beyond it being visible) though somewhat lower and remoter than the parts of the horizon adjoining to each side of it. And this I take to be the cause of the deception. For when any object, or interval between two objects, is judged to be farther off, than according to our common conceptions, we judge it also to be larger than we should do if we conceived it to be nearer.

319. At the distance of half a mile from the north side of the town, the road begins to ascend northwards; and from the bottom of this ascent the said interval between the inns and the church appears also very large: the tops of the inns and of the steeple being seen in the horizon beyond a long ridge of corn-lands rising gently from the right hand of the road. And from this lower station the said interval appears much greater than from the top of the ascent above it, though but little farther from the town. Because from this higher station, those intermediate ridges do not intercept the view of the little field between them and the town, which field from the lower station was imagined too great; and also because the town does not now appear in the horizon; the fields beyond it being plainly visible.

320. I have since taken notice of the like deceptions in some other places; as of the interval between the church and the castle at *Scarborough*, seen from some parts of the road between it and *Hacknes*.

Upon Art. 162.

321. Since the printing of this explanation of the apparent concavity of the sky, I have had an opportunity or two of making an observation that seems to be no small confirmation of it. Standing upon the sea-shore under the cliffs, that were extended on both hands, pretty straight, as far as I could see, and looking at the horizon of the sea, it did not appear to be a semicircle having my eye for its center; but the distance of its parts seen on each hand by the sides of the cliffs, seemed to be the greatest of all; and the distances of the intermediate parts seemed to decrease gradually, while I carried my eye from the cliffs towards the middle of the horizontal arch. I also took notice of a certain distant ship, which appeared almost to bisect one half of the horizontal arch intercepted between the line of the cliffs on one hand, and the perpendicular to it on the other. And yet the angle under the line of the cliffs and a line drawn to the ship, appeared much smaller than its complement, contained under the said line drawn to the ship and the said perpendicular to the line of the cliffs: which agrees with the like observations upon the sky mentioned in the next article; and shews that our idea of distance is inlarged by a variety of objects. Figure of the semi-horizon at Sea seen from the shore.

In the 272d Figure the line of the cliffs is AOD, the perpendicular to it is OC; the station of the observer is O; the horizon of the sea is $ABCD$; the apparent place of the distant ship is B, which I judged to bisect the arch ABC in B pretty nearly, though the angle AOB appeared much smaller than the angle BOC.

322. At another time and place I made the like observation, standing upon the top of some very high cliffs; and then the margin of the sea seemed to rise up a little towards the clouds in the horizon, like the margin of a tea-saucer, especially where it appeared the faintest and darkest.

Upon Art. 163.

323. Upon communicating this determination of the figure of the sky with my highly esteemed friend *Martin Folkes* Esquire, he was pleased to approve of it; and added that he had frequently observed the sky to appear of a conchoïdal shape, as represented by the back line $ABCD$ in fig. 96. Which I find also is frequently very evident; but it is easy to understand, that neither the proportion of the apparent distances of its vertical and horizontal parts Mr. *Folkes*'s remarks upon the conchoïdal figure of the sky, &c. Fig. 96.

parts, nor the method of determining it, is ſenſibly altered thereby.

324. He alſo obſerved to me, that the apparent diſtance of the ſun or moon at any given place B, might be computed within certain limits, by conceiving a perpendicular BP to fall upon the ground OA, and by taking notice of ſome mark near P, where it is imagined to fall; (upon which I find that ſeveral perſons ſtanding together at O will agree in their judgments very nearly;) then having meaſured the diſtance OP and taken the ſun's altitude or the angle BOP, the ſide OB of the triangle OBP may be found ſeveral ways. The angle BOP may be had exactly enough for this purpoſe by erecting a ſtaff pb and by meaſuring the length of its ſhadow pO.

a Art. 163. 325. Hence from the figure of the concave ABC determined above[a], we have the apparent diſtance of the ſun at any other altitude, and alſo the apparent height of the vertical clouds or ſky at C.

Fig. 97. 326. He alſo informed me, that the parallel ſides of a very long and broad walk, avenue, road, or the like, do not appear to converge like two ſtraight lines tending to a very diſtant point; but rather like the legs of two hyperbola's abc, def converging towards each ſide of an aſſymptote op drawn from the eye at o parallel to the ſides of the walk, as repreſented in the figure.

327. He has alſo told me, when he rides very faſt by the ends of ſeveral long and ſtraight ploughed lands, and looks ſideways upon them; that they appear convex towards him, before he comes over againſt them; then ſtraight, and after that convex again towards his eye moved paſt them. I never had an opportunity of obſerving this phænomenon, but this I think is the deſcription that he gave of it. So far Mr. *Folkes*.

Upon ART. 164, &c.

Horizontal moons of an extraordinary bigneſs conſidered.

328. In theſe articles I haveſhewn in general why the moon appears always larger in the horizon than in the meridian, and have confirmed the reaſon of it by an experiment deſcribed in Remarks 297, &c. I ſay in general, becauſe it is agreed, that at different times the horizontal moon appears of different magnitudes even in the ſame horizon, and now and then of a ſize extraordinary large. This, I am inclined to believe, is chiefly owing to an extraordinary largeneſs of her picture on the retina at thoſe times; which picture in the preſent theory was ſuppoſed invariable. This might beſt be examined by taking its diameters with a micrometer; or, becauſe this inſtrument is ſeldom at hand, by regiſtring the year and day of the month, together with the heights of the barometer and thermometer. For if it ſhould appear by many ſuch obſervations, that the largeſt horizontal moons happen generally at her perigee in the warmeſt ſummer-evenings, the barometer being low and the thermometer high; and on the contrary, that the ſmalleſt horizontal moons happen generally at her apogee in the coldeſt winter-mornings, the barometer being high and the thermometer low; ſince theſe cauſes are independent on one another, and all conſpire to increaſe the moon's picture in the former caſe, and to diminiſh it in the latter, we might reaſonably conclude that thoſe extraordinary moons are chiefly owing to the concurrence of theſe three circumſtances.

329. In my remarks upon Art. 170, relating to aſtronomical refractions, I have ſhewn why the horizontal full moon ſeldom appears oval like the horizontal ſun, (notwithſtanding the ſame refractions of their rays) but generally of a roundiſh figure, to which the refracted oval is reduced by a defect of illumination at one end of it, for a day or two before or after the full, eſpecially in the evenings of the autumnal months. Conſequently in comparing together the areas of theſe roundiſh diſks of different full moons, we may ſuppoſe them equal to circles whoſe diameters are equal to the vertical diameters of the diſks: becauſe theſe laſt diameters are ſubject to ſmaller and more regular changes than any other, and about the full are as often bigger as they are leſs than the horizontal, (or rather the ecliptical diameter,) according as the horizontal moon happens at different diſtances from the exact full moon. Now ſince theſe apparent vertical diameters, and the pictures of them, are varied both by the moon's diſtance from the earth and by her horizontal refractions depending upon the air's denſity, that is upon its gravity and warmth together; I find by a courſe computation of their greateſt variations in our climate, that the ſaid apparent diameters of the greateſt and leaſt full moons are nearly in the ratio of 36 to 25, or almoſt as 3 to 2.

330. But it muſt be conſidered that we dont compare in our minds an extraordinary large moon with an extraordinary ſmall one, (the latter being ſeldom ſeen, and even then but little regarded, as being not different from every common moon ſomewhat elevated,) but with an eſtabliſhed idea acquired from the moſt frequent horizontal moons; whoſe mean diameter muſt therefore be aſſumed about a geometrical mean between the ſaid diameters of the greateſt and leaſt moons, rather than an arithmetical mean; becauſe we ſee more moons in cold weather than in hot, and therefore more of the ſmaller than of the larger ſize, by reaſon of a longer continuance of greater refractions which contract thoſe diameters. Therefore the diameter of the largeſt

largest moon, is to that of the commonest middle sized moon, in a subduplicate ratio of 36 to 25 abovementioned, that is (since 36, 30 and 25 are continual proportionals) in the ratio of 6 to 5; which is nearly the ratio of the diameters of a shilling and a sixpence or of a crown and a half-crown; by which we maybetter judge, how near this hypothesis will answer our idea of those extraordinary large moons. And this hypothesis I find is somewhat farther confirmed by the only observation of the kind that I ever took down in writing; viz. of an extraordinary large moon of the common colour like that of bright copper, which I saw on the 27th of July 1732, rising about 8 o' clock in the evening over the town of *Cambridge* from the distance of half a mile, the wind being west and the barometer somewhat above changeable. Now by the almanack it appears that the moon was near her perigee at that time, and the wind being west, in that month, is a sufficient argument for the warmth of the air and smalness of its refractive power. For I did not take notice of the thermometer, not thinking of this hypothesis at that time. Here then we have two of the three causes conspiring with the theory, and the third (viz. the barometer) nearly at a medium; but this I reckon has rather a less effect upon the moon's apparent magnitude than either of the other two.

The moon's apparent diameters at various heights are in a less ratio than that of her apparent distances.

331. Since the picture of the horizontal moon is constantly more or less contracted by refractions, and that of the meridional moon at her greatest height with us, of 60 or 65 degrees, scarce sensibly, it follows, that in these two positions, the diameters of her disks considered as portions of the concave sky, must be in a lesser ratio than that of their apparent distances in the sky. Because in the present theory in Art. 164, the moon's picture was supposed invariable during her ascent or descent. The ratio of those apparent distances by the table in Art. 164 is about 1 to 3 or 3 to 9; and this would be the ratio of the diameters of her circular disks in the concave but for refractions; by which alone I reckon the horizontal disk is generally contracted in the ratio of two circles whose diameters are nearly as 9 to 8. Therefore the highest meridional disk is generally to the horizontal one but as 3 to 8 in diameter. These disks are represented by the circles 3 and 8 below Fig. 174 in Plate 20, for the eye to judge of their proportion. In this comparison the defect of illumination from the full, is not considered, seeing it affects the meridional moon as much as the horizontal, taking one time with another.

Objection answered.

332. Since the horizon, seen from the top of a mountain or any elevated place, appears remoter than it does from below, it seems to follow, from the present theory, that the horizontal moon should also appear larger from above than from below; which I believe is not at all agreeable to experience and I think it ought not to be. For it must be considered that any present prospect of the moon and horizon together cannot so much suggest a new idea of a certain determinate magnitude of her disk (the power of estimation by the eye being too weak and inaccurate for this purpose) as it revives an old established idea of it; resulting from a long experience of innumerable prospects of various moons and horizons seen from various situations. Our idea of the magnitude of the horizontal moon is therefore about a medium taken from all our observations, and is so rooted in our minds by repeated views as not to be altered but only confirmed by any single one. The same may be said of our idea of the magnitude of the meridional moon; and of the moon at intermediate degrees of altitude. Which ideas, though gained originally from our ideas of the moon's distances in the sky, may yet become connected with and suggested by our ideas of her corresponding altitudes, For which reason the former ideas may continue much the same though the present view of the horizon be quite intercepted, by the interposition of any near objects, and even that of the ambient sky too, by the sides of an empty tube, when the moon is seen through it. For by memory we retain very distinct ideas of the moon's magnitudes for a longer time. Nevertheless in a general course of experience, if it happens very rarely that the magnitude of the picture of the horizontal moon is considerably altered by any means whatever[a]; this powerful cause will immediately alter our established idea of her usual magnitude, as any one will find by viewing the moon through any spectacle-glass, moved a little from his eye.

[a] Rem. 328.

Various solutions of the horizontal moon considered.

333. That the most ancient astronomical observations upon the positions and distances of stars were not made with instruments but barely by estimation, or judgment of the eye, is sufficiently evident not only from the nature of things, but from history too. And since the first astronomers were not aware that the intervals of stars seem much greater near the horizon than in the meridian, no doubt their astronomy was much embarrassed by so great a fallacy in their estimates. How long it was discovered before *Ptolemy*'s time I have not enquired; but that he was aware of it, is very plain by the caution he gives to allow something for it, upon every recourse to those ancient observations[b]. But whenever it was that the fallacy was discovered, this is certain that the cause of it has ever since been dubious and disputed; and therefore I have been the fuller upon it, to place it, if possible, beyond dispute. The variety of authors and opinions

[b] Almagest lib. 1. cap. 9.

nions upon it, is so very great, that to avoid all needless prolixity, I chuse to refer the curious to *Ricciolus*'s Almagest *, where many of them are mentioned and referred to; and also to a curious dissertation upon them written by the ingenious Mr. *Will. Molyneux* †, with an intent, as he tells us, "not so much to establish any thing of his own that may be satisfactory in solving this admirable appearance, as to detect the errors of those that have offered at a solution thereof, and yet have come short of being satisfactory: that thereby he might again set the minds of Philosophers to work, and rouse them up to enquire a-new after this surprising phænomenon.' Since the publication of these discourses, I do not remember to have met with any farther advances of a different kind, except in the ingenious *Essay*, abovementioned [a], *towards a new Theory of Vision*; in which there is the following solution. Sect. LXVIII.

* Tom. 2. lib. 10. sect. 6. Quæst. 13. pag. 643.

† Phil. Transf. N°. 187.

[a] Rem. 217.

Why the moon seems greater in the horizon than in the meridian.

334. "In order to explain the reason of the moon's appearing greater than ordinary in the horizon, it must be observed that the particles which compose our atmosphere, intercept the rays of light proceeding from any object to the eye; and by how much the greater is the portion of the atmosphere interjacent between the object and the eye, by so much the more are the rays intercepted, and by consequence the appearance of the object rendered more faint: every object appearing more vigorous or more faint in proportion as it sendeth more or fewer rays into the eye. Now between the eye and the moon, when situated in the horizon, there lyes a far greater quantity of atmosphere, than there does when the moon is in the meridian. Whence it comes to pass that the appearance of the horizontal moon is fainter; and therefore by sect. LVI. it should be thought bigger in that situation than in the meridiân, or in any other elevation above the horizon.'

By what means tangible magnitude is perceived by sight.

335. In the LVIth section here referred to by the author, he tells us by what means the tangible magnitude of objects is perceived by sight. "First by the magnitude or extension of the visible object, which being immediately perceived by sight is connected with the other which is tangible and placed at a distance. Secondly, by the confusion or distinctness. And thirdly, by the vigorousness or faintness of the aforesaid visible appearance. *Ceteris paribus* by how much the greater or lesser the visible object is, by so much the greater or lesser do I conclude the tangible object to be. But be the idea perceived by sight never so large, yet if it be withal con fused, I judge the magnitude of the thing to be but small. If it be distinct and clear I judge it greater; and if it be faint I apprehend it to be yet greater.'

The foregoing solution examined.

336. After this the Author proceeds to confirm his solution of this phænomenon, by an examination and exclusion of some other causes and opinions *. But to avoid the entering into a particular consideration of his arguments, I chuse to shew from experience only, that these various degrees of the moon's faintness, make no sensible variations of her apparent magnitude: which I think will be evident by the following observations.

* Ibid. sect. 70. &c. item sect. 75. &c.

Is found contrary to experience.

337. First, that the moon appears much fainter in the day time than in the night, and therefore according to our author's principle, should appear larger by day than by night at the same height: which I could never perceive, though I have often viewed the moon for this purpose.

338. Secondly I observe that the horizontal moon, being much fainter than the horizontal sun, viewed by the naked eye, should in consequence of that principle (though the fact is otherwise) appear much larger than the horizontal sun: because these two bodies differ in no other apparent circumstance than this of faintness or brightness. For the angles at the eye subtended by their diameters, are generally as nearly equal to each other as the naked eye can distinguish; as is evident by astronomical tables of these angles and also by total eclipses of the sun: which the intermediate moon sometimes just covers entirely, and sometimes leaves a very small lucid ring of light uncovered; according to the different proportions of the distances of the sun aud moon from the eye. Again the horizontal sun, notwithstanding a very great obstruction of his rays by the atmosphere, is still less faint than the meridional moon; (the eye being generally offended by the brightness of the former, but not at all by that of the latter) and therefore the horizontal sun should appear smaller than the meridional moon, if the author's principle were true. But these consequences from it are manifestly contrary to experience.

339. Thirdly, I observe, that the moon, when totally eclipsed, appears much fainter than she does at the same elevation when not eclipsed; but does not appear larger than usual, as I am fully satisfied by the total eclipse of the moon on Nov. 20. 1732. For though I did not think of this comparison when I saw this eclipse, yet I was put in mind of it the very next morning (which was full as well) upon seeing the moon but a few degrees high just before her setting. She then appeared to me much larger than she did 8 or 9 hours before, when eclipsed very high and near the meridian; at which time she appeared of a much fainter and heavier colour, and more inclined to a dull red, than at the time of her setting. Besides, upon asking several persons that had seen her eclipsed over night, whether they took notice of any other unusual

circumstance of her appearance, besides that of her dusky colour, they told me, that they did not perceive any. Then I asked them directly as to her apparent magnitude; to which they replyed, that it seemed to them as usual, that is, not near so large as that of the horizontal moon. But had this greater faintness of the meridional moon eclipsed, than that of the horizontal moon uneclipsed, produced an idea of magnitude proportionable to it, or even no greater, than that of the usual horizontal moon; so remarkable an appearance could hardly have passed unobserved in this and in many other total eclipses that have happened at very great altitudes above the horizon: and yet I have never met with any memorandum of it in any observations of eclipses; though the dusky colour of the moon be frequently mentioned and described. Now the ordinary degrees of the moon's faintness are caused by the obstruction of the atmosphere to various quantities of refracted light after its reflection from the moon, as explained by our author[a]; and the degrees of this extraordinary faintness of the moon eclipsed, are caused partly by the absence of the sun's direct rays intercepted by the earth, and partly by the like obstruction of the atmosphere to other refracted rays both before and after their incidence upon the moon, as explained in the remarks on Art. 170; and therefore the unusual faintness differs from the usual one, in no other sensible circumstance but in degree only; and yet as we have seen, it occasions no difference in the degrees of apparent magnitude. This faintness therefore is not liable to those objections raised by our author against the like degrees of faintness of the sun and moon, produced by the interposition of smoked or coloured plane glasses[b]; which are well known to cause no sensible alterations of apparent magnitude.

a Rem. 334.

b Ibid. sect. 72.

340. Lastly, I observe that this hypothesis of faintness can scarce be thought sufficient by any means, to account for the like variety of the apparent magnitudes of the constellations; that is, of the intervals of the same fixt stars at various altitudes; which yet is allowed to be a phænomenon of the same kind and degree as that of the sun and moon, and consequently to depend upon the same cause.

This phænomenon of the moon not incapable of computation.

341. To conclude, "if this phænomenon of the horizontal moon be a clear instance of the insufficiency of lines and angles, for explaining the way wherein the mind perceives and estimates the magnitude of outward objects, as the author is pleased to assert[c], I would leave it to be considered, whether the said phænomenon be not as clear an instance of the insufficiency of faintness for the same purpose; the degrees of which may yet be defined and computed by lines and angles, to the same degrees of exactness in all cases, as the eye can distinguish them in the most simple and advantageous ones: which exactness would be quite sufficient either for establishing or overthrowing any general arguments and conclusions drawn from faintness; as might easily be shewn in the present phænomenon, if so much exactness were requisite, and if (for a foundation of a *calculus*) any certain observation or experiment could be produced to shew, that *cæteris paribus* any two given degrees of faintness did really and plainly suggest any two distinguishable and given degrees of apparent magnitude.

c Ibid. sect. 78.

342. But no more of this, because the author is pleased to allow some use of mathematical computation in opticks. Though this in general may, he thinks, be observed concerning it; namely "that it can *never* be very precise and exact, since the judgments we make of the magnitude of external things, do *often* depend on several circumstances, which are not proportionable to, or capable of being defined by lines and angles[d].' Upon this I would only observe, that had he been pleased to recollect, how many various kinds of quantities, (colours not excepted) which not long ago were judged incapable of any measures, have nevertheless been accurately reduced to the rules of mathematical computation, he might have had less reason to despair of the like reductions of quantities of many other kinds. But I ought to ask pardon for this digression, were it not incumbent upon philosophers to remove all difficulties and discouragements, that may lye against the pursuit of the most approved and successful, if not the only method of acquiring just and true apprehensions of the nature of things: which are all made *in number, weight and measure*.

d Ibid. sect. 78. Edit. 2.

343. In the second edition of this Essay and also in a vindication and explanation of it (called *the Visual Language*) very lately published, the author has made some additions to his solution of the said phænomenon; but seeing it still involves, and depends upon, the principle of faintness, I may leave the rest of it to the reader's consideration.

Upon Art. 167.

Oval figure of halo's confirmed.

344. Since the printing of this article I have had many opportunities, for some years together, of observing the shapes of halo's about the moon, which appeared to me always oval, more or less according as the moon was lower or higher, without any exception. I have never seen a halo about the sun since I had this notion of them; but the Rev. Dr. *Walker*, Fellow of our College, tells me that he had taken notice of these ovals for many years; and remembers one or two that he saw about the sun when elevated very high in summer time; and consequently when

when the oval figure was less discernible, according to this theory, than if the sun had been lower.

Upon Art. 170.

Upon the apparent divergence and convergence of the sun's beams. Fig. 98.

345. Among all the phænomena which serve to demonstrate, that very high and remote objects in the heavens do not appear to us in their real shapes, places and positions, but according to their perspective projections upon the apparent concavity of the sky, as above explained; there is none more apposite and convincing than that beautiful appearance of those broad diverging beams from the sun's apparent place among the clouds, which happen in a warm summer's evening to lye quietly and distinctly interspersed about that region of the sky. This application of this common phænomenon was first suggested to me by an odd appearance, somewhat like it, which I once saw upon *Lincoln*-Heath, and will here describe and account for.

Their apparent convergence. Fig. 99.

346. It was an apparent convergence of long, whitish beams towards a point diametrically opposite to the sun. For, as near as I could estimate, it was situated as much below the horizon as the sun was then elevated above the opposite part of it. This part is here represented by the line tDt, and the point below it in opposition to the sun is E; towards which all the beams vt, vt, &c. appeared to converge.

The real beams are nearly parallel.

347. Observing that the point of convergence was opposite to the sun, I began to suspect that this unusual phænomenon was but a case of the usual apparent divergence of the beams of the sun from his apparent place among the clouds, as represented in fig. 98. I say an apparent divergence; for though nothing is more common than for rays to diverge from a luminous body, yet the divergence of these beams in such large angles is not real but apparent. Because it is impossible for the direct rays of the sun to cross one another at any point of the apparent concavity of the sky, in a greater angle than about half a degree. For the diameter of the earth being so extreamly small, in comparison to the distance of the sun, as to subtend an angle at any point of his body of but 20 or 22 seconds at most[a]; and the diameter of our visible horizon being extreamly smaller than that of the earth; it is plain that all the rays which fall upon the horizon, from any given point of the sun, must be inclined to each other in the smallest angles imaginable; the greatest of them being as much smaller than that angle of 22 seconds, as the diameter of the visible horizon is smaller than that of the earth. All the rays that come to us from any given point of the sun may therefore be considered[a] as parallel to each other; as the rays eBg from the point e, or fBb from the opposite point f; and consequently the rays of these two pencils that come from opposite points of the sun's real diameter and cross each other in the sun's apparent place B among the clouds, can constitute no greater an angle with each other than about half a degree; this angle of their intersection eBf being the same as the sun would appear under to an eye placed among the clouds at B, or (which is much the same) to an eye at O upon the ground. Because the sun's real distance OS is inconceivably greater than his apparent distance OB. Therefore the rays of the sun as Bg, Bb do really diverge from his apparent place B in no greater angles gBb than about half a degree. Nevertheless they appear to diverge from the place B in all possible angles and even in opposite directions. Let us proceed then to an explanation of this apparent divergence, which is not self-evident by any means; though at first sight we are apt to think it is, by not distinguishing the vast difference between the true and apparent distances of the sun.

[a] Art. 875. Fig. 96. Fig. 98.

Their apparent divergence explained. Fig. 100.

348. What I am going to demonstrate is this. Supposing all the rays of the sun to fall accurately parallel to each other upon the visible horizon, as they do very nearly, yet in both cases they must appear to diverge in all possible angles. Let us imagine the heavens to be partly overcast with a spacious bed of broken clouds, v, v, v, &c. lying parallel to the plane of the visible horizon, here represented by the line AOD. And when the sun's rays fall upon these clouds in the parallel lines sv, sv, &c. let some of them pass through their intervals in the lines vt, vt, &c. and fall upon the plane of the horizon at the places t, t, &c. And since the rest of the incident rays sv, sv, are supposed to be intercepted from the place of the spectator at O by the cloud x, and from the intervals between the transmitted rays vt, vt, &c. by the clouds v, v, &c.; a small part of these latter rays vt, vt, when reflected every way from some certain kind of thin vapours floating in the air, may undoubtedly be sufficient to affect the eye with an appearance of lights and shades, in the form of bright beams in the places vt, vt, &c. and of dark ones in the intervals between them: just as the like beams of light and shade appear in a room by reflections of the sun's rays from a smoky or dusty air within it: the lights and shades being here occasioned by the transmission of the rays through some parts of the window, and by their interruption at other parts.

349. Now if the apparent concavity of this bed of clouds v, v, to the eye at O, be represented by the arch $ABCD$, and be cut in the point B by the line OBx drawn parallel to the beams tv; it will be evident by the rules of perspective, that these long beams will not ap-

pear in their real places, but upon the concave *ABCD* diverging every way from the place *B* *, where the sun himself appears, or the cloud *x* that covers his body, as represented separately in full view in fig. 98.

* Art. 156. and 170. pag. 69.

Their apparent convergence explained. Fig. 100.

350. And for the same reason, if the line *BO* be produced towards *E*, below the plane of the horizon *AOD*, and the eye be directed towards the region of the sky directly above *E*, the lower ends of the same real beams *vt*, *vt*, will now appear upon the part *DF* of this concave; and will seem to converge towards the point *E*, situated just as much below the horizon as the opposite point *B* is above it: which is separately represented in full view in fig. 99.

351. For if the beams *vt*, *vt* be supposed to be visible throughout their whole lengths, and the eye be directed in a plane perpendicular to them, here represented by the line *OF*; they and their intervals will appear broadest in and about this plane, because these parts of them are the nearest to the eye; and therefore their remoter parts and intervals will appear gradually narrower towards the opposite ends of the line *BE*. As a farther illustration of this matter, we may conceive the spectator at *O* to be situated upon the top of so large a descent *OHI* towards a remote valley *IK*, and the sun to be so very low, that the point *E*, opposite to him, may be seen above the horizon of this shady valley. In this case it is manifest that the spectator at *O* would now see these beams converging so far as to meet each other at the point *E* in the sky it self.

Why converging beams are less common than diverging ones,

352. I do not remember to have ever seen any phænomenon of this kind by moon-light; not so much as of beams diverging from her apparent place. Probably her light is too weak, after reflections from any kind of vapours, to cause a sensible appearance of lights and shades so as to form these beams. And in the unusual phænomenon I well remember, that the converging sun-beams towards the point below the horizon, were not quite so bright and strong as those usually are that diverge from him; and that the sky beyond them appeared very black, (several showers having passed that way) which certainly contributed to the evidence of this appearance. Hence it is probable that the thinness and weakness of the reflected rays from the vapours opposite to the sun, is the chief cause that this appearance is so very uncommon in comparison to that other of diverging beams. For as the region of the sky round about the sun, is always brighter than the opposite one; so the light of the diverging beams ought also to be brighter than that of the converging ones. For, though rays are reflected from rough unpolished bodies in all possible directions, yet it is a general observation that more of them are reflected forwards obliquely, than are reflected more directly backwards. Besides, in the present case, the incident rays upon the opposite region to the sun, are more diminished by continual reflections from a longer tract of the atmosphere, than the incident rays upon the region next the sun.

353. The common phænomenon of diverging beams, I think, is more frequent in summer than in winter, and also when the sun is lower than when higher up; probably because the lower vapours are denser and therefore more strongly reflective than the higher; and because the lower sky-light is not so bright as the upper, and because the air is generally quieter in the mornings and evenings than about noon-day; and lastly because many sorts of vapours are exhaled, in greater plenty in summer than in winter, from many kinds of volatile vegetables; which vapours when the air is cooled and condensed in the mornings and evenings, may become dense enough to reflect a sensible light.

354. I will conclude my remarks upon these optical appearances in the heavens with a popular explanation of the refractions of rays through the atmosphere, and of the principal phænomena thence resulting. Upon astronomical refractions.

355. *Albazen* the Arabian, who is reckoned to have lived about A. D. 1100, seems to have been more inquisitive into the nature of refractions than the more ancient writers; insomuch that having made experiments upon them at the common surface betwixt air and water, air and glass, water and glass or crystal, and being prepossessed with the ancient opinion of crystalline orbs in the regions above the atmosphere, he had the boldness to suspect a refraction there also. Which he tells us [a] may be proved by taking the distance of a star from the pole of the equator (with an Armillary sphere,) both when it is very low and when very high near the zenith; and affirms that the former polar distance will be found less than the latter by reason of the refraction of the rays. And this, *Fryar Bacon* tells us [b], was taken from *Ptolemy*'s 5th Book of Aspects; where he attributes it to a refraction of the rays at the common surface betwixt the air and æther. If this be so, his Book of Aspects (which I never could meet with) must have been written after his Almagest. For here he tells us [c], that *Hipparchus*, in order to determine the times of the equinoxes, by observing at what day and hour the sun's rays coincided exactly with the plane of his equatoreal *Armilla*, (or brass circle fixt in the plane of the equator,) was astonished to find that it happened twice in the same day at the interval of 5 or 6 hours, the sun being high at one

History of the discovery.

a Opt. lib. 7. prop. 15.

b Opus Majus p. 398. Lond. 1733.

c Lib. 3. cap. 2. See also *Riciol.* Almagest. Tom. I. p. 113.

one time and low at the other. He also tells us, that he himself had observed the same thing; and he attributes it to the inaccuracy or unsteadiness of the *Armilla*: so that he had then no suspicion that this effect was caused by a refraction of the sun's rays.

356. But however this may be, it is certain that *Alhazen* proceeds to deduce several properties of this kind of refraction; as that it increases the altitudes of all objects in the heavens; that it contracts their diameters and distances from each other; and that it causes the twinkling of stars. But still we do not find that he or his follower *Vitello*, knew any thing of its just quantity; which being but small could not be determined but by very accurate instruments; the want of which may be the reason that we hear little more of it till about the year 1500; at which time it was revived and cultivated by *Bernard Walther*, *Mœstlin* and others, but chiefly by the noble *Tycho Brahe*; who perceiving the importance of it to the perfection of Astronomy, could not rest till after incredible diligence in contriving instruments and observing with them, he had settled the quantity of these refractions at all altitudes to a tolerable degree of exactness.

357. But still he could not discover where and in what manner the rays were refracted, nor even *Kepler*, who wrote so much upon this subject[a]. *Tycho* thought the refraction was chiefly caused by dense vapours very near the earth's surface; and *Kepler* places it wholly at the top of the atmosphere, which he took to be uniformly dense; and thence he determines its altitude, from the observed quantity of refraction, to be but little more than that of the highest mountains. But the true constitution of the density of the atmosphere, deduced afterwards from the *Torricellian* experiment, afforded a juster idea of these refractions, especially after it appeared by a repetition of Mr. *Lowthorp*'s experiment that the air's refractive power is proportionable to its density.

a Paralipom. in Vitellionem

Refraction of a ray through the atmosphere explained. Fig. 101.

358. In order to give an idea of the course of a ray refracted through the atmosphere, let the plane of this figure be conceived to pass through it, and through *Y* the earth's center; about which let innumerable circles *Oo*, *Pp*, *Qq*, &c. be drawn at any small intervals from one another, continued from the bottom to the top of the atmosphere. Now though it be evident from the consideration of the air's gravity and elasticity, that its density decreases continually in ascending upwards, (the spring of every higher particle being less pressed upon by the weight of a shorter column of air incumbent upon it) yet at first we may conceive it to be interrupted in a manner; or, that between the two lowest circles *Oo*, *Pp*, the air is every where equally dense, but somewhat denser than the air next above it; and that this air, between the circles *Pp*, *Qq*, is also uniformly dense, but somewhat denser than the air next above it; and so on to the top of the atmosphere: whose expansion will be limited, where its density is so much diminished, and consequently its spring so much weakened, as to be just balanced by the weight of a surface of single particles.

359. Now let *O* be the eye of a spectator, *YOZ* a line drawn up to his zenith, *OPQRS* the course of a ray, which came from any star *S*, and which we may consider as going backwards from the eye to the star, because it will describe the very same course over again[b]. Then the ray *OP* in emerging at *P* out of a denser air into a rarer, will be refracted from the perpendicular *YP* produced*; and in emerging again at *Q* out of this last air into another still rarer, it will again be refracted from the next perpendicular *YQ* produced; and so on, till at last it emerges from the top of the atmosphere into empty space.

b Art. 11.

* Art. 12.

All stars appear elevated by refractions.

360. Let the last emergent ray *RS* be produced backwards, till it meets the vertical line *YOZ* in *V* and the visual ray *OP*, produced, in *X*; and it is manifest, from what has been said, that the star's apparent distance from the zenith, measured by the angle *XOZ**, is less than its true distance, measured by the angle *XVZ*; and therefore that *OXV*, the difference of these two angles[c], is the total effect of all the refractions; whereby the star appears so much higher than it would have done without them. Because what has been proved of these discontinued refractions at *P*, *Q*, *R*, through this fictitious atmosphere, will hold good in the true atmosphere, by conceiving the intervals of the circles *Oo*, *Pp*, *Qq* to be diminished and their number to be increased indefinitely, till the ray becomes continually refracted at every point; and thereby changes its course from a polygon to a continued curve, to which the lines *SX*, *OX* will then become tangents at the top and bottom of the atmosphere.

* Art. 101.

c Eucl. I. 32.

361. I have here supposed the star's true distance from the zenith to be measured by the angle *SVZ*, because a straight line *ON* drawn from the eye to the star, would make an insensible angle with the ray *SRV*, the distance of the star being exceeding great, and the curvity of the ray but very small, especially near the top of the atmosphere. If its curvity were circular, I reckon that the greatest distance *OV*, between the lines *ON*, *VS*, even then would scarce be three miles.

The total refraction of a ray decreases in ascending.

362 The total refraction of an horizontal ray is the greatest of all, and while the star ascends, the refraction is continually diminished, till it vanishes at the zenith. For supposing an ascend-

ascending star to be seen successively in the directions OP, Op, and the rays to return back in the same lines $OPQRS$, $Opqrs$ in which they came to the eye; then since the ray Op is less inclined than OP to the refracting circle pP, the refraction at p will be less than at P *; and therefore the next ray pq will also be less inclined than PQ to the next refracting circle qQ; and consequently the next refraction at q will be less than at Q; and so the sum of all the refractions at p, q, r, &c. will be less than the like sum of all those at P, Q, R, &c.; that is, the angle OXV will be continually diminished till it becomes nothing at the zenith. On the other hand, when the ray OP becomes horizontal, it will be the most refracted, as passing most obliquely through all the refracting circles.

* Art. 14. &c.

The refractions of stars, their true and apparent places and intervals, what.

363. The eye being the center of any spherical surface, to which the relative places of stars are referred; the point where it is cut by a straight line drawn from the eye to a star, is called *the star's true place*; and the point where it is cut by a tangent to the visual ray at its point of incidence upon the eye, is called *the star's apparent place*; and the arch of a great circle lying betwixt the true and the apparent place, or the angle NOX, (at the eye,) measured by it, or the equal angle OXV * under the tangents to the curvity of the visual ray at its two extremities, is called *the star's refraction*. Accordingly the arch of a great circle betwixt the true places of two stars, or the angle at the eye measured by it, is called *the true interval of the stars*; and lastly the arch of a great circle betwixt their apparent places, or the angle at the eye measured by it, is called *the apparent interval of the stars*. A great circle is well known to divide the spherical surface into two hemispheres, and the plane of it to pass through the eye at the center.

Fig. 101. * Rem. 361.

The intervals of all stars appear contracted by refractions.

364. The apparent interval between two stars situated on the same side of the zenith and in the same vertical circle or plane, is so much less than their true interval, as the refraction of the higher star is less than that of the lower. For if the refractions of the stars were equal, their apparent interval would continue equal to their true interval. Therefore as much as the higher star is less elevated by refraction than the lower, so much their apparent interval falls short of the true; that is, by the difference of the two refractions.

365. If the two stars lye on opposite sides of the zenith and in the same vertical circle their apparent interval will be less than the true by the sum of their refractions.

366. Secondly, the apparent interval between any two stars equally high, is always less than their true interval. For the stars being equally high, are equally elevated above their true places in two vertical circles that converge to the zenith; and therefore their true interval is somewhat contracted by the refractions; though but little, because near the horizon, where the refractions are the greatest [a], the two vertical circles are nearly parallel to each other; and near the zenith, where they converge quickest, the refractions are very small.

a Rem. 362.

367. And lastly, the apparent interval between two stars in any oblique position, is always less than their true interval. Let the true places of the stars be a, b, and their apparent places α, β in the two vertical circles $a\alpha z$, $b\beta z$; and since the refraction of the lower star, is greater than that of the higher, that is, $a\alpha$ greater than $b\beta$; the apparent interval $\alpha\beta$ will be less oblique to the two verticals than the true interval ab; and being higher too, will be shorter than ab upon both accounts.

Fig. 102.

Table of refractions of stars.

368. The principal use of a Table of refractions, is to deduce readily the true place of a star from its apparent place, by deducting the refraction belonging to its apparent altitude observed by an instrument. The following Table, made by Sir *Isaac Newton*, is taken from the Philosophical Transactions N°. 368, where Dr. *Halley*, the publisher, has given some useful remarks upon some allowances to be made in astronomical observations for the refractions of the air, without the trouble of trigonometrical calculations.

A Table

A Table of the Refractions of Stars adapted to their apparent altitudes.

Appar. Alt. deg.	m.	Refraction. m.	sec.	Appar. Alt. deg.	Refraction. m.	sec.	Appar Alt. deg.	Refraction. m.	sec.
0	0	33	45	16	3	4	46	0	52
0	15	30	24	17	2	53	47	0	50
0	30	27	35	18	2	43	48	0	48
0	45	25	11	19	2	34	49	0	47
1	0	23	7	20	2	26	50	0	45
1	15	21	20	21	2	18	51	0	44
1	30	19	46	22	2	11	52	0	42
1	45	18	22	23	2	5	53	0	40
2	0	17	8	24	1	59	54	0	39
2	30	15	2	25	1	54	55	0	38
3	0	13	20	26	1	49	56	0	36
3	30	11	57	27	1	44	57	0	35
4	0	10	48	28	1	40	58	0	34
4	30	9	50	29	1	36	59	0	32
5	0	9	2	30	1	32	60	0	31
5	30	8	21	31	1	28	61	0	30
6	0	7	45	32	1	25	62	0	28
6	30	7	14	33	1	22	63	0	27
7	0	6	47	34	1	19	64	0	26
7	30	6	22	35	1	16	65	0	25
8	0	6	0	36	1	13	66	0	24
8	30	5	40	37	1	11	67	0	23
9	0	5	22	38	1	8	68	0	22
9	30	5	6	39	1	6	69	0	21
10	0	4	52	40	1	4	70	0	20
11	0	4	27	41	1	2	71	0	19
12	0	4	5	42	1	0	72	0	18
13	0	3	47	43	0	58	73	0	17
14	0	3	31	44	0	56	74	0	16
15	0	3	17	45	0	54	75	0	15

369. As to the methods taken by astronomers for making a table of refractions, it is here sufficient to say in short, that they chiefly consist, first in finding the latitude of the observer's place by observations made so near the zenith, as not to be affected by a sensible refraction; secondly in taking the greatest altitude of a star passing so near the zenith as to suffer no sensible refraction, whereby its true distance from the pole is given; thirdly in observing as many other altitudes, as you please, of that same star, noting the sidereal times of each observation, reckoned from its transit over the meridian; and lastly, from these (or the like) *data*, in computing its true altitudes at those given times of observation; which, being severally deducted from the observed altitudes, will give the corresponding refractions. An observer living under the Equator might make a table of refractions with much less trouble by means of any star that goes over his zenith.

Inconstancy of these refractions.

370. I will only observe farther, that though an accurate astronomy cannot be obtained without a correct table of refractions, yet that no one table can serve precisely either for the same place, the same season or even the same time of the day[a], especially when the objects observed are low; it being well known that the air's density and refractive power are increased by cold and diminished by heat[b], not to mention the changeable mixture of vapours and exhalations with the air near the horizon. There is a famous observation of this kind made by some *Hollanders* that wintered in *Nova Zembla* in the year 1596, and were surprised to find, that after a continual night of three months, the sun-

[a] Art. 591.

[b] See Dr. *Jurin's* Notes on *Varenius's* Geography Chap. 9. prop. 21.

sun began to rise seventeen days sooner than according to computation, deduced from the altitude of the pole observed to be 76°: which cannot otherwise be accounted for, than by an extraordinary quantity of refraction of the sun's rays passing through the cold and dense air in that climate. *Kepler* computes that the sun was almost five degrees below the horizon when he first appeared[a]; and consequently the refraction of his rays was about nine times greater than it is with us. On the other hand, it has been observed that the horizontal refractions at places near the equator are near a third part less than at *Paris*. But in ascending equally their difference diminishes gradually, till it becomes insensible at the height of about 60 degrees. Reg. Sci. Acad. Histor. per *du Hamel*. p. 600.

a Paralipom. in Vitellio. p. 138.

Oval figures of the horizontal sun and moon.

371. Since the apparent vertical diameters of the horizontal sun and moon (by reason of the unequal refractions of the highest and lowest rays) are much more contracted than their horizontal ones, their pictures upon the retina and consequently their apparent figures become oval; their longest and shortest apparent diameters being frequently as 5 to 4*, especially in the mornings when the rays are most refracted through a colder, denser, and moister air. The circle *adce*, whose center is *b*, represents the sun's true disk projected upon the spherical surface abovementioned[b], and the oval $\alpha\delta\gamma\epsilon$ its apparent one. Now supposing $b\beta$, the refraction or elevation of its center, to be equal to its true diameter *ac*, the elevation $a\alpha$ of its lowest point will be greater, and $c\gamma$ that of its highest point will be less, than $b\beta$; which will cause $\beta\alpha$ to be less than $\beta\gamma$, and the lower segment of the oval to be flatter than the upper. Their proportions may be easily computed by the table of refractions and observed through a smoaked glass or with a telescope*. But unless the horizontal moon be exactly in the full, the defect of the illumination of her disk, towards the right hand or left will partly destroy that oval figure and reduce it to those rounder, though somewhat irregular, shapes with which it frequently appears. And the rising full moons about the time of the autumnal equinox, and the setting full moons about the vernal, will be the roundest of all, because the ecliptick is then very oblique to the horizon; so that the defect of illumination falls fuller upon one end of the horizontal diameter. For these reasons the horizontal moon appears oval but very rarely, especially in the evenings of warm weather, the refractions being then smaller.

* *Balthasaris* micrometria p. 101.
Fig. 103.

b Rem. 363.

* Rem. 128.

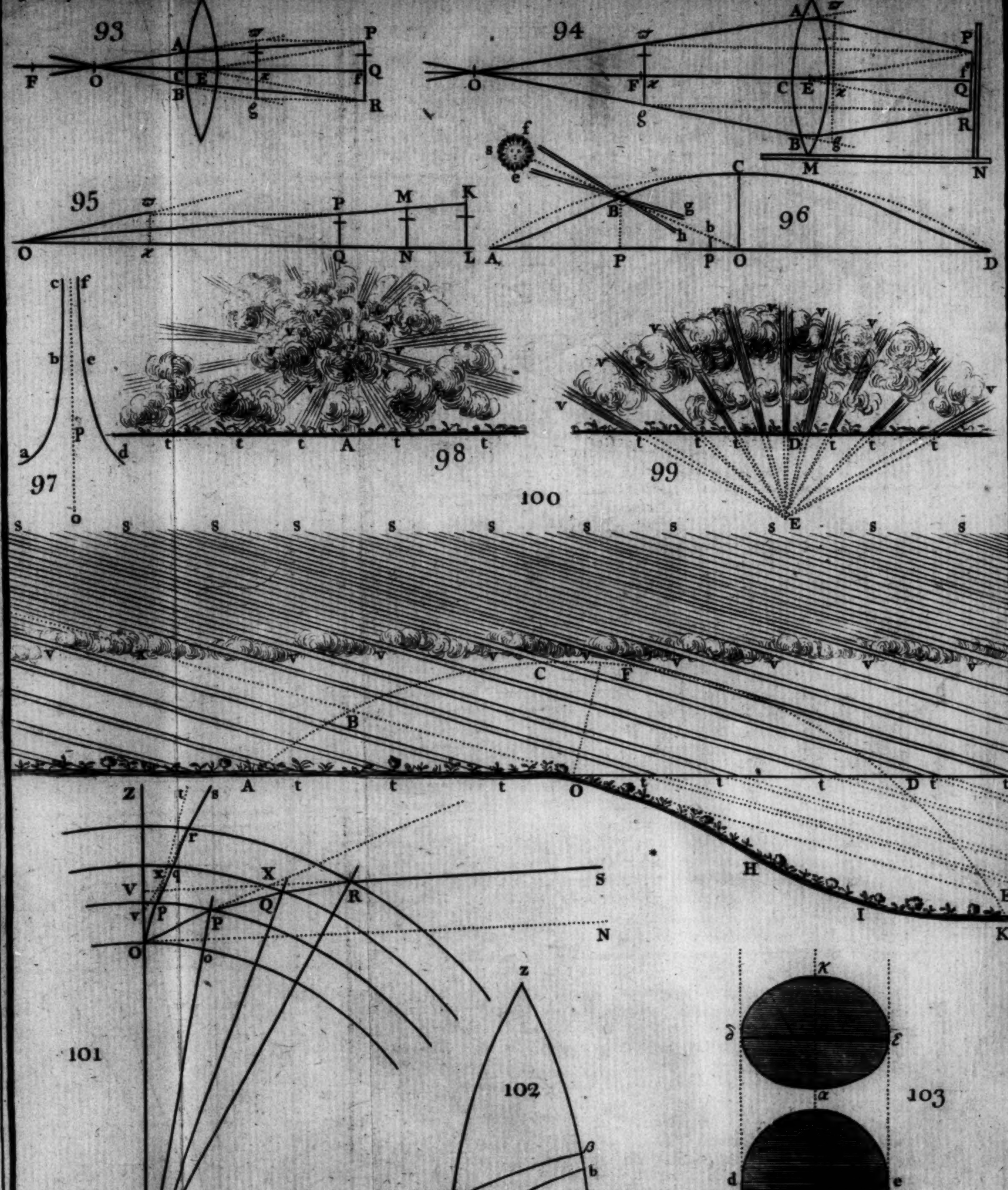
Visible effects of refractions in the moon's eclipses.

372. The refraction of an horizontal ray being generally equivalent to the apparent diameter of the sun or moon, it is manifest that all the heavenly bodies are entirely visible when in reality they are below the plane of the horizon produced. For this reason several total eclipses of the moon have been seen in the horizon, when the sun was also visible in the opposite part of it. The ancient philosophers that knew nothing of the air's refractive power, were very much perplexed with this strange phænomenon; and also to give a reason why the moon should be visible at all, even at any altitude, when totally immersed within the earth's shadow, as she always is; appearing generally of a dusky, red colour, not unlike that of tarnished copper, or of iron almost red hot. This they thought was the moon's native light, by which she became visible when hid from the brighter light of the sun. *Plutarch* indeed, in his Discourse upon the Face in the Moon, attributes this appearance to the light of the fixt stars reflected to us from the moon; but this is much too weak to produce that effect. The true and only sensible cause of it, is the scattered beams of the sun bent into the earth's shadow by refractions through the atmosphere, after the following manner.

The earth's shadow how much contracted by refractions.
Fig. 104.

373. Let the body of the sun be represented by the greater circle *ab*, and that of the earth by the lesser one *cd*; and let the lines *ace* and *bde* touch them both on their opposite sides and meet in *e* beyond the earth; then the angular space *ced* will represent the conick figure of the earth's shadow, which would be totally deprived of the sun's rays, were none of them bent into it by the refractive power of the atmosphere. Let this power just vanish at the circle *hi*, concentrick to the earth, so that the rays *ah* and *bi*, which touch its opposite sides may proceed unrefracted and meet each other at *k*. Then the two nearest rays to these, that flow within them, from the same points *a* and *b*, being refracted inwards through the margin of the atmosphere, will cross each other at a point *l*, somewhat nearer to the earth than *k*; and in like manner, two opposite rays next within the two last will cross each other at a point *m*, somewhat nearer to the earth than *l*, having suffered greater refractions, by passing through longer and denser tracts of air lying somewhat nearer to the earth. The like approach of the successive intersections *k*, *l*, *m*, is to be understood of innumerable couples of rays; till you come to the intersection *n*, of the two innermost; which we may suppose just to touch the earth at the points *o* and *p*. It is plain then, that the space bounded by these rays *on*, *np*, will be the only part of the earth's shadow wholly deprived of the sun's rays. Let *fmg* represent part of the moon's orbit, when it is nearest to the earth at a time when the earth's dark shadow *onp* is the longest; in this case I will shew that the ratio of *tm* to *tn* is about 4 to 3, and consequently that the moon, though centrally eclipsed at *m*, may yet

Page [illegible] of Remarks Plate E



yet be visible by means of those scattered rays abovementioned, first transmitted to the moon by refractions through the atmosphere, and from thence reflected to the earth.

Demonstration. Fig. 105.

374. For let the incident and emergent parts *aq*, *rn* of the ray *aqorn*, that just touches the earth at *o*, be produced till they meet at *u*, and let *aqu* produced meet the axis *st* produced in *x*; and joining *us* and *um*, since the refractions of an horizontal ray passing from *o* to *r* or from *o* to *q* would be alike and equal, the external angle *nux* is double the quantity of the usual refraction of an horizontal ray; and the angle *aus* is the apparent measure of the sun's semidiameter seen from the earth; and the angle *ust* is that of the earth's semidiameter *tu* seen from the sun (called his horizontal parallax); and lastly the angle *umt* is that of the earth's semidiameter seen from the moon, (called her horizontal parallax;) because the elevation of the point *u* above the earth, is too small [a] to make a sensible error in the quantity of these angles; whose measures by astronomical tables are as follow.

a Rem. 361.

		′ ″
The sun's least apparent semidiameter	= ang. *aus*	= 15 — 50
The sun's horizontal parallax	= ang. *ust*	= 00 — 10
Their difference [b] is	= ang. *txu*	= 15 — 40
Double the horizontal refraction	= ang. *nux*	= 67 — 30
Their sum [c] is	= ang. *tnu*	= 83 — 10
The moon's greatest horizontal parallax	= ang. *tmu*	= 62 — 10

b Eucl. I. 32.

c Eucl. I. 32.

Therefore we have *tm* : *tn* :: (ang. *tnu* : ang. *tmu** :: 83′—10″ : 62′—10″ ::) 4 : 3 in round numbers, which was to be proved. It is easy to collect from the moon's greatest horizontal parallax of 62′—10″, that her least distance *tm* is about 55 $\frac{1}{3}$ semidiameters of the earth, and therefore the greatest length *tn* of the dark shadow, being three quarters of *tm*, is about 41 $\frac{1}{2}$ semidiameters.

* Art. 60.

Illumination of the moon eclipsed how great.

375. The difference of the last mentioned angles *tnu*, *tmu* is *mun* = 21′, that is, about two thirds of 31′—40″, the angle which the whole diameter of the sun subtends at *u*. Whence it follows that the middle point *m* of the moon centrally eclipsed, is illuminated by rays which come from two thirds of every diameter of the sun's disk, and pass by one side of the earth; and also by rays that come from the opposite two thirds of every one of the said diameters, and pass by the other side of the earth. This will appear by conceiving the ray *aqorn* to be inflexible, and its middle point *o* to slide upon the earth while the part *rn* is approaching to touch the point *m**; for then the opposite part *qa* will trace over two thirds of the sun's diameter [d]. The true proportion of the angles *num*, *aus* could not be preserved in the scheme, by reason of the sun's immense distance and magnitude with respect to the earth.

* Art. 44.

d Art. 59.

Images of the sun how formed by the atmosphere. Fig. 106.

376. Having drawn the line *atu*, it is observable that all the incident rays as *aq*, *ax* flowing from any one point of the sun, to the circumference of the earth, will be collected to a focus *u*, whose distance *tu* is less than *tm* in the ratio of 62 to 67 nearly; and thus an image of the sun will be formed at *uβ*, whose rays will diverge upon the moon. For the angle *tau* is the difference of the angles *xun*, *uat* found above; and *tu* : *tm* :: ang. *tmu* : ang. *tau** :: 62′—10″ : 67′—30″.

* Art. 60.

How they increase in magnitude and brightness.

377. The rays that flow next above *aq* and *ax*, by passing through a thinner part of the atmosphere, will be united at a point in the axis *atu* somewhat farther from the earth than the last focus *u*; and the same may be said of the rays that pass next above these, and so on; whereby an infinite series of images of the sun will be formed, whose diameters and degrees of brightness will increase with their distances from the earth.

The colour of the moon eclipsed explained.

378. Hence it is manifest why the moon eclipsed in her perigee is observed to appear always duller and darker than in her apogee. The reason why her colour is always of the copper kind between a dull red and orange, I take to be this. The blue colour of a clear sky shews manifestly that the blue-making rays are more copiously reflected from pure air than those of any other colour; consequently they are less copiously transmitted through it among the rest that come from the sun, and so much the less as the tract of air through which they pass is the longer. Hence the common colour of the sun and moon is whitest in the meridian and grows gradually more inclined to diluted yellow, orange and red as they descend lower, that is, as the rays are transmitted through a longer tract of air; which tract being still lengthened in passing to the moon and back again, causes a still greater loss of the blue-making rays in proportion to the rest; and so the resulting colour of the transmitted rays must lye between a dark orange and red; according to Sir *Isaac Newton*'s rule for finding the result of a mixture of colours*. We have an instance of the reverse of this case in leaf-gold, which appears yellow by

* Opt. p. 134. 8°.

by reflected and blue by transmitted rays. The circular edge of the shadow in a partial eclipse appears red, because the red-making rays are the least refracted of all others a, and consequently are left alone in the conical surface of the shadow, all the rest being refracted into it.

a Art. 172.

Other effects of the reflective power of the air.

379. The reflective power of the air is the chief cause that enlightens objects so uniformly on all sides. The absence of this power would occasion a strange alteration in the appearances of things; their shadows would be so very dark and their sides enlightened by the sun so very bright, that probably we could see no more of them than their bright halves; so that for a view of their other halves we must turn them half round, or if immoveable, must wait till the sun could come round upon them. Such a pellucid unreflective atmosphere would indeed have been very commodious for astronomical observations upon the course of the sun and planets among the fixt stars, visible by day as well as by night; but then such a sudden transition from darkness to light and from light to darkness immediately upon the rising and setting of the sun, without any twilight, would have been very inconvenient and offensive to our eyes.

Twilight explained. Fig. 107.

380. The better to understand the beginning, increase and ending of twilight, let the sun's rays coming in the direction *sab*, enlighten a segment of the atmosphere represented by the shaded segment *abga*, and terminated below by the line *ab*, that touches the earth's surface at *d*, and above by the arch *agb*. From the end *b*, opposite to the sun, draw a line *be*, touching the earth at another point *e*; and supposing no refractions of rays, a spectator at *e* might just perceive a faint light in his horizon *eb*, reflected to him from the air or vapours at *b*. By the earth's diurnal rotation let the spectator be carried from *e* to *f*, and his horizon *be* into the position *gf* cutting *db* in *b*; and from *f* he will see the part *bgb* of the luminous segment *bgab*, by rays reflected every way from every point of it *. And lastly when the earth has carried him to *d*, the whole luminous segment *agba* will become visible to him, together with the sun in his horizon *da*.

* Art. 56.

The old way of finding the height of the atmosphere.

381. From this consideration the ancient mathematicians determined the heigth of the atmosphere to be about 50 miles *, as follows. Immediately upon perceiving the first and faintest light in the eastern horizon *eb*, they observed the altitudes or positions of some of the brightest stars; from which they computed how many degrees the sun was then below the horizon, and found them at a medium to be about 18 degrees; which being the measure of the angle *dbm* between the two horizons *db*, *eb* produced to *m*, or between their perpendiculars *cd*, *ce*, drawn from the earth's center, they rightly concluded, that the enlightened vapours at *b*, were situated in the line *cb* which bisects the said angle *dce* of 18 degrees. Now in the right-angled triangle *cdb*, the radius *cd* is to the secant *cb*, of the angle *dcb* of 9 degrees, as 10000 to 10125, or (multiplying both by 4 and dividing by 10) as 4000 to 4050. Therefore, the earth's semidiameter *cd* being about 4000 miles, we have *cb* equal to 4050; and consequently the height of the vapours at *b* above the earth's surface is 50 miles: supposing as I said that the horizontal rays *db*, *be* were not refracted; which *Alhazen* did not consider, not knowing how much to allow for it, as I observed above.

* Alhazen prop. ult.

A correction of it by refractions. Fig. 108.

382. But since those rays *db*, *be* do each of them suffer a continual refraction inwards along the curves $d\beta$, βe; the utmost height of the reflective matter at β above the earth's surface, will be reduced to about $44\frac{1}{2}$ miles, according to the following rule given by Dr. *Halley**. From the aforesaid angle *dce* of 18 degrees, substract twice the usual refraction of an horizontal ray, or about a degree; and half the remainder will be about $8\frac{1}{2}$ degrees; whose secant being 10111, it will follow that as 10000 to 111, so is the semidiameter of the earth or 4000 miles to 44, 4 miles.

* Phil. Transf. N°, 181.

Demonstration of it.

383. For supposing two rays to set out from *d* and *e* along the horizontal lines *db*, *eb*, and after they have described the curves $d\beta$, $e\beta$, to cross each other at β near the top of the atmosphere; they will soon depart from it in straight lines $\beta\gamma$, $\beta\delta$, each declining from the respective horizons *db*, *eb* in an angle of about half a degree. And therefore the perpendiculars *cp*, *cq* to the lines $\beta\gamma$, $\beta\delta$ produced back, will severally decline from the perpendiculars *cd*, *ce*, to the said horizons, in the like angles *pcd*, *qce* of half a degree; and will be nearly equal to the semidiameter of the earth; because the curvity of the rays βd, βe near β is exceeding small *. Consequently $c\beta$ is the secant of the angle βcp, to the radius *cp* or *cd* nearly, that is, of the angle *dcb* diminished by *dcp*, the refraction of an horizontal ray. It is farther to be observed, that the ray $\alpha d\beta$ came from the sun not situated in the horizontal line *adb*, but in the tangent $s\alpha$ of the curve αd, inclined to *adb* in an angle of half a degree. Therefore since the tangent $\beta\delta$ to the reflected ray βe, is also inclined to the other horizon *eb* in the like angle, the angle under the tangents $s\alpha$, $\beta\delta$ must be equal to the angle *abm* under the two horizontal lines *bd*, *be*.

* Rem. 361.

Dimensions of the visible segment of the atmosphere

384. Hence it follows, that the height of the reflective atmosphere, (being about $44\frac{1}{2}$ miles,) is about $\frac{1}{90}$ part of the earth's semidiameter; and that

that a ray of light $\alpha d\beta$, passing horizontally by any place d on the earth's surface, is so refracted along the curve $d\beta$, as, at the point β where it departs from the reflective atmosphere, to sink about $5\frac{1}{2}$ miles below the tangent db to the place d; and that the distance $d\beta$ is about 600 miles; and consequently that any place d is constantly enlightened in the day time by rays reflected from every part of a segment of the atmosphere, whose height is about $44\frac{1}{2}$ miles and whose circular base is about 1200 miles in diameter.

The air's refractive power. * Rem. 410.

385. The sine of incidence *in vacuo* is to the sine of refraction into common air, as 1000000 to 999736*; and therefore when the angle of incidence is a right one or the nearest to it, the greatest angle of deviation, contained under the refracted ray and the incident one produced, is almost 79 minutes. And this angle being but small will be diminished very nearly in a subduplicate ratio of the air's density as Sir *Isaac Newton* shews in his Opticks[a]; and in his *Principia* he shews[b], that at the height of one semidiameter of the earth above her surface, if the air reaches so high, it must become rarer than the air with us, in a far greater ratio than that of all the space within the orbit of Saturn to a globe of space of an inch in diameter.

a pag. 247. 80. b pag. 512. Edit. 3.

All the heavenly bodies are equally raised by refractions.

386. Hence we may reasonably conclude that the sun and all the planets, at any given altitude, appear equally raised by refractions; because they are all far above that rarefied air, and because their lights, as being all derived from the sun, are all equally refrangible. And it appears by astronomical observations that the rays of the fixt stars are also equally refracted with those of the sun and planets. *Tycho Brabe* was a long time of a different opinion, holding the refractions of the fixt stars to be less than those of the sun and moon; but at last he was sensible of his mistake, occasioned by making the parallaxes of the sun and moon too great; by which he depressed them too much among the fixt stars, and was therefore obliged to raise them too much by too great an allowance for refractions.

Twinkling of stars owing to refractions.

387. The twinkling of stars is another effect of the refractive power and trembling motion of the air and interspersed vapours; which cause the successive rays to fall upon the eye in different directions, and consequently upon different parts of the retina at different times, and also to hit and miss the pupil alternately. These tremors of the air are manifest to the eye by the tremulous motion of shadows cast from high towers; and by looking at objects through the smoke of a chimney or through steams of hot water, or at objects situated beyond hot sands, especially if the air be moved transversely over them[a]. But when stars are seen through telescopes that have large apertures, they twinkle but little and sometimes not at all. For (as Sir *Isaac Newton* has observed) " the rays of light which pass through divers parts of the aperture, tremble each of them apart, and by means of their various and sometimes contrary tremors, fall at one and the same time upon different points in the bottom of the eye, and their trembling motions are too quick and confused to be perceived severally. And all these illuminated points constitute one broad lucid point, composed of those many trembling points confusedly and insensibly mixed with one another by very short and swift tremors, and thereby cause the star to appear broader than it is, and without any trembling of the whole.'

a *Hook's* Micrography p. 231. Opt. pag. 98.

Upon Chapter 6. Concerning the origine and cause of colours.

Design.

388. Sir *Isaac Newton* has not only obliged us with his own discoveries of the true origine and causes of colours, but also with an account of the notions which the world had formerly of them[a]. But as these are very unsatisfactory, I chuse to pass them by, and to transcribe some of those practical methods by which he determined the constant Ratio of refraction both in fluids and solids; and thereby confirmed this fundamental property of light, from which all the certainty and accuracy of our knowledge in opticks is chiefly derived.

a Lectiones Optic. p. 146.

Description of an instrument for finding the ratio of refraction in fluids. Fig. 109.

389. Let HK be a square beam of wood two or three yards long, or rather a hollow tube, to prevent its bending by its own weight; and let its opposite sides be perfectly plane and parallel; and let HI and KL be two square boards fixt perpendicular to one of its sides; the one KL at the very end of the beam, and the other HI about four inches from the other end. Then let the base of a small deep vessel CF, of any shape and substance, be fixt upon the board HI by hard cement, and rest against the end of the beam continued beyond HI; and let a small hole about a tenth of an inch broad be bored through the middle of the bottom of the vessel at F, and also through the board it rests upon: and the opposite board being painted white, let a mark R be made upon it at the same distance from the beam as the center of the hole F, so that the line FR may be exactly parallel to the sides of the beam; and let a piece of glass ground equally thick and well polished be applyed to the under side of the board HI, and be fastened to it with cement dawbed round about the hole F, to prevent any of the fluid contained in the vessel from running out; and let this glass be placed exactly perpendicular to the

the sides of the tube by means of a square. Then let two cylindrical pins of brass or iron be fixt in the middle of the opposite sides of the beam, and be lodged in two angular notches cut in two parallel boards placed upon a firm pedestal, that the beam may turn easily upon the pins, like the beam of a ballance, and be readily fixt at any given inclination to the horizon.

390. The instrument being thus prepared and the beam being placed in a vertical plane passing through the sun, let the vessel *CF* be about half filled with water; and when the beam is so inclined that the refracted rays shall pass through the hole *F* and fall on the board *KL*, by moving the beam a little it will be easy to make any colour you please fall upon the mark *R*; then let the inclination of the beam to the horizon be taken with a large quadrant, whose side *ek*, being applyed to the under side of the beam, will give the angle of refraction *ekr* of that ray, and its sine *er*; because the plumb-line *kr* is perpendicular to the surface of the water. Then let the sun's altitude be immediately taken; and its complement *AkD* to 90 degrees will be the angle of incidence, and *AD* its sine. Which sines being compared together, after several trials of the experiment at different heights of the sun, will always be found to be in the same ratio, when a ray of the same colour falls upon the mark *R*. If it be desired to make many repetitions of the experiment in a little time, and for smaller angles of incidence than those of the sun's nearest distance from the zenith, a piece of looking-glass may be inclined over the mouth of the vessel, to reflect the rays downwards, partly into the vessel and partly by the side of it, to be examined as before instead of the sun's direct rays. Lect. Opt. Par. I. §. 2.

How to place a prism so as to refract rays equally at its sides. a Ibid. Par. I. §. 2.

391. But the most accurate way of all for finding the exact quantity of the ratio of refraction, is this other of Sir *Isaac Newton*[a]. It was observed in Art. 171 that when the axis of the prism is held perpendicular to the sun's rays, and the refractions are made upwards, that by turning the prism slowly about its axis, the refracted light, or the sun's coloured image cast upon a wall, will first descend and then ascend during the same continued rotation of the prism. Between the ascent and descent when the image is stationary, let the prism be stopt and remain fixt in this posture; and then the refractions of the rays at their ingress and egress on each side of the prism will be equal.

392. For during the descent of the image, it is plain that the sum of these two refractions continually decreases, and then increases again during its ascent; and therefore there are two positions of the prism, before and after the image is stationary, wherein the sum of the refractions at its sides are equal; and cause the image to fall upon one and the same place of the wall. In these two positions, the ray *DE* in the one and *δε* in the other, lying within the refracting angle *ABC*, are equally inclined to its sides *AB*, *BC*, but contrary ways; that is, the triangles *BDE*, *Bεδ* are equiangular. For supposing it so, and the rays to go both ways, along the lines *DE*, *δε*, the refractions in going out at *D* and *ε* will be equal to each other, and also in going out at *E* and *δ*; and consequently the sum of the unequal refractions at *D* and *E* will be equal to the sum of those at *δ* and *ε*: and hence it is that the image falls upon the same place on the wall in both positions of the prism. But according as this common place of the image comes nearer to the limit of its contrary motion, experience shews that the two positions of the prism come nearer to that intermediate position which throws the image to its limit. Therefore at the same time the angles at the bases *DE*, *δε* of those similar triangles *BDE*, *Bεδ* approach gradually to equality, and become equal when the image is at its limit; and consequently the refractions at *D* and *E* are then equal. This will also appear by the rays *STV* reflected from the top of a glass-prism provided the legs *AB*, *BC* of the refracting angle be equal, as they generally are; for then, as the incident rays *ST* and *SD* are parallel, so the reflected ray *TV* and the refracted one *EP* will also be parallel in this position of the prism; as will be evident to the eye by the small distance or rather coincidence of the two images *V*, *P* on the wall, which in all other positions of the prism will be widely distant. Sir *Isaac* has proved the whole mathematically, Lect. Opt. Par. I. §. 3. prop. 25.

Fig. 110. 111.

Fig. 112.

The angle of refraction is then half the refracting angle of the prism. Fig. 113. b Eucl. l. 32.

393. In this position of the prism the angle of refraction at the ingress of a ray is equal to half the refracting angle *ABC*. For let *LDK* be perpendicular to *AB*, and since the line *BQ*, that bisects the vertical angle *B* of the isoscelar triangle *DBE*, is perpendicular to its base *DE*; in the right angled triangle *BQD*, its two acute angles *QBD*, *QDB* will be equal to its right angle[b], or to the right angle *BDK*, consisting of the angles *QDB* and *QDK*. Therefore by taking away the common angle *QDB*, there remains *QBD* equal to *QDK*, that is, half the refracting angle of the prism is equal to the angle of refraction.

How to measure it. Fig. 114.

394. Now the refracting angle of the prism may be measured by laying two rulers cross each other upon the side of a smooth table; and by altering their inclination till the edges of the parts that project over the table, coincide with the sides of the refracting angle of the prism interposed between them. For then two lines drawn upon the table, by the sides of each ruler, will give the refracting angle, to be measured by a sector; as appears by the figure, where the rulers are *ab*, *cd*; and the prism is *e*.

395.

How to find the angle of incidence. Fig. 113.

395. The prism being placed as above, let the altitudes of the incident and emergent rays, SD, EP, be taken with a quadrant; and the angle of incidence SDL, will be equal to half the sum of these altitudes. For let these rays be produced back till they intersect one another in I, and any horizontal line in M and N; and the angles at M and N will be the altitudes of these rays above the horizon; and both together will be equal to the external angle MIE*, which is equal to the two internal angles of the triangle IDE; and therefore half the sum of the altitudes is equal to one of these equal angles IED or IDE, which added to the angle of refraction EDK found before, gives the angle of incidence IDK or SDL.

* Eucl. I. 32.

396. If the sun be higher elevated, till the emergent ray EP becomes parallel to the horizon, then the angle at N will vanish; and if the sun be still higher, the emergent ray will tend downwards; and then the angle at N will become negative: therefore in this last case, half the difference of these altitudes must be added to half the refracting angle of the prism, to get the angle of incidence.

An example.

397. Sir *Isaac Newton* has given us the following example of this method*. In a glass-prism whose refracting angle was $62\frac{1}{2}$ degrees, the half of it $31°. 15'$ is the angle of refraction into the prism, whose sine is 5188 the radius being 10000. When the axis of the prism was parallel to the horizon, and the sun's image upon the wall was at its limit of regression, he observed, with a quadrant, the angle which the mean refrangible rays (that is those which went to the middle of the coloured image) made with the horizon; and by adding this angle to the sun's altitude observed at the same time, he found the angle PIM, which the emergent rays contained with the incident ones, to be $44°. 40'$; whose half $22°. 20'$, being added to the angle of refraction $31°. 15'$, makes the angle of incidence $53°. 45'$., whose sine is 8047; and the ratio of these sines in round numbers* is 20 to 31.

* Opt. p. 71.

* See *Cotes*'s Harmonia Mensurarum p. 7. Schol. 3.

The excellencies of the method.

398. The excellency of this method appears from these considerations, that it requires no other instruments but a quadrant and a prism; that the refraction of the ray being doubled, an error in the practice is but half of what it would be in a single refraction; that it is very easy to place the prism in the required position; and that a small deviation from it does not alter the place of the image, or the sum of the two refractions, as is evident upon trial and because this sum is then the least of all others. For it is a known thing that the variations of quantities generated by motion are generally insensible, when the quantities become the greatest or least possible, that is, in the moment between their increase and decrease.

Properties of refractions thro' several parallel planes.

399. Sir *Isaac* informs us of some other properties of refractions well worth observing[a]; to wit, *if a ray of light goes out of air through several contiguous mediums terminated by parallel planes, as through water and glass, and thence goes out again into air, the emergent ray will always be parallel to the incident one.* For let a piece of coach-glass of an equal thickness, be smeared over with a little water, or any other fluid; and be held parallel to the horizon, that the water may be of an equal thickness; then you will find that the rays of the sun, transmitted through both mediums, will go parallel to his unrefracted rays; as will appear by observing the places where they fall upon any distant plane beyond the glass.

a Lect. Opt. Par. I. §. 2. No. 31.

400. Hence if a ray passes through many refracting mediums terminated by parallel planes, it will have the same inclination to the surfaces of the last medium as if it had suffered but a single refraction in passing immediately out of the first medium into the last. For example let Aa, Bb, Cc be the parallel surfaces of water lying upon glass, and let the ray DE be refracted into the line EF in water, and then into FG in the glass; and let another ray PQ parallel to DE, fall immediately upon the glass and be refracted into the line QR; then the refracted rays FG, QR will be parallel. For let these rays emerge into air in the lines GH, RS; and because GH is parallel (to DE* or to PQ by hypothesis or) to RS*, it follows that the refractions at G and R are equal*; and consequently the rays FG and QR are parallel and therefore equally inclined to the incident rays DE, PQ; that is, the sum of the refractions of the one at E and F is equivalent to the single refraction of the other at Q.

Fig. 115.

* Rem. 399.

* Art. 36.

* Art. 12.

401. The conclusion will be the same though the number of different mediums be increased to infinity and the intervals of their parallel surfaces be as much diminished; so that the ray may be continually refracted into a curve line.

402. Hence the proportion of the sine of incidence to the sine of refraction of one and the same sort of rays out of one medium into another, is compounded of the proportion of the sine of incidence to the sine of refraction out of the first medium into any third medium, and of the proportion of the sine of incidence to the sine of refraction out of the third medium into the second*. So that by this Theorem the refraction out of one medium into another is gathered as often as you have the refractions out of them both into any third medium.

* Newt. Opt. pag. 113.

403. For let the perpendiculars to the surfaces through the points of refraction E, F, G be IK, LM, NO. Then the sine of the angle

EFL or FEK, is to the sine of DEI, as 3 to 4 *; and the sine of the same angle DEI or HGO * is to the sine of FGN or GFM as 31 to 20 *; and by compounding these proportions the sine of EFL, is to the sine of GFM, as 3×31 to 4×20, as 93 to 80; which measures the refraction out of water into glass.

* Rem. 404. * Rem. 399. * Rem. 397.

The exactest way of finding the refractions of fluids.

404. If a prismatick vessel be made of wood, and two opposite holes, for light to pass through, be made any where in the sides containing the refracting angle, and if pieces of broken looking-glass unfoiled be cemented over the out sides of the holes, and be set exactly to a right angle, as being easiest to be examined by a square, and the vessel be filled with water, poured into a hole made in its third side, or with any fluid whose refractive power is to be found out, and the hole be stopped up with a cork; and the like experiment be repeated with this prism as was tryed before with that of glass *, you will have the ratio of the refraction of water. For the incident and emergent rays in the air will be inclined to the intermediate rays within the water, in the very same angles, as they would have been if the water had been contiguous to the air [a]. Sir *Isaac Newton* by this accurate method, found the ratio of refraction of red rays out of air into water to be 4 to 3.

* Rem. 395.

a Rem. 400.

Refraction of air determined by experiment. Fig. 113.

405. While the rays are passing through the prism to the place P upon the opposite wall, if you suppose the water to be let out, the image at P will immediately sink down to M, where the line SD continued straight on would fall upon the wall. Because the refractions at the outward and inward surfaces of the glasses correct one another [b]. In like manner supposing the prism to be full of condensed air, and that this air has a greater refractive power than the outward air, the like appearance would happen in proportion to that power, excepting that the image P in this case would not sink all at once but gradually, as the air within becomes thinner, by escaping gradually. Consequently if we suppose the remaining air to be exhausted from the prism, the refractions would now be made downwards; and while the air was suffered to re-enter the prism, the image P would appear to ascend gradually. During these experiments if we suppose the rays to go backwards to the eye of a spectator placed at S, at first he would see the place P upon the wall, and during the time that the condensed air was escaping from the prism in the first case, or the outward air was re-entering in the second, he would see all the points of the line PM successively appearing in the same direction SD: and lastly if you suppose the wall PM to be very remote and the rays DS to come through a fixt telescope to his eye, he would see the same appearances more plainly and distinctly, especially if cross hairs be placed in the focus of the telescope for him to take aim by.

b Rem. 400.

Fig. 116.

406. An experiment of this kind was first tryed by Mr. *Lowthorp* * who made his vacuum between two glass planes by the help of quicksilver, and found the ratio of the sine of incidence to the sine of refraction out of air into the vacuum as 100000 to 100036; and the Royal Academy at *Paris* having afterwards attempted the same without success *, it was again repeated by order of our Royal Society at *London*. The apparatus was now made by the direction of Dr. *Halley* and consisted of a strong brass prism, two sides of which, had sockets to receive two plane glasses; and the third side had a pipe and stop cock, whereby the air in the prism might either be exhausted or condensed; the prism had also a mercurial gage fixt to it, to discover the density of the contained air; and was contrived to turn upon its axis in order to make the refractions equal on each side when it was fixt to the end of the telescope. The refracting angle was near 64 degrees and the length of the telescope about 10 feet, having a fine hair in its focus.

* Phil. Trans. No. 257.

* Hist. An. 1700. p. 144. 8o.

407. The event of the tryals is related by Mr. *Haukshee* in these words *. "Having chosen a proper and very distinct erect object whose distance was 2588 feet (June 15. S. V 1708. in the morning, the barometer being then at 29, 7 $\frac{1}{2}$ and the thermometer at 60) we first exhausted the prism; and then applying it to the telescope, the horizontal hair in the focus covered a mark on our object distinctly seen through the vacuum, the two glasses being equally inclined to the visual rays. Then admitting the air into the prism, the object was seen to rise above the hair gradually, as the air entered, and in the end the hair was found to hide a mark 10 $\frac{1}{4}$ inches below the former mark. This often repeated as often succeeded.

* Phys. Mech. Exper. p. 225. 8o.

408. "This done we applyed the condensing engine to the prism; and having pumped in another atmosphere, so that the density of the included air was, by the mercurial gage, double to that of the outward, we again placed it before the telescope and then letting out the air by the cock, the object, which before seemed to rise, now appeared gradually to descend; and the hair at length rested on an object higher than before by the same interval of 10 $\frac{1}{4}$ inches. And this likewise often repeated never failed.

409. "We again crouded in another atmosphere, and upon discharging the condensed air, the object was seen near 21 inches lower than the hair. But in this the great pressure forcing the cement, would not permit us to make so frequent repetitions as in the former."

409.

Fig. 116. 410. "Now the radius being 2588 feet the interval $10\frac{1}{4}$ inches subtended an angle *PIM* of 68″. The half of which gives 34″ for the angle *QDI*; which being taken from the angle *QDK* or *QBD* of 32°, gives the angle *KDI* or *LDS* equal to 31° — 59′ — 26″. And so the sine of incidence in vacuo, is to the sine of refraction into common air as 1000000 to 999736." So far Mr. *Hauksbee*.

Whether heat alone may not alter the air's refractive power.

411. It appears by these experiments that the subtenses of the angles of deviation *PIM*, and consequently the angles themselves [a], generated by the air's refractive power, were proportionable to its densities. And since the density of the atmosphere is as its weight directly and as its heat inversely, the ratio of its density at any given times may be had by the heights of the barometer and of Mr. *Hauksbee*'s thermometer, as described in pag. 220. ibid. And thence he concludes that this will also be the ratio of the air's refractions, that is, of the said angles of deviation at those given times. But before we can depend upon the accuracy of this conclusion, I think it ought to be examined whether heat and cold alone may not alter the refractive power of air while its density continues the same. This may be tryed by heating the condensed or the rarified air shut up in the prism, just before it is fixed to the telescope; and by observing whether the hair in its focus will continue to cover the same mark all the while the air is cooling.

a Art. 59.

Tables of refractions referred to.

412. A Table of the sines of incidence and refraction through a great variety of solid and fluid bodies may be seen in Sir *Isaac Newton*'s Opticks pag. 247. from which he computes another table of the Forces by which those bodies refract and reflect light; and finds them nearly proportionable to the densities of the same bodies; excepting that unctuous and sulphureous bodies refract more than others of the same density. Mr. *Hauksbee* has also given another Table of the ratio of the refractions of many other liquors chiefly chymical [b]; and when he says "that bodies do not refract light in proportion to their specifick gravities or densities," he means only that when the sines of incidence upon different bodies are the same, the ratio of the sines of refraction is not the same as the inverse ratio of their specifick gravities; which is very true, but not at all contrary to Sir *Isaac Newton*'s rule for the ratio of their forces. By which rule and by the foregoing experiments I find that the refractive force of air is as its density, its degree of heat being given.

b Ibid. pag. 192.

Upon Chapter 7. Concerning the cause of the refraction and reflection of light.

Design.

413. Having explained Sir *Isaac Newton*'s sentiments upon this subject, in pursuit of my general design I must here say something upon the opinions of other philosophers, some of which may be seen in Dr. *Barrow*'s and in Dr. *Gregory*'s opticks. But as most of them are too precarious and arbitrary to require a formal confutation, I will content my self with the mention of Mr. *Leibnitz*'s theory as having been received with great approbation. This gentleman being dissatisfied with *Des Cartes*'s theory, built upon an hypothesis that the motion of light is more resisted in a rarer medium than in a denser, advances another principle, by which he says the direct, reflected and refracted motion of light may be accounted for even to a degree of mathematical exactness. His principle and theory built upon it, is thus translated from the *Leipsick* Acts [c] by Mr. *Molyneux* [d] "*Light proceeds from the radiating point to the point to be enlightened, that way, which is of all the most easy*; and this is first to be determined in respect to plane surfaces, and then to be accommodated to concave and convex surfaces, by considering the planes that are tangents to these surfaces.

c Anni 1682. p. 185.
d Dioptr. pag. 192.

Fig. 117. 414. "*Hence in plane or simple opticks, the direct ray proceeds from the point* C *to the point to be illuminated* E, *by the shortest direct way; the same medium continuing all along, that is, in the right line* CE.

415. "*In catoptricks the angle of incidence* CEA *and of reflection* DEB *are equal.* Let *C* be the radiating point, *D* the point to be illustrated, and *AB* a plane speculum; it is required to find in the speculum the point *E* that reflects the ray to *D*. I say it shall be such a point, that the whole progress, way or journey of the ray $CE + ED$ may be the least or shortest that is possible; or less than $CF + FD$ supposing *F* any other point of the speculum. And this shall be obtained, if *E* be taken such, that the angles, *CEA*, *DEB* may be equal, as is manifest from geometry. For produce *DE* to the point *Z* in the circumference *CDZ* whose center is *E*, and join *FZ*; then the arch $AZ = (DB =) AC$; and $FZ = FC$; therefore $CE + ED$ $(= DZ)$ is less than $CF + FD$ or $ZF + FD$ *.

* Eucl. I. 20.

416. "*In dioptricks the sines* CI *and* GK *of the angles of incidence and refraction.* CEI *and* GEK, *are to each other reciprocally as the resistances of the mediums.* Let *IE* be air and *EK* water

water or glass, more dense than air, C a radiating point in air, G the point to be illustrated within the glass. It is required by what way or path the point C shall radiate to G, or it is required to determine, in the surface of glass AB, the point E which refracting the ray that comes from C sends it to G. Here the point E must be taken such that the way which the ray takes may be of all ways the *easiest*. But now in different mediums the difficulties of the way or progress are in a ratio compounded of the length of the way and of the resistance of the mediums. Let the right line m represent the resistance that light finds in its passage through air, and n the resistance of its passage through glass; the difficulty of the way from C to E shall be as the rectangle CE and m, and from E to G as the rectangle under EG and n. Therefore that the difficulty of the whole way CEG may be the least possible, the sum of the rectangles $CE \times m + EG \times n$ ought to be the least of all possible, or less than $CF \times m + FG \times n$ supposing any other point F taken besides E. The point E is now required." So far Mr. *Leibnitz*.

417. Then by arguing mathematically he finds the point E will be so situated, that CI the sine of incidence in air, will be to GK the sine of refraction into glass, as m to n, that is in the assumed ratio of the resistance in glass to the resistance in air.

418. This is Mr. *Leibnitz*'s theory, who tells us farther that the catoptrical demonstration abovementioned was insisted upon by *Ptolemy* and other ancients, and is still extant in *Heliodorus Larissaeus* and elsewhere. But had he proceeded the least step farther than they did, to accommodate this principle to concave and convex surfaces, as he talked of doing at first; he might soon have perceived the insufficiency of it, and that a ray takes several other courses from one point to another besides the shortest.

Fig. 118. 419. For example let $AEBF$ be a great circle of a concave sphere or speculum whose diameter is AB and center C; produce the radius CB to any point D, and let CD be the diameter of any other circle $CEDF$ cutting the former circle in E and F; and in the arches CE, CF let G and H be any two points equidistant from C; I say that rays flowing from G will be reflected to H from every one of the four points A, E, B, F; and consequently three of their courses will be of different lengths. For joining EG, EC, EH, the angles of incidence and reflection CEG, CEH will be equal, because they stand upon equal arches CG, CH of the circumference EGH *. The reason of the other two cases is very plain; and also that three of the courses of the rays are unequal; because they all approach towards equality as the points G and H approach to C; and become most unequal when G and H approach nearest to E and F.

* Eucl. I. 27.

420. As to the dioptrical demonstration, where Mr. L. says "the difficulty of the way from C to E shall be as the rectangle under CE and the assumed resistance m", his notions of difficulty and resistance are not easy to be understood. In all known cases the motion of a body and the resistance it suffers do constantly decrease together, action and reaction being always equal; and therefore the difficulty of the way from C to E can scarce be rightly expressed by the rectangle under CE and a constant resistance m. But be the difficulty and resistance what you please, no doubt of it, both of them must be nothing at all in *vacuo*; and therefore the easiest way for a ray to pass from a given point in any resisting medium into a vacuum, is to go in the perpendicular to the refracting surface, as being the shortest way through any difficulty or resistance whatever; which being quite over at the refracting surface, it may then take any other course in vacuo without any farther difficulty: and on the contrary, in returning back from the vacuum into the dense medium, it must take the shortest course through the same perpendicular as before: and thus when the sun shines upon the atmosphere all his rays should be refracted into lines tending to the earth's center as being the shortest and easiest way through the atmosphere; and then we should see the sun exactly over our heads in all places and at all times. But it is no wonder that strange consequences should follow from an arbitrary hypothesis. I will only observe farther, that all other Theories for solving the reflection and refraction of light, except Sir *Isaac Newton's*, do also suppose that it strikes upon bodies and is resisted by them; which yet has never been proved by any deduction from experience. On the contrary it appears by Art. 183, &c. and might be shewn by Mr. *Molyneux*'s and Professor *Bradley*'s observations upon the parallax of the fixt stars *, that their rays are not at all impelled by the rapid motion of the earth's atmosphere, nor by the object-glass of the telescope through which they pass. And by Sir *Isaac Newton*'s theory of refraction, which is grounded upon experience only, it appears that light is so far from being resisted and retarded by refraction into any dense medium, that it is swifter here than in vacuo in the ratio of the sine of incidence in vacuo to the sine of refraction into the dense medium; and on the contrary *.

Fig. 117.

* Phil. Trans. N. 406.

* Philos. Princip. lib. I. prop. 95.

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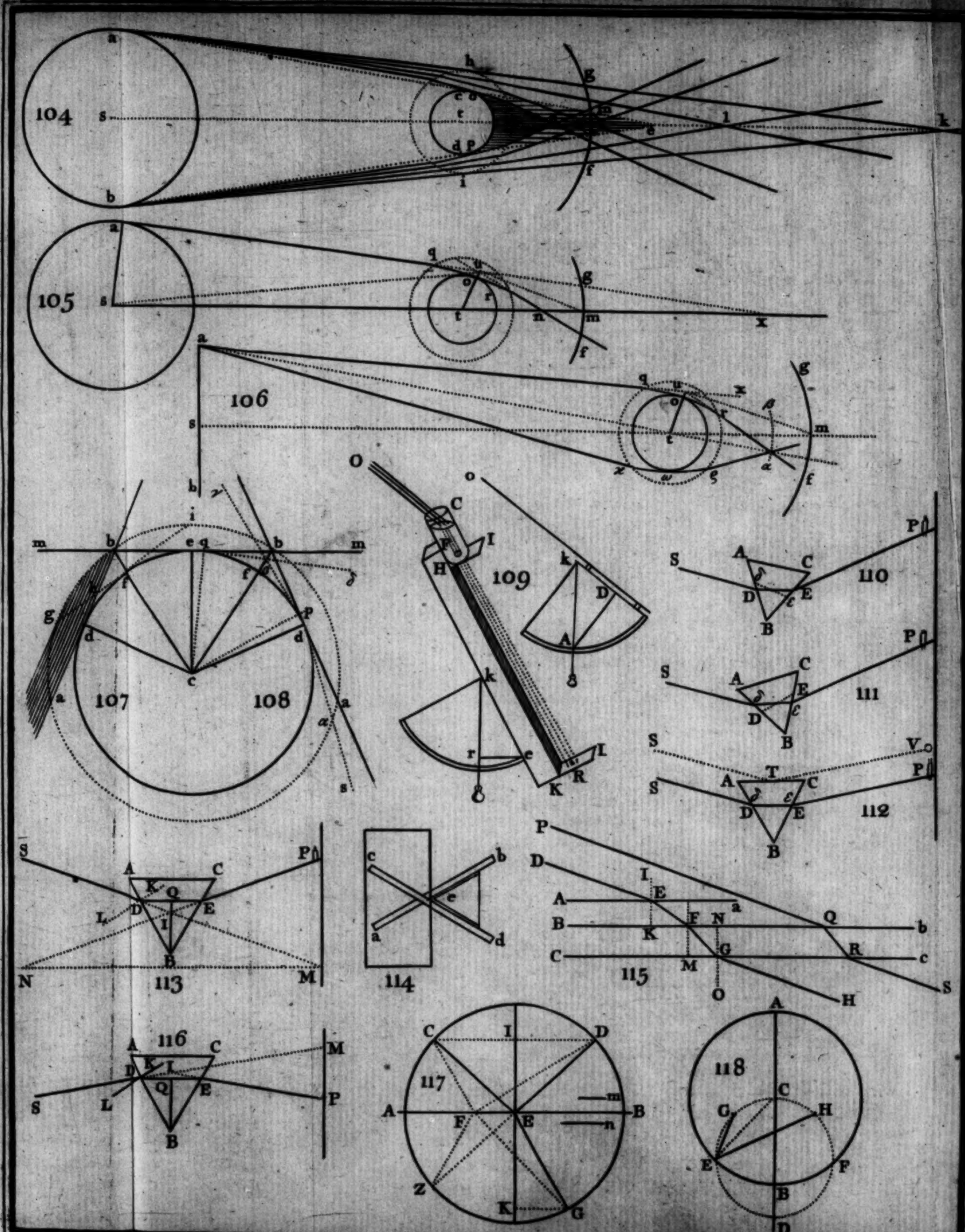
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THE

AUTHOR'S REMARKS

UPON BOOK II.

Upon Chapter 1. 2. 3. 4. *Containing the Geometrical Elements of Opticks.*

fign. 421. THE elementary propoſitions for finding focuſs's and images, are contained in the four firſt chapters of this Book, and are taken partly from Sir *Iſaac Newton*[a] and partly from Mr. *Huygens*[b]. But as Sir *Iſaac* has given us no demonſtrations at all, and as thoſe by Mr. *Huygens* are generally very tedious and intricate, by reaſon of too many compoſitions and reſolutions of ratio's, I was therefore obliged to contrive others in a ſhorter and eaſier way. Now though the proportions expreſſed in theſe propoſitions and their corollaries, are the beſt rules for determining the place of a focus to the utmoſt exactneſs, it will yet be of ſome uſe to do the ſame thing by geometrical Conſtructions, that is, by drawing lines only, and laſtly by a general algebraick Theorem or two comprehending the whole of theſe Elements. For as to the method uſed by ſome authors, of computing the place of a focus by trigonometry, it is not only the leaſt ſcientifick, but of the leaſt uſe too; unleſs to determine the courſe of a ray when the angles of incidence are very great. But for this I have given ſome rules in Lem. 3 and 4 Book 2.

[a] Opt. Ax. pag. 8.
[b] Dioptr. prop. 1. to ...

Upon ART. 207.

Geometrical conſtructions for finding the focus of reflected rays. Fig. 119, 120.

422. The focus Q of incident rays being given, their focus after reflection from a ſpherical ſurface C, whoſe center is E, may be found by this conſtruction. Through the given points Q and E draw a line QE cutting the concave or convex ſurface in C, then biſect its radius CE in T, and at the points T, C erect the perpendiculars TG, CH, cutting any line drawn through Q in the points G, H; join the points G, E and draw a line Hq parallel to the line GE, and it will cut the axis QE in q the focus of the reflected rays.

423. For the triangles TQG, CQH being equiangular, and alſo GQE and HQq; we have $TQ : TE$ or TC ($:: GQ : GH$) $:: EQ : Eq$*; and disjointly we have $TQ : TE :: TE : Tq$, the ſame proportion as was proved in the Article it ſelf.

* Eucl. VI. 2.

424. But this next conſtruction is ſtill ſimpler. In a perpendicular IEK to the axis QEC, take any two points I, K equidiſtant from E, and draw QI cutting the perpendicular CH in H; then draw the line KH and it will cut the axis in the focus q.

425. For let the perpendicular TG cut QI in G and join G, E; then ſince $TC = TE$, we have $GH = GI$*; and conſequently, ſince we took $EK = EI$, we have KH parallel to EG*, as in the foregoing conſtruction.

** Eucl. VI. 2.

426. Hence when the focus Q is infinitely remote, the line IH is parallel to EC, and conſequently the focus q coincides with T. Fig. 121.

Upon ART. 211.

427. For a proof of this Article ſee the Remarks upon Art. 339.

Upon ART. 216.

428. The determination of the conick ſections which I promiſed in this article may be ſeen with ſome other determinations in the Remarks upon Art. 246.

Upon ART. 236.

429. In order to make the ſolution of this propoſition ſerve for a ſingle ſurface as well as for a lens and a ſphere, I forgot to alter part of it in the following manner. In the 4th line of the Roman print read thus; *then taking* Ef *equal to* EF in the lens or ſphere, but equal to CF in the ſingle ſurface; ſay as &c. And in the 9th line read thus. *For with the center* E *and ſemidiameters* EF and Ef deſcribe &c.

Upon ART. 239.

Geometrical conſtructions for finding the focus of rays refracted through a ſphere or lens. Fig. 122. 123.

430. The focus Q of incident rays being given, the focus of the refracted rays through a ſphere or a thin lens, whoſe center is E, may be found in this manner. At the principal focus F of rays that came parallel to the axis QE, the contrary way to the incident ones that belong to Q, and alſo at the center E, erect the perpendiculars FG, EI to the axis, cutting any line drawn through Q, in G and I reſpectively. Join E, G and the line Iq drawn parallel to it, will cut the axis in the focus q of the refracted rays. For the triangles QFG, QEI being equiangular

angular, and also QGE, QIq, we have QF : QE :: (QG : QI* ::) QE : Qq*, the same proportion as was proved in the article it self.

** Eucl. VI. 4.

And thro' a single surface. Fig. 124.

431. The focus q of rays refracted through a single spherical surface C may be found by erecting one of the perpendiculars FG at its principal focus of rays that came parallel to QE, the contrary way to those that belong to Q, and the other perpendicular CH at its vertex C, cutting any line QG in H, and by joining GE, and drawing Hq parallel to it. For we have QF : QC :: (QG : QH* ::) QE : Qq*, the same proportion as was proved in Art. 238.

** Eucl. VI. 4.

432. Or the focus q may be found by this construction. Let the perpendiculars CH, EI cut any line drawn through Q in H and I, and in EI take EK to EI as the sine of incidence to the sine of refraction of any ray belonging to Q the focus of incident rays. And the line HK produced will cut the axis in the focus q of the refracted rays. For since EK : EI :: (as the sine of refraction, to the sine of incidence, that is, :: FC : FE* ::) GH : GI*; it follows that the line HK is parallel to the base GE of the triangle IGE*, as it ought to be by the foregoing construction.

* Art. 224.
** Eucl. VI. 2.

And thro' any two surfaces. Fig. 125.

433. Hence having Q the focus of incident rays we may find their focus x after refraction through any two surfaces whose vertex's are C and c and centers E and e. For having made the foregoing construction for the first surface C, let qKH cut the perpendicular at c in h, and the perpendicular at e in i, in which taking ek to ei as EI to EK and drawing kh, it will cut the axis Cc produced in the focus x. For this second operation is only a repetition of the first, because q is the focus of incident rays upon the second surface c. But in practice it is easier to determine the point k by a line drawn through the points q, I. Dr. *Barrow* tells us* that this construction was communicated to him by his friend Mr. *Isaac Newton*, to whom he soon after resigned his Mathematical Professorship in 1669.

* Lect. Opt. pag. 103. and Preface.

Upon Art. 246.

An accurate determination of images by reflections from a spherical surface. Fig. 126. to 129.

434. Other things remaining as in Art. 215, now let the object PQ be a straight line perpendicular to QE, and let QE produced cut the reflecting circle CA continued round in γ opposite to C, and the circle TS continued round in τ opposite to T; and let the incident rays which diverge from or converge towards Q, belong to the focus x after reflection from the nearest points to γ; and according as the perpendicular EQ is longer or shorter than ET, with the Focus E and transverse axis qx let an ellipsis or an hyperbola $qp x \varpi$ be described, and let it cut any line EP produced in p and ϖ, and the reflecting circle in A and α; then the conick arch pq will be the image of the object PQ, formed by rays reflected from the arch AG; and the conick arch ϖx the image of the same object PQ, formed by rays reflected from the opposite arch $\alpha\gamma$: and the whole conick section will be the image of the infinite line ΩPQZ formed by reflections from the whole circle.

435. When the perpendicular EQ is equal to ET, the ellipsis or the hyperbola will be changed into a parabola, whose focus is E and whose vertex is x; and when EQ is increased to infinity, the ellipsis will coincide with the circle whose diameter is $T\tau$ or $x\omega$; which is the parameter of all the curves.

436. Those that are conversant in the analysis of geometrick Places may soon be satisfied of the truth of these constructions and of all that follow, by supposing a focus of incident rays P to move along the line PQ, and by seeking the geometrick place described by its conjugate focus p, while the line PE is revolving about the center E; or they may see a demonstration of the like constructions of most of these cases in the 17th and 18th of Dr. *Barrow*'s Optick Lectures, who first discovered this remarkable figure of the image of a straight line.

And by refractions. Fig. 130. 131. 132.

437. Other things remaining as in Art. 245, now let the object PQ be a straight line perpendicular to a line QEq drawn through E the center of a refracting sphere; and let q be the focus of a slender pencil of rays which before their refractions through the sphere diverged from Q, and x the focus of another pencil of rays which before their refractions through the sphere converged towards Q; and according as the perpendicular EQ is longer or shorter than EF, the focal distance of the sphere, with the focus E and transverse axis qx let an ellipsis or an hyperbola $qp x \varpi$ be described, and let it cut any line EP produced in p and ϖ; and the conick arch pq will be the image of the object PQ, formed by rays diverging from PQ; and the opposite conick arch ϖx the image of the same PQ formed by rays converging towards PQ; and the whole conick section will be the image of the infinite line ΩPQZ.

438. When the perpendicular EQ is equal to EF, the ellipsis will be changed into a parabola, whose focus is E and whose vertex is x; and when EQ is increased to infinity, the ellipsis will coincide with the circle whose diameter is Ff or $x\omega$; which is the parameter of all the curves.

439. Hence when the angle PEQ, which a straight object subtends at the center of a thin lens, is but small, the image of it coincides very nearly with the arch of a conick section, determinable in the same manner as for a sphere.

Because

Because the relation of the conjugate focus's $\mathcal{Q}$, q is given by the very same proportion in both cases.

Fig. 133. 440. Now let the rays that diverge from $\mathcal{Q}$ be but once refracted through a spherical surface AC, and then belong to the focus q; and supposing this surface continued round till it cuts the axis again in c, let other rays which converge towards $\mathcal{Q}$ be refracted at c only, and then belong to the focus κ; and according as $E\mathcal{Q}$ is longer or shorter than EF, with the focus E and transverse axis $q\kappa$ let an ellipsis or an hyperbola $qp\kappa$ be described, and let it cut any line PE produced in p; and the conick arch pq will be the image of the perpendicular $P\mathcal{Q}$, formed by rays diverging from it and once refracted at the arch AC.

Fig. 133. 134. 441. When $E\mathcal{Q}$ is equal to EF the ellipsis will become a parabola whose focus is E and vertex κ; and when $E\mathcal{Q}$ is infinite the ellipsis will coincide with the circle whose diameter is $2Ef$ or $f\varphi$ or $\kappa\omega$; which is also the parameter of all the curves.

PROPOSITION I.

Algebraick theorems for finding focus's. *Having the focus of rays falling almost perpendicularly upon a given spherical surface, to find their focus after refractions.*

Fig. 135. 442. Let OI be the given surface whose center is S, and in any radius OS produced let $\mathcal{Q}$ be the given focus of the incident rays as $\mathcal{Q}I$; it is required to find the focus q of the refracted rays. Call OS, $O\mathcal{Q}$, Oq respectively S, $\mathcal{Q}$, q; and let the given ratio of the sines of refraction be m to n, and let m be bigger than n. Then joining SI, since very small angles are very nearly proportionable to their sines, we
* Art. 221. have the ang. OSI : ang. $SI\mathcal{Q}$ :: $\mathcal{Q}$: $\mathcal{Q}-S$*, and the ang. $SI\mathcal{Q}$: ang. SIq :: m : n. And by compounding these proportions we have the ang. OSI : ang. SIq :: $m\mathcal{Q}$: $n\mathcal{Q}-nS$; and
* Eucl. I. 32. disjointly we have the ang. OSI : ang. SqI*,
* Art. 221. 204. that is, $q : S$* :: $m\mathcal{Q}$: $\overline{m-n}$, $\mathcal{Q}+nS$. Whence (putting $\theta = m-n$) we have this first Theorem,

$$q = \frac{m\mathcal{Q}S}{\theta\mathcal{Q}+nS} = \frac{\frac{m}{\theta}S\mathcal{Q}}{\mathcal{Q}+\frac{n}{\theta}S}.$$

443. This is the value of q in the given case where the lines $O\mathcal{Q}$, OS, Oq lye all on the same side of the surface OI; and thence the theorem for q may be easily adapted to any other given case, by considering $O\mathcal{Q}$ as being always affirmative, and by changing the sine of S when OS and $O\mathcal{Q}$ lye on contrary sides of their origin O; and by changing the sign of θ when the sine of incidence is less than the sine of refraction; and lastly by placing Oq the contrary way to $O\mathcal{Q}$ when the value of q comes out negative by the theorem so changed.

444. *Corol.* 1. When the line S becomes infinitely great, the surface OI will become a plane; and then we have $q = \frac{m}{n}\mathcal{Q}$, (because, in the theorem, the limit of the variable ratio of $\frac{m}{\theta}S$ to $\mathcal{Q}+\frac{n}{\theta}S$ is m to n.) Therefore $qO : \mathcal{Q}O :: m : n$, that is, as the sine of incidence, to the sine of refraction; which is the rule in Art. 223.

445. *Corol.* 2. When the line $\mathcal{Q}$ is infinite, let the point q come to t; and then the line q or $t = \frac{m}{\theta}S$, (because, in the theorem, the variable ratio of $\mathcal{Q}$ to $\mathcal{Q}+\frac{n}{\theta}S$ becomes a ratio of equality.) Also when q is infinite and consequently $\mathcal{Q}+\frac{n}{\theta}S = 0$*, let the point $\mathcal{Q}$ * Rem. 442.
come round to T; then the line $-\mathcal{Q}$ or $T = \frac{n}{\theta}S$. But $t-S = (\frac{m}{\theta}S-S=)\ \frac{n}{\theta}S$. And so the continuations St, OT of SO to the principal focus's t, T are equal to each other; and are severally in proportion to SO, as n to θ or $m-n$; which is the rule in Art. 224.

446. *Corol.* 3. In the theorem substitute T and t for their values just now found, and we have $q = \frac{t\mathcal{Q}}{\mathcal{Q}+T}$. Hence $\mathcal{Q}-q = (\mathcal{Q}-\frac{t\mathcal{Q}}{\mathcal{Q}+T} = \frac{\mathcal{Q}+T-t}{\mathcal{Q}+T}\mathcal{Q} =)\ \frac{\mathcal{Q}-S}{\mathcal{Q}+T}\mathcal{Q}$. Therefore the line $\mathcal{Q}T : \mathcal{Q}O :: \mathcal{Q}S : \mathcal{Q}q$, which is the rule in Art. 239.

447. *Corol.* 4. In the Theorem substitute $-m$ Theorem for reflected rays. Fig. 136.
for n, and consequently $2m$ for θ, and we have $q = \frac{\frac{1}{2}S\mathcal{Q}}{\mathcal{Q}-\frac{1}{2}S}$; which theorem gives the focus of rays reflected from the spherical surface OI. For the calculation continues the same whether the ray goes forwards or backwards in the line Iq; and to change the angle of refraction SIq Fig. 135. 136.
into an angle of reflection, it (and its sine n) must be diminished to nothing and then become negative and equal to ($-m$ the sine of) the angle of incidence $SI\mathcal{Q}$.

448. *Corol.* 5. Hence when S becomes infinite, the surface OI will become a plane; and then we have $q = -\mathcal{Q}$; which is the rule in Art. 202. Fig. 136.

449. *Corol.* 6. When $\mathcal{Q}$ is infinite let the point q come to T; then the line q or $T = \frac{1}{2}S$, which is the rule in Art. 205.

450. *Corol.* 7. In the theorem of corol. 4. substitute T for its value found in the last corol.

and

and we have $q = \frac{TQ}{Q-T}$. Hence $q - T = \frac{TQ}{Q-T} - T = \frac{TT}{Q-T}$; that is, TQ, TS, Tq are continual proportionals; which is the rule in Art. 207.

PROPOSITION II.

Having the focus of rays falling almost perpendicularly upon a given lens, to find their focus after refractions.

Fig. 137. 451. Let $OIEo$ be the given lens, whose vertex's are O and o; R the center of the first surface OI; r the center of the second oE; P the given focus of incident rays in the axis $oOrR$ (produced,) and p the focus of the emergent rays required. Let ϖ be their focus after the first refraction at the surface OI, and m to n the ratio of the sines as above, and call Oo, or, OR, OP, op respectively o, r, R, P, p. Then for Q, S, m, n, θ in Theorem 1*, write P, R, m, n, θ, and we have $O\varpi = \frac{mPR}{\theta P + nR}$, to which adding Oo or o, we have $o\varpi$ or $\varpi = \frac{mPR + \theta Po + nRo}{\theta P + nR}$. Again, for Q, S, m, n, θ in Theorem 1*, write $\varpi, r, n, m, -\theta$; and we have $p = \frac{nr\varpi}{-\theta\varpi + mr}$; in which by substituting the value of ϖ, we have

* Rem. 442.

$$p = \frac{mnPRr + n\theta Pro + nnRro}{m\theta Pr - m\theta PR + mnRr - \theta\theta Po - n\theta Ro}.$$

General theorem for any lens.

452. This theorem for a meniscus lens, having its concave surface exposed to P, is easily adapted to a lens of any given form, by conceiving one or both its semidiameters OR, or to decrease or to increase or to become infinite and then negative, till the meniscus acquires the form of the given lens; and by changing the sign of R or r when the semidiameters lye on opposite sides of their surfaces O, o to the focus P; and lastly by placing p on the opposite side of o to P, when its value in the theorem so changed comes out negative.

453. Thus by writing ∞ (which signifies infinite) for R, this theorem is easily adapted to a plano-convex lens, having its first surface plane; and by writing $-R$ for R it is adapted to a double convex lens; and by writing $-R$ for R and ∞ for r, it is adapted to a plano-convex lens whose first surface is convex; and by writing $-R$ for R, and $-r$ for r, and $o+r$ for R it is adapted to a lens of concentrick surfaces, whose first surface is convex: and by writing $R+o$ for r, it is adapted to a lens of concentrick surfaces whose first surface is concave; and by writing ∞ for r it is adapted to a plano-concave lens whose first surface is concave; and by writing $-r$ for r, it is adapted to a double concave; and by writing ∞ for R and $-r$ for r, it is adapted to a plano-concave whose first surface is plane; and lastly it is adapted to a sphere whose semidiameter is R and diameter Oo, by writing $-R$ for R, and R for r, and $2R$ for o; and by substituting given numbers for the ratio of refraction, the bigger for m and the lesser for n, it is adapted to lens's of any given substances.

454. If all the quantities in the theorem be given in magnitude and position except one, this one, whatever it be, may be found by the known rules of algebraick reductions. And thus also the ratio of the sines of incidence and refraction may be found, by substituting an unite either for n or for m, and by seeking the value of the other. I think the first general theorem of this kind was published by Dr. *Halley* * and looked upon as a notable instance of the great use and comprehensiveness of algebraick Theorems. * Phil. Transf. No. 205.

455. *Corol.* 1. Since the thickness of a lens for most optical uses is generally but small in comparison to the semidiameters of its surfaces; and since the accuracy of this theorem depends on the smalness of the angles of incidence and refraction, that is, *cæteris paribus*, on the smalness of the breadth and consequently of the thickness of the lens; the theorem will still be sufficiently accurate though we reject all its terms multipyed by o, as inconsiderably small in comparison to the rest; which being all divided first by m and then by $\theta r - \theta R$, will give

$$p = \frac{nPRr}{\theta Pr - \theta PR + nRr} = \frac{\frac{n}{\theta} \times \frac{Rr}{r-R} \times P}{P + \frac{n}{\theta} \times \frac{Rr}{r-R}}.$$

456. *Corol.* 2. When the line p is infinite, let the point P come to F; then the line P or $F = -\frac{n}{\theta} \times \frac{Rr}{r-R}$: Likewise when the line P is infinite, let the point p come round to f; then the line $-p$ or $f = \frac{n}{\theta} \times \frac{Rr}{r-R}$. This will appear by the reasons given in Rem. 444, 445. Therefore the focal distances OF, of are equal; and $r - R : r :: \frac{n}{\theta} R : F$ or f; which is the rule in Art. 232, because $\frac{n}{\theta} R$ is the Continuation of R mentioned in that article.

457. *Corol.* 3. In the theorem of corol. 1. substitute F for its value found in corol. 2. and we have $p = \frac{-FP}{P-F} = \frac{FP}{F-P}$. Hence $p - P = \frac{FP}{F-P} - P = \frac{PP}{F-P}$; therefore the lines PF, PO, Pp are continual proportionals, of which the

the first and third tend the same way from P; which is the rule in Art. 239.

458. *Corol.* 4. Hence $p+f=\frac{FP}{F-P}+F=\frac{FF}{F-P}$; therefore $PF:FO::of:fp$; which is the rule in Art. 236.

Focus of an oblique pencil considered. Fig. 138.

459. *Corol.* 5. If $\mathcal{Q}$ the focus of incident rays be placed at any small distance from either side of the axis of a lens of any thickness, the focus q of the emergent rays may be found in this manner. Through $\mathcal{Q}$ and R draw a line $\mathcal{Q}RI$ perpendicular to the first surface; and the focus $\varkappa$ after the first refraction will therefore be in some point of it. Again through $\varkappa$ and r drawing $\varkappa rE$ perpendicular to the second surface, the focus q after the second refraction will be in some point of this perpendicular. And thus the successive focus's $\varkappa$ and q might be found by the theorem in Rem. 442.

460. *Corol.* 6. But supposing two focus's P and $\mathcal{Q}$ to be at equal distances from the first surface, their conjugate focus's p and q will be at other equal distances from the second surface very nearly, provided the interval between the focus's P, $\mathcal{Q}$, or rather the angle $PR\mathcal{Q}$, be very small. For since we suppose $OP=I\mathcal{Q}$, we have $O\varpi=I\varkappa$ and consequently $R\varpi=R\varkappa$; and by adding Rr to both, we have $r\varpi=(rR+R\varkappa=)r\varkappa$ very nearly, by reason of the smalness of the angle $\varpi R\varkappa$; and consequently $o\varpi=E\varkappa$ very nearly, and therefore $op=Eq$ very nearly.

461. *Corol.* 7. Therefore a small object $P\mathcal{Q}$ perpendicular to the axis of any lens, will also have its image pq very nearly perpendicular to the axis. Images determined.

462. *Corol.* 8. Hence the small object $P\mathcal{Q}$ will be to its image pq, in the given ratio of $\overline{\theta P+nR},r$ to $\overline{\theta p+nr},R$ very nearly. For joining $\varpi\varkappa$, the figures $PR\mathcal{Q}$, $\varpi R\varkappa$ will be very nearly similar; and also the figures $\varpi r\varkappa$, prq. And since we had $O\mathcal{T}=\frac{n}{\theta}R$*, and $P\mathcal{T}:PO::PR:P\varpi$*, disjointly we have $P\mathcal{T}:\mathcal{T}O::(PR:\varpi R::)\,P\mathcal{Q}:\varpi\varkappa$. Again taking $ot=\frac{n}{\theta}r$, since t will be the focus of parallel rays, after refraction at oE only, that come the contrary way to those which we may suppose to converge towards p, we have also $pt:po::pr:p\varpi$*, and disjointly $ot:pt::(\varpi r:pr::)\,\varpi\varkappa:pq$. And by compounding these two proportions we have $P\mathcal{Q}:pq::(P\mathcal{T}\times to:pt\times\mathcal{T}O::\overline{P+\frac{n}{\theta}R}\times\frac{n}{\theta}r:\overline{p+\frac{n}{\theta}r}\times\frac{n}{\theta}R::)\,\overline{\theta P+nR},r:\overline{\theta p+nr},R$. Here I conclude the algebraic Elements of Opticks.

* Rem. 445.
* Rem. 446.
* Rem. 446.

Upon Chapter 5. Appearances through any number of lenses &c.

Upon Art. 256.

463. Line 16. *Hence it is easy to collect &c.* For we had

$$fA=\frac{OA,a}{OA+a}$$

$$gB=\frac{\overline{fA+AB}\times b}{\overline{fA+AB}+b}=\frac{\overline{OA,a+\overline{OA+a},AB}\times b}{OA,a+\overline{OA+a},AB+\overline{OA+a},b}=\frac{nb}{m}\text{ for shortness.}$$

$$bC=\frac{\overline{gB+BC},c}{gB+BC+c}=\left(\frac{\frac{n}{m}\overline{b+BC},c}{\frac{n}{m}b+BC+c}=\right)\frac{\overline{nb+mBC},c}{nb+mBC+mc}$$

Hence

$$\frac{AB}{fA}=\frac{OA+a}{OA,a}AB$$

$$\frac{BC}{gB}=\frac{m}{nb}BC.$$

$$\frac{CP}{bC}=\frac{nb+mBC+mc}{\overline{nb+mBC},c}CP$$

But

But we had

$$O\Pi = OA \times \overline{1+\frac{AB}{fA}} \times \overline{1+\frac{BC}{gB}} \times \overline{1+\frac{CP}{hC}} \times \text{\&c. in the present article.}$$

Therefore

$$O\Pi' \text{ or } O\beta = OA \times \overline{1+\frac{AB}{fA}} = OA \times \overline{1+\frac{OA+a}{OA,\,a}\,AB} = OA + \frac{OA+a}{a}\,AB = \frac{n}{a}.$$

$$O\Pi'' \text{ or } O\gamma = OA \times \overline{1+\frac{AB}{fA}} \times \overline{1+\frac{BC}{gC}} = \frac{n}{a} \times \overline{1+\frac{BC}{gB}} = \frac{n}{a} \times \overline{1+\frac{m}{nb}BC} = \frac{n}{a} + \frac{m}{ab}BC.$$

$$O\Pi''' \text{ or } O\delta = OA \times \overline{1+\frac{AB}{fA}} \times \overline{1+\frac{BC}{gC}} \times \overline{1+\frac{CP}{hC}} = \overline{\frac{n}{a}+\frac{m}{ab}BC} \times \overline{1+\frac{CP}{bC}} =$$

$$= \frac{nb+mBC}{ab} \times \overline{1+\frac{nb+mBC+mc}{\overline{nb+mBC},\,c}\,CP} = \frac{nb \times mBC}{ab} + \frac{nb+mBC+mc}{abc}\,CP$$

$$= \frac{n}{a} + \frac{m}{ab}BC + \frac{m}{ab}CP + \frac{n}{ac}CP + \frac{m}{abc}BCP.$$

Restore the values of m and n, then

$$O\beta = OA + \frac{OAB}{a} + AB = OB + \frac{OAB}{a}.$$

$$O\gamma = OB + \frac{OAB}{a} + \left(\frac{m}{ab}BC = \right) \frac{OA,\,BC}{b} + \frac{OA,\,AB,\,BC}{ab} + \frac{AB,\,BC}{b} + \frac{OA,\,BC}{a} + BC =$$

$$= OC + \frac{OAC}{a} + \frac{OBC}{b} + \frac{OABC}{ab}.$$

$$O\delta = OC + \frac{OAC}{a} + \frac{OBC}{b} + \frac{OABC}{ab} + \left(\frac{m}{ab}CP + \frac{n}{ac}CP + \frac{m}{abc}BCP = \right) \text{\&c.}$$

Upon ART. 270.

464. The distance Ob is here supposed to be finite in comparison to OP and bP. For if Cb was not finite, the ratio of the angles AOr, Cbt could not be finite. This and the following article may be demonstrated several other ways, which for shortness I omit.

After ART. 280.

465. That noble and beautiful theorem in Art. 249, from which I deduced all these corollaries, was the last invention of that great Mathematician Mr. *Cotes*, just before his death at the age of 32: upon which occasion I am told Sir *Isaac Newton* said, *if* Mr. COTES *had lived we might have known something*. His demonstration of it is likewise so elegant and clear, that it highly deserved to have followed the theorem it self. But since the method of it cannot easily be extended to the like general Theorems which I have given in the next propositions, for the sake of uniformity I was obliged to substitute another in its stead, and to give Mr. *Cotes*'s by it self, as follows. Where the reader need only observe, that what he calls the apparent place of the object, is only the real place of its last image, according to the received opinion and language of Opticians at that time.

PROPOSITION.

" 466. *To find the apparent magnitude, situation, apparent place and degree of distinctness with which an object is seen through any number of glasses of any sort, at any distances from each other and from the eye and object.*

467. Let PM be an object viewed by the eye at O through any number of glasses at A, B, C whose focal distances are the lines a, b, c. The distance OP may be considered as divided by the glasses A, B, C into two parts such as PA, AO; PB, BO; PC, CO; or into three parts such as PA, AB, BO; PA, AC, CO; PB, BC, CO; or into four parts such as PA, AB, BC, CO; and so on as far as the number of glasses permits. All the several products of such corresponding parts applyed respectively to the focal distance, or to the product of the focal distances Fig. 155.

ſtances of the glaſſes which are placed at the point or points of diviſion, will give ſo many ſeveral lines, which muſt be looked upon as negative if there be an odd number of convex glaſſes at the points of diviſion, otherwiſe as affirmative. Let $P\Omega$ be the ſum of PO and thoſe ſeveral lines according to their ſigns, and let $P\Omega$ and PO lye the ſame way if the ſum be affirmative, but contrary ways if the ſum be negative, and Ω will be the point at which the naked eye being placed ſhall ſee the object under the ſame magnitude with which it appears through all the glaſſes the eye being at O, and therefore the apparent magnitude of the object
* Art. 100. will bear the ſame proportion to the true * magnitude, as the diſtance PO bears to the diſtance $P\Omega$.

468. The apparent ſituation of the object is alſo determined by the point Ω. For if Ω and O be placed on the ſame ſide of the object, it will appear erect, otherwiſe inverted.

469. Imagine now the eye to be removed from O to C, ſo that its diſtance from the laſt glaſs may vaniſh, at the ſame time the point Ω will move to another place ω, which may be found as above. Let $O\gamma$ bear the ſame proportion to OC which ΩP bears to $\Omega\omega$, and let the order of the points O, C, γ be the ſame as the order of the points Ω, ω, P, then will γ be the apparent place of the object viewed at O through all the glaſſes.

470. And from the ſituation of this point γ a judgment may be formed of the degree of diſtinctneſs with which the object appears. For the rays flowing from the point P, by paſſing through the glaſſes, are diſpoſed to fall upon the eye in the ſame manner, as if the glaſſes being removed they tended from the point γ when it is before the eye, or towards the point γ when it is behind the eye.

Demonſtration. Fig. 140. 471. For let αr be the firſt image of the object made by the glaſs A; βs its ſecond image made by the glaſs B; γt its third image made by the glaſs C. It is evident that the object PM and its image αr will be terminated by the ſame
* Art. 245. lines $PA\alpha$, MAr*; that the image αr and its image βs will be terminated by the ſame lines $\alpha B\beta$, rBs; that the image βs and its image γt will be terminated by the ſame lines $\beta C\gamma$, sCt. Now if the eye be placed at O and γt be the laſt image, it is manifeſt that the object ſeen through all the glaſſes will appear under the ſame angle which is really ſubtended by the
* Art. 105. image γt*. Draw therefore $M\Omega$ parallel to tO making the angle $P\Omega M$ equal to γOt, and the naked eye being placed at Ω ſhall ſee the object PM under the ſame magnitude with which it appears through all the glaſſes, the eye being at O. And conſequently the apparent magnitude ſhall be to the true magnitude, as the angle $P\Omega M$ to the angle POM*, or as the * Art. 108.
diſtance PO to the diſtance $P\Omega$*. Let us at firſt * Art. 222.
ſuppoſe all the glaſſes to be concaves. The diſtance $P\Omega$ will be to PA, as the angle PAM to the angle $P\Omega M$, or as αAr to γOt, or in the compounded ratio of αAr to αBr, βBs to βCs, γCt to γOt, that is in the compounded
ratio of $\alpha A + AB$ to αA*, $\beta B + BC$ to βB, * Art. 221.
$\gamma C + CO$ to γC. Therefore $P\Omega = PA \times \frac{\alpha A + AB}{\alpha A} \times \frac{\beta B + BC}{\beta B} \times \frac{\gamma C + CO}{\gamma C}$ &c; by which theorem the diſtance $P\Omega$ is given ſo ſoon as αA, βB, γC, can be found. Theſe may be found by Art. 239 as follows, $\alpha A = \frac{PA \times a}{PA + a}$, $\beta B = \frac{\overline{\alpha A + AB} \times b}{\alpha A + AB + b}$, $\gamma C = \frac{\overline{\beta B + BC} \times c}{\beta B + BC + c}$.
Whence it is eaſy to conclude * that if the eye * As in
at B views the object PM through one glaſs at Rem. 463.
A, $P\Omega$ will be equal to $PB + \frac{PAB}{a}$; that if the eye at C views the object PM through two glaſſes A and B, $P\Omega$ will be equal to $PC + \frac{PAC}{a} + \frac{PBC}{b} + \frac{PABC}{ab}$; that if the eye at O views the object PM through three glaſſes A, B and C, $P\Omega$ will be equal to $PO + \frac{PAO}{a} + \frac{PBO}{b} + \frac{PCO}{c} + \frac{PABO}{ab} + \frac{PACO}{ac} + \frac{PBCO}{bc} + \frac{PABCO}{abc}$; and ſo forwards continually as the ſolution of the problem directs. Now if any of the glaſſes be convex, the focal diſtances of ſuch glaſſes muſt be looked upon as negative, ſince they are contrary to thoſe of con-
cave glaſſes*. and therefore the terms which * Art. 230.
involve an odd number of convex glaſſes at the points of diviſion muſt be taken as negative. *Q. E. D.* 1°.

472. The point M is ſeen immediately through the glaſſes by the ray tO, which enters the eye at O in a direction parallel to $M\Omega$ by conſtruction. If therefore Ω falls on the ſame ſide of the object as O, the ray tO muſt advance towards the eye from the ſame ſide of the common axis OP as the point M, and conſequently the object will appear through the glaſſes erect. But if Ω falls on the contrary ſide of the object to the point O, the ray tO which advances towards the eye in a direction parallel to ΩM, muſt appear to come from the contrary ſide of the axis to the point M, and conſequently the object will appear through the glaſſes inverted. *Q. E. D.* 2°.

473. By conſtruction the lines $M\Omega$, $M\omega$, MP, $P\Omega$ are reſpectively parallel to the lines tO, tC, $t\gamma$, γO and therefore the figure $M\Omega\omega P$

is

is similar to the figure $tOC\gamma$. Hence it follows that $O\gamma$ the distance of the last image from the eye, or the distance of the apparent place of the object from the eye, is to OC as ΩP is to $\Omega\omega$; and that the order of the points O, C, γ will be the same as the order of the points Ω, ω, P. Q. E. D. 3°.

474. The point γ being the last image of the point P, the rays which flow from P, after they have passed through all the glasses, will flow from or towards the point γ. *Q. E. D.* 4°." So far Mr. *Cotes*.

475. Mr. *Huygens* has demonstrated a good many cases of this general theorem in as many distinct propositions of his Dioptricks*; which is more than I have found in any other author. * Prop. 36. to 47.
But notwithstanding his great invention and accuracy in geometry, he has generally embarrassed his readers with so many formal compositions and resolutions of ratio's, that they cannot have a better idea of the excellency of Mr. *Cotes*'s theorem, than by comparing it with those of this other great Geometer.

Upon Chapter 6. Aberrations of rays from the geometrical focus.

Upon Art. 339.

The density of rays in the focus of a speculum. Fig. 382. Plate 30.

476. Hence supposing the density of the reflected rays in the circle of aberrations to be uniform, it is to the density of the incident rays falling perpendicularly upon a plane AP, as the whole surface of the sphere of which the speculum is a portion, to the area of a circle whose diameter is the versed sine PC of the small arch AC very nearly, and the more exactly as this arch is smaller; supposing also that all the incident rays are reflected.

477. For since the very same rays pass through two circles described by the lines AP and XY turned about EC; their densities in these circles are reciprocally as the circles themselves; that is, the density of the reflected rays, is to the density of the incident rays, as AP^2 to XY^2* or $\frac{1}{16}FG^2$* or $\frac{AP^6}{16\times 4CE^4}$*; that is, putting D for $2CE$, as $4D^4$ to AP^4; that is, as $4D^2$ to PC^2, (because D, AP, PC are very nearly continual proportionals*;) that is, as the area of 4 great circles of the sphere, or the whole surface of the sphere*, to the area of a circle whose diameter is PC very nearly.

* Eucl. XII. 2.
* Art. 339.
* Art. 337.
* Eucl. VI. 8. Cor.
* Archim. de Sph. & Cyl.

478. Therefore the greatest density of the reflected rays is at the focus F, considered as a physical point; and is immensely greater than the density of the incident rays. For the proposition above becomes geometrically exact when AP is infinitely diminished, and XY comes to its limit at F; and the density at F is always the same whether a slender pencil falls upon the speculum or a large one, because the outward rays are reflected wide of the focus F.

Density of rays in the focus of a lens. Fig. 381. Plate 30.

479. In like manner when rays fall parallel upon the plane side of a plano-convex lens, (putting m to n for the ratio of majority of the sines of incidence and refraction) their greatest density at their focus F, is to the density of the incident rays, as the whole surface of a sphere whereof the lens is a portion, to the area of a circle whose diameter is $\frac{mm}{nn}PC$, or in glass $\frac{9}{4}$ of the versed sine of the smallest aperture of the lens; that is immensely great. It follows from Art. 332.

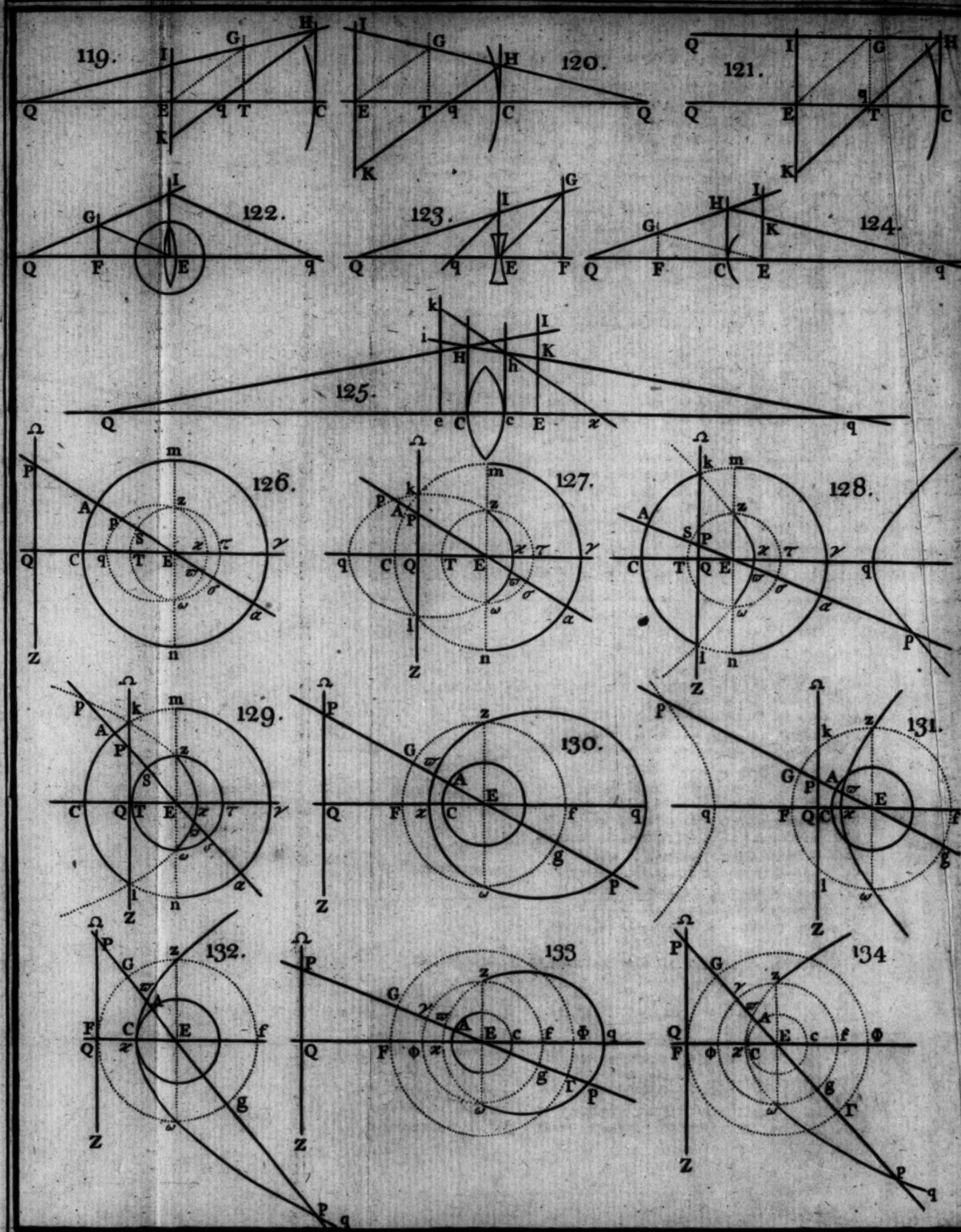
480. Therefore the density of reflected or refracted rays in the several points of an image of a very remote object, is also immensely greater than the density of the incident rays of any one pencil. For it would be immensely great, if all the rays of every pencil were rejected, except a few that go near to their axes, and those outward rays being scattered upon points collateral to each point of the image, help to increase the density of the rays in the whole image.

481. I had almost forgot to acknowledge that I borrowed the first and fourth propositions of this chapter from Sir *Isaac Newton*'s Optical Lectures, Part 1. Sect. 4. where the second is demonstrated by his method of infinite series.

Upon Chapter 7. Of the magnifying powers of Telescopes &c.

Upon Art. 364, 365.

482. When I printed this Table of the magnifying powers of refracting telescopes from Mr. *Huygens*'s Dioptricks, I was not aware of its being calculated by the Editors of that work; who have not allowed them so great a magnifying power as the Author himself intended, or as the best object-glasses now made will admit of. For the Author (in his *Astroscopia Compendiaria*) mentions an object-glass of 34 feet focal distance, which in astronomical observations bore an eye-glass of $2\frac{1}{2}$ inches focal distance*, and consequently magnified 163 times. According to this standard, a telescope of 35 feet ought to magnify 166 times, and of one foot, 28 times; whereas * Art. 909.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whereas the table allows but 118 times to the former and but 20 to the latter. Now $\frac{166}{118}$ or $\frac{28}{20} = 1,4$; by which if you multiply the numbers in the given column of magnifying powers, you will gain a new column, shewing how much those object-glasses ought to magnify if wrought up to the perfection of this standard.

The new apertures and eye-glasses must also be taken in the same proportions to one another as the old ones have in the table; or the eye-glasses may be found by dividing the length of each telescope by its magnifying power. And thus a new table may easily be made for this or any other more perfect standard when offered.

Admirable performance of reflecting telescopes.

483. As an undoubted evidence of the great excellency of reflecting telescopes, I will add, that upon a just comparison, made by Mr. *Bradley Savilian* Prof. of Astron. and the late Dr. *Pound*, of Mr. *Hadley*'s telescope, whose focal distance is not quite 5 feet and $\frac{1}{4}$, with the *Hugenian* object-glass, whose focal distance is 123 feet; they found that the former magnified objects as many times as the latter, and represented them as distinct, though not altogether so clear and bright; which they attributed partly to the difference of their apertures, (that of the *Hugenian* being somewhat the larger) and partly to several little spots in the concave surface of the object-metal which did not admit of a good polish. But, notwithstanding this difference in the brightness of the objects, they were able, with this reflecting telescope, to see whatever they had discovered by the *Hugenian*: particularly the transits of *Jupiter*'s satellites, and their shades, over the disk of *Jupiter*, the black list in *Saturn*'s ring, (an argument of its being double,) and the edge of the shade of Saturn cast on his ring *, and lastly the five satellites of Saturn; in viewing of which, this telescope had the advantage of the *Hugenian* at the time of comparison; which being in summer, and the *Hugenian* telescope being managed without a tube* the twilight hindered them from seeing in this, some of those small objects, which at the same time they could discern with the reflecting telescope. This account was sent by Dr. *Pound* to Dr. *Jurin*. Phil. Transf. N°. 378.

* See Fig. IV. Phil. Transf. N°. 376.

* Art. 892.

484. Notwithstanding this admirable performance of Mr. *Hadley*'s telescope, which magnified between 228 and 230 times, I am well assured that an object-metal of 3 feet and $\frac{1}{4}$ focal distance was wrought by Mr. *Hauksbee* to so great a perfection as to magnify 226 times; and therefore seems to be scarce inferior to Mr. *Hadley*'s of 5 feet $2\frac{1}{2}$ inch focal distance; since with the same eye-glass that gave it this power, it shewed not only the minute parts of the new moon exceedingly distinct, but also the belts of Jupiter and the black list or division of Saturn's ring. For the latter objects it bore an aperture of $3\frac{1}{2}$ or 4 inches; and in cloudy weather shewed land-objects best when the whole surface of the speculum was open, whose breadth was $4\frac{1}{2}$ inches.

I computed my table of the magnifying powers of reflecting telescopes from the module of Mr. *Hadley*'s, using its middle sized eye glass, and aperture, long before I had heard of Mr. *Hauksbee*'s. But if this be taken for a new standard, it follows, from Art. 361, that a speculum of one foot focal distance ought to magnify 93 times, whereas our table allowed it but 60. Now $\frac{93}{60} = 1.55$, and the given column of magnifying powers multiplyied by this number gives a new column, shewing how much the object-metals ought to magnify if wrought up to the perfection of Mr. *Hauksbee*'s. And thus a new table may easily be made for this or any other more perfect standard; taking also the new eye-glasses and apertures in the same ratio's to one another as the old ones have in the present table.

To find by experiment how much a telescope magnifies.

485. The magnifying power of Mr. *Hauksbee*'s telescope was determined by the following experiment made by himself, Mr. *Folkes* and Dr. *Jurin*. Having fixt a paper-circle of one inch diameter upon a wall, at the distance of 2674 inches from the eye-glass of the telescope, they viewed it in the telescope with one eye, while with the other eye naked, they viewed two parallel lines, drawn upon paper, 12 inches asunder, moving them gradually to and fro' till they appeared to touch two opposite points of the circle seen in the telescope; and then the perpendicular distance of the lines from the eye was found to be 132 inches. The telescope being of Sir *Isaac Newton*'s form, the observer was obliged to incline his head and neck in a posture nearly horizontal and parallel to the length of the tube, that his naked eye might see the two lines cross the under side of it.

In this position of the objects, the angle at the eye made by the rays that came from the extremities of the diameter of the one inch circle, was equal to the angle subtended at the other eye by the 12 inch interval of the parallel lines; and therefore the ratio of this angle to that which the said circle would subtend at the naked eye viewing it at the said distance of 2674 inches, is the magnifying power of the telescope; and is compounded of the direct ratio of the subtenses of these angles, and the inverse ratio of the distances of the subtenses from the

the eye, that is, of 12 to 1 and 2674 to 142; which make the ratio of 226 to 1 very nearly.

486. Suppofing a larger paper-circle had been placed at fo great a diftance, that its picture might have been formed by the fpeculum in its principal focus, the telefcope would have magnified it more than our one inch circle, in the ratio of the diftance of this latter circle from the principal focus, to its diftance from the center of the fphere of the fpeculum: becaufe the diameter of the picture of the remoter circle would have been greater, in this ratio, than that of our one inch circle, fuppofing thefe circles to fubtend the fame angle at the center of the fpeculum. But this ratio in the prefent experiment being only 2674 to 2671, gives but an inconfiderable increafe to that of the magnifying power already determined.

487. If this experiment fhould be further fufpected of inaccuracy, becaufe the pictures of the objects upon the two retina's of the obferver's eyes might poffibly be unequal, I will fhew that no inequalities of this kind can affect the conclufion. For let any ftraight object be viewed with both eyes, and fuppofe the vifual angle at one of the eyes to be alfo fubtended by any nearer object. Then it is plain, that the pictures of both objects in this one eye, will be exactly equal to one another, however they may differ from that of the remoter object in the other eye. And the nearer object will then cover the remoter in appearance. But if the magnitude or diftance of the nearer object be fo altered as to alter the vifual angle and picture upon the retina; this will of confequence be perceived by a neceffary alteration of the apparent magnitude of this object. Therefore when the apparent magnitudes of the two objects were equal, the vifual angles were alfo equal whether the pictures upon the two retina's were equal or not. The application of this hypothefis to the experiment with the telefcope I think is fufficiently obvious.

Method of examining the goodnefs of telefcopes.

488. Thus we have an eafy and accurate method of examining the goodnefs of a telefcope of any kind. Firft by giving it the leaft eye-glafs that will fhew the new moon, or rather Jupiter and Saturn with fufficient light and diftinctnefs when the air is quiet and pure; and then by finding how much it magnifies by the method above mentioned; by which it will appear how near it approaches to the perfection of thofe ftandards. But if feveral telefcopes of the fame kind have nearly the fame length, or the fame magnifying power though of different kinds, thofe are the beft in their kind with which you can read a given print at the greateft diftance. Experiments of this kind if made publick, would be very ufeful to the buyers of telefcopes, and would excite workmen to excell one another: fome few are mentioned, in the next remark.

Telefcopes made with glafs-fpeculums.

489. The method of making reflecting telefcopes with glafs fpeculums quickfilvered over on the convex fide, was firft recommended by Sir *Ifaac Newton* *, and has lately been executed with great fuccefs at *Edinburgh*, as will appear by the following extract of a letter from my honoured friend Mr. *Mac Laurin*, the worthy Profeffor of Mathematicks in that Univerfity.

* Opt. p. 94. 8o.

" Mr. *Short* an ingenious perfon well verfed in the theory and practice of making telefcopes, has improved the reflecting ones fo much, that I am fully fatisfied he has far outdone what has yet been executed in this kind. He has not only fucceeded in giving fo true a figure to his fpeculums of glafs quickfilvered behind, as to make the image from them perfectly diftinct, but has made telefcopes with metal fpeculums, which far furpafs thofe I have feen of any other workman.

He has made fix reflecting telefcopes with glafs fpeculums, three of 15 inches focal diftance and three of nine inches. One of the firft is at prefent in my Lord *Ilay*'s hands; with which it is eafy to read in the philofophical tranfactions at the diftance of 230 feet. Another of them is in the hands of Mr. *Alexander Bayne* our Profeffor of Law; with which he eafily reads the philofophical tranfactions at the diftance of 280 feet. I made fome tryals with one of the fpeculums of nine inches, and can read with it very eafily in the philofophical tranfactions at the diftance of 138 feet; but at that time had not an opportunity to try it at a greater diftance; at another time I read with it a much fmaller print crofs the ftreet at the diftance of 125 feet. It coft him a great deal of trouble to make thefe of a true figure and with parallel furfaces*, and feveral when finifhed were found ufelefs, by reafon of veins that then appeared in the glafs.

* Newt. Opt. p. 94, 95. 8o.

In the glafs fpeculums every thing elfe was very well, only the light was fomewhat faint compared with that reflected from his metal-fpeculums. This I take to have been owing to the fpeculum's not having been well quickfilvered, and partly to the thicknefs of the glafs. For one of them, I obferved, had a brighter reflection when fluid quickfilver was applied to its back furface than after it was foiled.

After he found the light in thefe glafs-fpeculums fainter than he expected, and alfo becaufe of the great difficulty in finifhing them, he applied himfelf to improve the telefcopes with metal fpeculums. By taking care of the figure he finds himfelf able to give them larger apertures than other workmen do; and by adjufting the fpeculums and the whole inftrument he has much improved it. He executes every part himfelf,

ſelf, and takes vaſt pains to make the inſtruments as perfect as poſſible, and has made them of focal diſtances of two inches and ſix tenths, of four inches, of ſix inches, of nine inches, and of fifteen inches. He perforates the large ſpeculum and uſes a concave little ſpeculum. By thoſe of four inches focal diſtance he ſaw the Satellits of Jupiter very well, and read in the philoſophical tranſactions at above 125 feet diſtance. By thoſe of ſix inches focal diſtance he read at 160 feet diſtance. By thoſe of nine inches focal diſtance he read at 220 feet diſtance. By thoſe of fifteen inches focal diſtance he and Mr. *Bayne* have read in the Tranſactions at 500 feet diſtance, and have ſeveral times ſeen the five Satellits of Saturn together, particularly on the 24th of November and the 7th of December laſt; which very much ſurpriſed me till I found that Mr. *Caſſini* had ſometimes ſeen them all with a ſeventeen foot refracting teleſcope.

I have compared ſome of theſe with ſuch as have been brought from *London*; and find one of Mr. *Short*'s* of ſix inches focal diſtance, compared with one of the beſt I have ſeen from *London* of nine inches and three tenths focal diſtance, to exceed it in brightneſs, diſtinctneſs and magnifying power; and when I called an indifferent perſon, who knew not who had made the inſtruments, to give his opinion, he very readily preferr'd that of ſix inches focal diſtance. It alſo manifeſtly exceeded another I had from *London* of eleven inches and a half focal diſtance. The ſame was the reſult of ſome other compariſons.

Upon the whole I am convinced he has much improved this excellent Invention and that his inſtruments are by far the beſt of their lengths that have yet been executed. I am &c.

Edinburgh Dec. 28. 1734.

Colin Mac Laurin.

Upon Chapter 9. Determinations of the focus's of rays very obliquely reflected or refracted at any number of ſurfaces, and of cauſticks.

History of the Subject.

490. Before the diſcovery of the law of refraction, according to the given ratio of the ſines of incidence and refraction of any given magnitudes, opticians could only conſider the refractions of ſuch rays as fell almoſt perpendicularly upon the refracting ſurfaces, where the angles of incidence and refraction, being but ſmall, were known by experience to be nearly in a given ratio to each other; and theſe rays they found would all belong to one focus pretty nearly. But Dr. *Barrow*, obſerving that the ſeveral ſmall portions of a large pencil of rays, flowing from a given focus, would diverge after refraction or reflection from ſeveral different focus's, according as they fell with different obliquities upon the ſeveral parts of a ſpherical ſurface, and being of opinion that the eye receiving a certain ſmall portion of theſe rays would judge the object to appear in the place from which they diverged, and conſequently to appear in different places according as the eye received a different portion; took occaſion from thence to determine theſe places geometrically by means of the law of refraction then newly diſcovered*; and conſequently to handle the ſubject of dioptricks and catoptricks in a more extenſive manner than any writer had then done. The focus's of rays obliquely refracted and reflected have alſo been touched upon by Sir *Iſaac Newton* in his Optical Lectures, in order to determine the diameters and breadths of the rain-bows and to make way for his admirable theorems concerning the ſeparations of heterogeneal rays. Theſe are the principal writers on the ſubject of the preſent chapter, in which I have not only comprehended the chief of their diſcoveries, but alſo have made them much more general; by ſhewing that the relation of the focus's of incident and emergent rays, to the focus's of parallel rays coming contrary ways, is always the ſame, after any number of oblique refractions or reflections, as when a pencil of rays is but once refracted or reflected at the vertex of a ſingle ſurface; as will appear more plainly in the propoſitions themſelves.

* Rem. 2, 3

Upon Art. 412.

Focus of a ſuperficial pencil.

491. Having yet ſaid nothing upon the focus of rays obliquely refracted through a plane ſurface, I will here tranſcribe a very elegant determination of it by Sir *Iſaac Newton**. Let ABD be one of the incident rays that diverge from or converge towards the focus A; and let the perpendicular AH to the refracting ſurface BH, cut the refracted ray EBG in G. To the rays AB, BG draw the perpendiculars HI, HR, and in BG take BF to BA as RG to IA; and F will be the focus of the refracted rays, that go the neareſt to EBG on both ſides of it.

* Lect. Opt. Par. I. §. 3. prop. 8. Fig. 141. 142.

492. The author having given no demonſtration of this conſtruction, I will here deduce one from the propoſition in the preſent article. Let any refracting circle BK, whoſe center is C, touch the refracting line BH in B on its ſide oppoſite to A, and let CD and CE be the ſines of incidence and refraction common to the

 plane

plane and the spherical surface; and putting m to n for the ratio of $CD \times BE$ to $CE \times BD$, if the rays were refracted at the arch BK, by the present article we should have $BF : EF (:: mBA : nDA) :: BA : \frac{n}{m} DA$ or $\frac{n}{m} BA + \frac{n}{m} BD$; and disjointly as $BF : BE :: BA : \frac{n}{m} BA + \frac{n}{m} BD - BA$, or $\frac{n}{m} BD$ when the rays are refracted through the plane BH. For the arch BK will coincide with its tangent BH, when its radius BC becomes infinite, at least before it becomes negative: and then the finite line $\frac{n}{m} BA - BA$ may be rejected from the infinite line $\frac{n}{m} BA + \frac{n}{m} BD - BA$. We have therefore $BF : BA :: mBE : nBD :: CD \times \overline{BE}^2 : CE \times \overline{BD}^2$ (by restoring the values of m and n) $:: \frac{\overline{BE}^2}{CE} : \frac{\overline{BD}^2}{CD} :: \frac{\overline{HK}^2}{BR} : \frac{\overline{HI}^2}{BI}$ (because the triangles BEC and HRB, and also BDC and HIB are similar) $:: RG : IA$, because RB, RH, RG and also IB, IH, IA are continual proportionals *. * Eucl. VI. 8. Cor.

Focus of a solid pencil of oblique rays considered.

493. Upon this occasion Sir *Isaac Newton* makes the following remark. "That F is the focus of those rays only which lye in the plane ABH. For no other refracted rays can cut BF either in F or any where else, excepting those which before refraction lay in a conical surface described by the revolution of AB about the axis AH, and which will all cut the said BF in G where the axis AH cuts it. Therefore the rays that go very near to the ray FBE diverge chiefly from two centers or focus's F and G; the point F being the focus of those that lye in the plane ABH, and G of those that lye in the conical surfaces, described by the lines AB, BG turned about AH. All other rays that surround AB, will be so refracted as to approach the nearest to FB somewhere between F and G. So that with respect to an eye, having the center of its pupil at any point O of the ray BE, the place of the image of the point A ought to be diffused through the whole space FG; or because the space FG is the image of a single point A, its sensible image ought rather to be placed at some single point near the middle of that space, and as it were in the middle of all the rays that diverge from it upon the eye. But an accurate determination of that point, regard being had to all the rays that flowed from A before their incidence on the pupil, is a very difficult problem, unless its solution be rather founded upon some probable hypothesis than upon the rigour of the case. For instance, since the number of rays flowing from G and the adjacent points, seems to be equal to the number of those that flow from F and its adjacent points, the place Z of the image of A ought to be so situated between these limits, that the angle under two rays converging from F and G to any given point of the pupil, may always be nearly bisected by a ray flowing from Z to that same point of the pupil. Upon this hypothesis, taking O for the center of the pupil, the point Z may be found by saying as $OF + OG : OG :: FG : GZ$. For disjointly we shall have $OF : OG :: FZ : GZ$; and therefore supposing three lines to be drawn from the points F, G and Z to any point of the pupil, lying very near to its center O, the angle under the two outward lines will always be very nearly bisected by the intermediate line *." * Eucl. VI. 3.

494. So far Sir *Isaac Newton*, whose doctrine is manifestly applicable to rays obliquely refracted through a spherical surface, taking a line drawn through its center and the focus of the incident rays for its axis. In his application of it to the sensible image or apparent place of the point A he follows Dr. *Barrow*'s opinion, upon which see the remarks upon Art 138. and Art. 614.

Upon Art. 477.

495. The focus A of incident rays as AB upon a refracting plane BH being given, Dr. *Barrow* has taught us how to draw the refracted rays as BO very expeditiously *. In the perpendicular AH to the refracting surface, on the same side of it as A, take HL to HA as the sine of incidence to the sine of refraction; and draw LM parallel to BH; then let any incident ray AB cut LM in M; and in the angle BHL inscribe a line BN equal to BM, and the line NBO will be the refracted ray. For by construction the sine of incidence is to the sine of refraction (as HL to HA, or as BM or BN to BA *, that is by Art. 221.) as the sine of the angle BAH, equal to the angle of incidence, to the sine of the angle BNH, which therefore is equal to the angle of refraction. Fig. 143, 144. * Opt. Lect. IV. Sect. 5. * Eucl. VI. 2.

496. When parallel rays fall upon a refracting spherical surface, he has also taught us how to draw any number of refracted rays by this construction *. Let I to R be the ratio of refraction, and through the center C of the refracting circle BCN draw the semidiameter BC parallel to any incident ray MN, and in BC produced take $BZ : CZ :: I : R$; and divide CZ in F by taking $FZ : FC :: I : R$; and with the center F and semidiameter FZ describe a circle ZGE; and through the given points N, C draw Fig 145, 146. * Opt. Lect. XI. Sect. 1.

draw NCG cutting this circle in G; and in the axis CZ take $CK=CG$, and drawing NK it will be the refracted ray. For having joined FG and BG, since $BZ : CZ :: (I : R ::) FZ : FC$ alternately we have $BZ : FZ :: CZ : FC$, and disjointly $BF : FZ :: FZ : FC$. Therefore since the sides of the triangles BFG, GFC, that contain the common angle GFC, are proportionable, those triangles are similar. Hence we have $BG : GF :: GC : CF$ and alternately $BG : GC :: GF : CF$, that is $BG : GC :: FZ : CF :: I : R$. But in the triangles BCG, NCK we have $BC=CN$ and $CG=CK$, and the angle $BCG=NCK$, and consequently $BG : GC :: NK : CK$; therefore $NK : CK :: I : R$. But in the triangle NCK the angle at C is equal to the angle of incidence or to its supplement, and therefore CNK is the angle of refraction or its supplement by Art. 221.

497. Hence while the point of incidence N approaches to B the point K will approach to Z; and therefore every two contiguous rays will cross each other before they cross the axis CZ. For while BN decreases it is plain that CG or CK must increase till they equal CZ. So far from Dr. *Barrow*.

Upon Chapter 11. *Concerning corona's and parhelia.*

History of the Subject.

498. Upon occasion of a corona observed at *Paris*, 12 May 1667, *Hugenius* gave in his thoughts on that subject to the Royal Academy of Sciences, in a discourse afterwards re-published in our Philosophical Transactions *; which I need not transcribe as containing little more than the heads and design of the present treatise. There indeed he gives a hint at the cause of an Anthelion, observed by *Hevelius*, in the intersection of two bright arches crossing one another at oblique angles, which here, I find, he intended to explain at large, but did not finish the treatise; for which reason, and especially as I here give a translation of a latin translation from the original dutch, the reader will excuse me, if, upon a critical examination, he should find I have any where mistaken the author's meaning; for now and then I was a little doubtful of it.

* Abr. by *Lowthorp*. Vol. II. p. 189.

499. I judged it best to throw all the mathematical materials into one view, to be seen in the Appendix *, tho' some few of them were before demonstrated in a different manner in the body of the treatise.

* Art. 579.

Upon Art. 530.

500. Line 34. *I judge the breadth of the corona to be equal to the apparent space through which the colours are spread* &c. The sun's apparent diameter must be added to the apparent space here mentioned, as in the Rain-bow; see Art. 502.

501. Line 42. *Now this happens when there is less plenty of such globules*, that is, whose diameters bear equal ratio's to their opake kernels; for a greater or smaller inequality of those ratio's will more or less dilate the breadth of the corona, so as to produce whiteness by a mixture of the generated colours.

A white bow in the place of the rain bow.

502. *Aug.* 26. 1730. at $7\frac{1}{2}^{h}$. a. m. travelling between *York* and *Tadcaster*, while the sun shone upon a foggy air, I observed a regular arch in the heavens in the usual situation of the primary rain-bow, or rather, as I judged, contracted into an arch of a smaller circle. But it seemed manifestly broader than the inner rainbow and all over of a strong white, which I took to arise partly from the weakness of the sun's rays in passing through the fogg, but chiefly from the extraordinary breadth of the arch; for a like reason to what I have just mentioned about the corona.

Upon Art. 532.

503. *At the end.* These corona's, if their diameters were small, are accounted for by Sir *Isaac Newton* from principles * quite different from these of *Hugenius*.

* Opt. 8o. pag. 288. Obs. 13.

Upon Chapter 12. *Concerning the apparent shapes of objects seen by reflected or refracted rays.*

Upon Art. 598 to 614.

Appearances through plane glasses.

Fig. 147.

504. From this theory of the apparent shapes of objects, it follows that a straight object seen through a glass bounded by parallel plane surfaces, should appear a little concave towards the eye, though scarce sensibly, unless the glass be very thick. For, supposing the same notation as before, let the surfaces of the glass be cut perpendicularly by the plane of the figure in the lines Ar, Bs; then we have AO to Af and also Bg to Bf in the given ratio of the sines that determine the refractions *, and alternately $AO : Bg :: (Af : Bf ::) Ar : Bs$. Which shews that the triangles AOr, Bgs are similar, and consequently that the fictitious * ray gsL emerges parallel to Or or OR; and so does the true ray $S\mathfrak{Q}$ *, which is more refracted both at R and S, as is plain by the position of the aberrations $f\varphi$, $g\gamma$ *. Consequently $S\mathfrak{Q}$ and sL are parallel; and

* Art. 225.

* Art. 614.

* Art. 36.

* Rem. 495.

and therefore PQ, being always less than PL, should appear always concave towards the eye *; but their difference QL, being always equal to δs, whatever be the distance between the glass and object, is so small, unless the glass be very thick, that the apparent curvity of the line $\Pi\varkappa$ may be quite insensible, as we find it is in looking through coach-glasses, whose surfaces are truly plane and parallel.

* Art. 610.

And thro' uneven glass.

505. Therefore the distortions of objects seen through windows made with unwrought glass, is owing to its curvity and uneven thickness. For the smallest angular deviations of the refracted rays from their due course, thence arising, will cause them to fall very wide of the eye placed at a good distance from the window; so that an object cannot be seen through it by such rays as being equally refracted in every part of the glass, come in due order to the eye, but by others, which by coming through improper places of the glass, cause a distorted image on the retina. And since the errors in those places are greatly augmented by the distance of the eye from the window, experience has taught us to go very near to it, in order to see things in their proper shapes.

Fig. 148.

506. It is well known, that a large plane object, as the bottom of a vessel, lying deep under water or any liquor, appears concave towards the eye; which agrees with our theory *. For supposing AR to be the surface of water, the object PQ is less than PL, because the aberration $f\varphi$ tends from the surface AR *; and when the angle AOR is given, QL and $\varkappa\lambda$ are as the depth AP. In the present case of a single refraction at a plane surface, I find the curve $\Pi\varkappa$ is of the third order.

* Art. 610.

* Rem. 495.

Apparent shapes thro' lenses.

507. As a farther confirmation of the truth of this theory, it is observable that a large plane object seen through a deep concave lens appears always convex towards the eye, agreeably to Fig. 513, and the more concave as the object is more remote and the lens at some distance from the eye.

508. Secondly, that a large plane object, seen inverted through a very thick convex lens or a sphere (their properties being alike) appears convex towards the eye placed at some distance from the glass; and concave when it is seen upright, agreeably to Fig. 514, 515. The parallel hairs of a micrometer are another known instance of this last case, as appearing always concave towards the eye and convex towards each other, when opened so wide as to be seen thro' the edges of a thick eye-glass. For conceiving the surface described by the revolution of the curve $\Pi\varkappa$ about its axis $O\Pi$, to be cut by two planes parallel to the axis and passing through the hairs, the sections seen by the naked eye would appear in the shape of the hairs seen through the glass *. If a scruple should arise from the place of the intersection d abovementioned *, it will vanish by supposing the surface AR to be plane instead of spherical. The phænomena of the bars of a sash-window viewed as above through concave and convex glasses are explicable in the same manner.

Fig. 515.

* Art. 599.

* Art. 611.

The shape of an object seen in an elliptical or hyperbolical speculum. Fig. 149. to 156.

509. Let O and f be the foci of an elliptical or hyperbolical speculum represented by AR, and PQ a straight line perpendicular to its axis OPA, in which taking $O\Pi = \frac{AO}{Af} \times fP$ and $\Pi\Omega = \frac{Af}{AO} \times fP$, place them both the same way from O as P lyes from f in the ellipsis, and both the contrary way in the hyperbola; then with the vertex Π and foci O, Ω describe another hyperbola $\Pi\varkappa$, and draw any line fQ cutting the object PQ in Q and the speculum AR in R, and let OR (produced) cut the hyperbola in $\varkappa$; I say the object PQ will appear in the speculum AR, to the eye at O, of the same shape and magnitude and at the same distance, with which the hyperbolick arch $\Pi\varkappa$ would appear to the naked eye at O; and in the same situation too, provided $\Pi\varkappa$ be inverted, when it falls behind the eye, and removed to an equal distance before it.

For, from the known property of an elliptical or hyperbolical speculum AR, *viz.* that the rays which flow from its focus O will all belong to its focus f after reflection, I find, that while the visual angle AOR is gradually varied, the hyperbolick arch $\Pi\varkappa$ is the geometrick place of the intersection $\varkappa$ of the line OR (produced) and the line $Q\varkappa$ drawn parallel to the axis. The truth of the proposition is therefore evident by Art. 599.

510. It is well known, that the hyperbolick arch $\Pi\varkappa$ is concave towards the lesser of the two lines $O\Pi$, $\Pi\Omega$, and that its curvity is greater according to their greater inequality. The position and curvity of the apparent object $\Pi\varkappa$ is therefore known by the construction *, which gives $O\Pi : \Pi\Omega :: \overline{OA}^2 : \overline{Af}^2$.

* Rem. 509.

And in a parabolick speculum. Fig. 151.

511. Hence, if the eye be at O in the focus of a parabolick speculum AR, draw any line QR parallel to its axis OA, and the object PQ will appear in the shape and place of the parabolick arch AR. For, while the vertex A and focus O of the concave hyperbola AR, keep fixt, let the focus f recede to an infinite distance, and the hyperbola's AR, $\Pi\varkappa$ will both be changed into one and the same parabola AR, as is evident by the construction above *.

Fig. 150.

* Rem. 509.

And in a spherical speculum. Fig. 153.

512. Hence also, if the eye be at O in the center of a spherical speculum AR, the object PQ will appear in the same shape and magnitude and at the same distance as it does to the naked

Fig. 152. naked eye at O. For by diminishing the interval Of, of the elliptical foci, to nothing, the ellipsis is changed into a circle and the hyperbola $\Pi\kappa$ into the straight line PQ, by the said construction *.

* Rem. 509, 510. Fig. 149. to 156.

513. Now let O and f be conjugate foci of rays reflected at the vertex A of a spherical speculum Ab, and the straight object PQ, if not too long, will appear to the eye at O nearly of the same shape and magnitude, and at the same distance, with which the hyperbolick arch $\Pi\kappa$ would appear to the naked eye at O. For according as the foci O, f lye on the same or contrary sides of the circular arch Ab, its curvature must be the same at the vertex A with that of an ellipsis or hyperbola that has the same points O, f for its foci: because the circular arch Ab reflects the rays, from the common vertex A, into the same lines as they do, by supposition: and therefore the apparent curvature of the object PQ seen in the spherical speculum will be nearly the same as if it were seen in the elliptical or hyperbolical speculum.

Fig. 148. to 156.

514. But if the straight object PQ be very long, its apparent shape in the spherical speculum will come nearer in most cases to that of a parabolical or elliptical arch Πk, of the same curvature with that of the hyperbola $\Pi\kappa$ at its vertex Π. First, because the hyperbola and ellipsis grow gradually straighter in going from their vertex, and therefore deviate more and more from the invariable curvature of the circle. This being so, let the ray OR (produced) fall upon the circle in b, and, after reflection, upon the object PQ in K, and let Kk, drawn parallel to the axis AO, cut the visual ray Ob (produced) in k. Then, supposing the reflected ray bK (produced) would pass through f, it is plain by the figures, that the intersection k would fall within the hyperbola $\Pi\kappa$; and the aberration of the ray bK from f towards φ, will generally carry the intersection k within the cavity of the hyperbola $\Pi\kappa$; and thus the shape of the apparent object or curve Πk, described by the intersection k, will deviate from the shape of the hyperbola $\Pi\kappa$ towards that of a parabola or an ellipsis.

515. The geometrick places described by the intersection k being curves of the third order, whose shapes are not familiar to us, I chose to compare them with the known shapes of conick sections.

Let us now proceed to determine the apparent shape, magnitude and place of an object seen with both eyes in a spherical speculum.

Phænomena of objects seen in spherical speculums with both eyes.

516. Other things remaining, let a straight object QS be perpendicular to the plane of the figures, and let the curved surface, described by the revolution of the curve Πk round its axis $P\Pi$, be cut in the curve kt by a plane passing through the lines QS, Qk; then drawing the line St parallel to Qk, the object QS will appear in the spherical speculum Ab in the same shape, magnitude and situation to the eye at O, as the section kt would do to the naked eye at O *; provided the section, when it falls behind the eye, be inverted and carried as far before it *, as in Fig. 160. This being premised, thro' the center E of the speculum Ab draw QE and make another angle QEA equal to the adjoining angle QEA, and in the new radius EA take another line EO equal to the given EO, then the phænomena of the object QS viewed with both eyes placed at the points O, O are reducible to the following cases.

Fig. 157. to 160.

* Art. 598.

* Art. 139.

517. *Cas.* 1. If the reflected ray bO and the line QE be not parallel, produce them, if need be, till they meet in q, and if q falls behind the eyes, the object QS will appear *double*, and the appearance to each eye will be the same as above described when the other was shut. We may easily adapt the 158th and 159th figures to this case, by conceiving the object drawn a little farther from the speculum.

518. *Cas.* 2. But if the intersection q falls before the eyes, and their axes be inclined to one another in an angle equal to OqO, the object QS will appear *single* in diverse manners.

519. *Cas.* 3. For, if the line Oq be less than Ok, the object QS will appear to both eyes *nearer and smaller* than to either eye alone. Fig. 157. 160.

520. *Cas.* 4. But if Oq be greater than Ok, the object QS will appear to both eyes *remoter and larger* than to either eye alone. Fig. 158. 159.

521. *Cas.* 5. Lastly, having compleated the rectangle $QStu$, let the line Ou (produced) meet Qq in x, or be parallel to it, and if the line Qx be longer than Qq, as in fig. 157, the object QS will appear *convex* towards both eyes, otherwise *concave*; and whether Qx be longer or shorter than Qq, will be evident from the following considerations.

522. According as the surface described by the revolution of the curve Πk, is convex or concave towards PQ, which was determined above *, its section kt will also be convex or concave towards the object QS, and the line Qu will accordingly be longer or shorter than Qk. Hence, and from the given position of the points Q, O to the subtenses ku, qx of the angle kOu or qOx, it will be evident in fig. 157, where Qu is longer than Qk, that Qx is also longer than Qq; and in fig. 158, 159, where Qu is shorter than Qk, that Qx is also shorter than Qq; but in fig. 160, though Qu is longer than Qk yet Qx is shorter than Qq.

* Rem. 513.

523. Let the the same construction be compleated for the second eye as we made for the first, and from the equality of the angles QEO, QEO and lines EO, EO, it is plain that all other

Demonstration of the phænomena.

other parts of the figures on opposite sides of the line *QE* are equal and similar to one another.

524. 1. Now if the reflected rays *bO, bO*, which came originally from *Q*, tend to meet in *q* behind the eyes, they must needs fall upon points of the two retina's which do not correspond * to each other; because the axes of the eyes produced forwards cannot diverge as the visual rays *Ob, Ob* are supposed to do. Consequently the point *Q* and object *QS* will appear double *.

* Art. 137.

* Art. 137.

525. 2. But if the point *q* falls before the eyes and their axes be directed towards it or any collateral point lying nearly at the same distance from the eyes as *q* does, their axes will be inclined to one another in the angle *OqO* or some other equal to it, and then the rays *bO, bO* will fall upon corresponding points of the retina's, and excite a single appearance of the object *QS* *.

* Art. 137.

A previous Experiment or two explained.

526. In order to explain the rest of the phænomena we must consider an experiment described in Art. 977. and add to it another case or manner of viewing the compasses there mentioned; whose legs being opened to any angle and held by the joint in a plane nearly perpendicular to a plane passing through the axes of the eyes, with their points in this latter plane, let the axis of the right eye be directed steadily towards the point on the left hand, and that of the left eye towards the point on the right hand, then, of the legs that appeared double, the two innermost will soon unite in one, appearing like a third leg in the middle between the other two, and will tend from the joint towards the intersection of the axis of the eyes; and this middle leg, if viewed attentively while the real legs are gradually opened, will decrease in apparent magnitude and approach towards the eyes *; and on the contrary will increase and recede from you while the angle of the legs is contracted. The like phænomenon is observable in viewing two lighted candles of an equal height and thickness at the distance of two or three feet from the eyes; but the apparent candle in the middle does not approach quite so near the eyes as the corresponding leg of the compasses, I mean in proportion to the given distance of the original objects from the eyes. In the figure, *aα* and *bβ* are the diameters of the candles, *d* and *e* the centers of the pupils, *aeα*, and *bdβ* two cones of rays crossing one another in *f*, where the candles appear united in one, of a size proportional to the thickness of the cones at *f* or a little beyond it. Now if the candles *a, b* be gradually drawn asunder, their apparent union near *f*, while the eyes are fixt upon it, will decrease in apparent magnitude and approach towards you. For the similar and equal pictures of the two candles upon corresponding places of the two retina's cause the same sensation as two such pictures of a single candle at *f* would do; and this sensation excites the usual idea of a single candle *.

* Rem. 301.

Fig. 161.

* Rem. 301.

527. And by the way, if the lines *afe, bfd* be drawn upon a plane board a foot or two long, and a pin be stuck upright at their intersection *f*, and the pupils of the eyes be placed near the edge of the board at any small height above the points *d, e*, while the pin is viewed steadily the two lines *fa, fb* will appear united and upright in the place or by the side of the pin. For in this case they cannot appear in two different places * and therefore must appear in the common intersection of two planes passing thro' the lines *af, bf* and the pupils of the eyes.

* Art. 137.

Application of the experiments.

528. 3. 4. By applying what has been said of the points of the compasses to any two points *k, k* or *t, t* of the curves *kt, kt* supposed to be viewed by the naked eyes at *O, O* directed to the point *q*, the reason of the third and fourth phænomena is very evident.

529. 5. Hence also we have the reason of the apparent convexity or concavity of the object *QS*. For the two triangular planes *Out, Out* (produced) will intersect one another in a line *xs* perpendicular to the plane of the scheme, because the lines *tu, tu* are perpendicular to it. And the point *S*, being seen by reflected rays coming to the eyes *O, O* in the same directions *tO, tO* in which they would come from two radiating points *t, t*, to the naked eyes at *O, O*, must appear in the same place in which the radiating points *t, t* would appear united to the naked eyes. And this place of apparent union will be nearer to, or farther from, the eyes than the place of apparent union of the two other points *k, k* to the naked eyes, or the apparent place of *Q* in the speculum, according as the visual angle *OsO* or even *OxO* is greater or less than *OqO*, that is, according as *Qx* is less or greater than *Qq*.

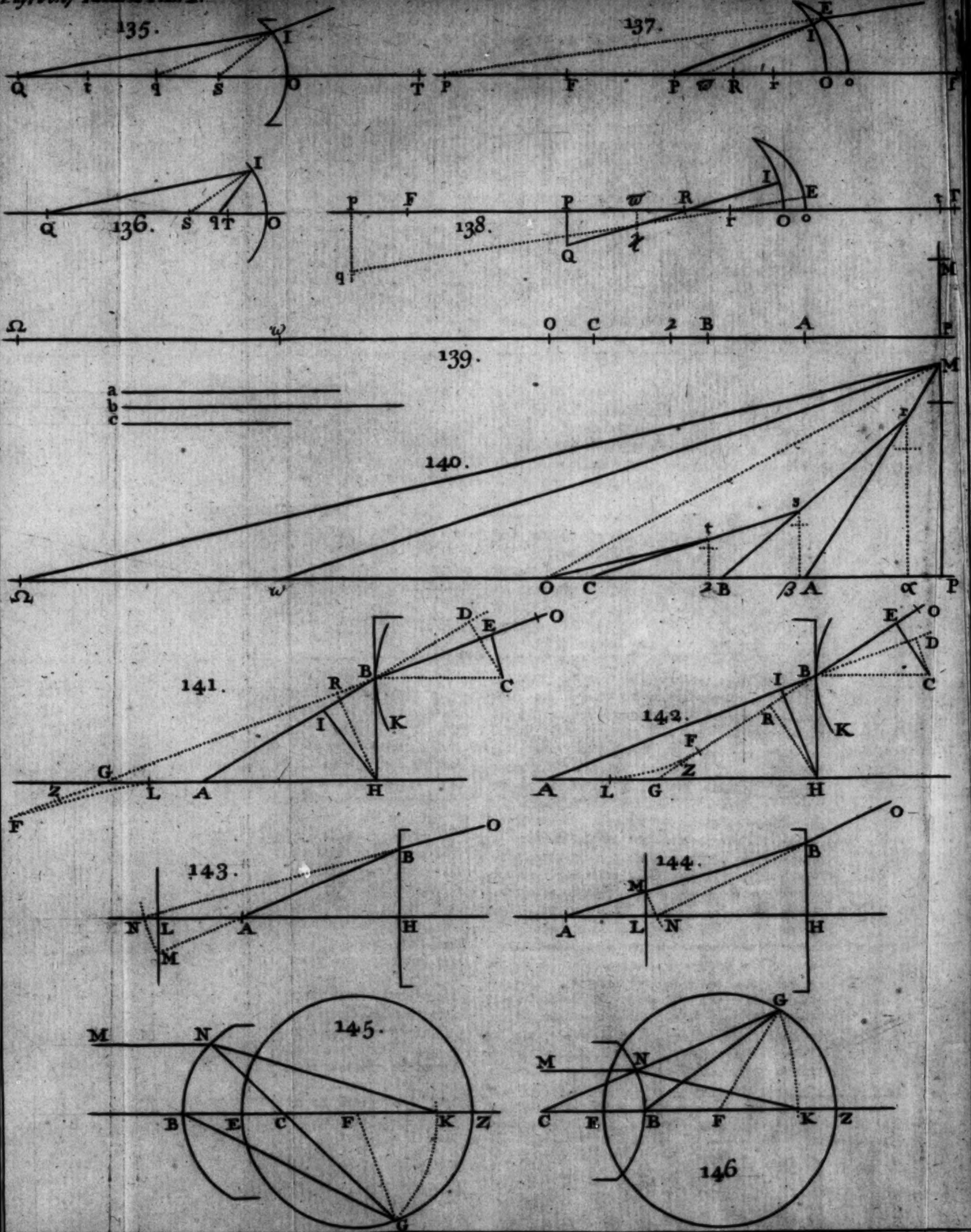
The theory discovers an extraordinary case.

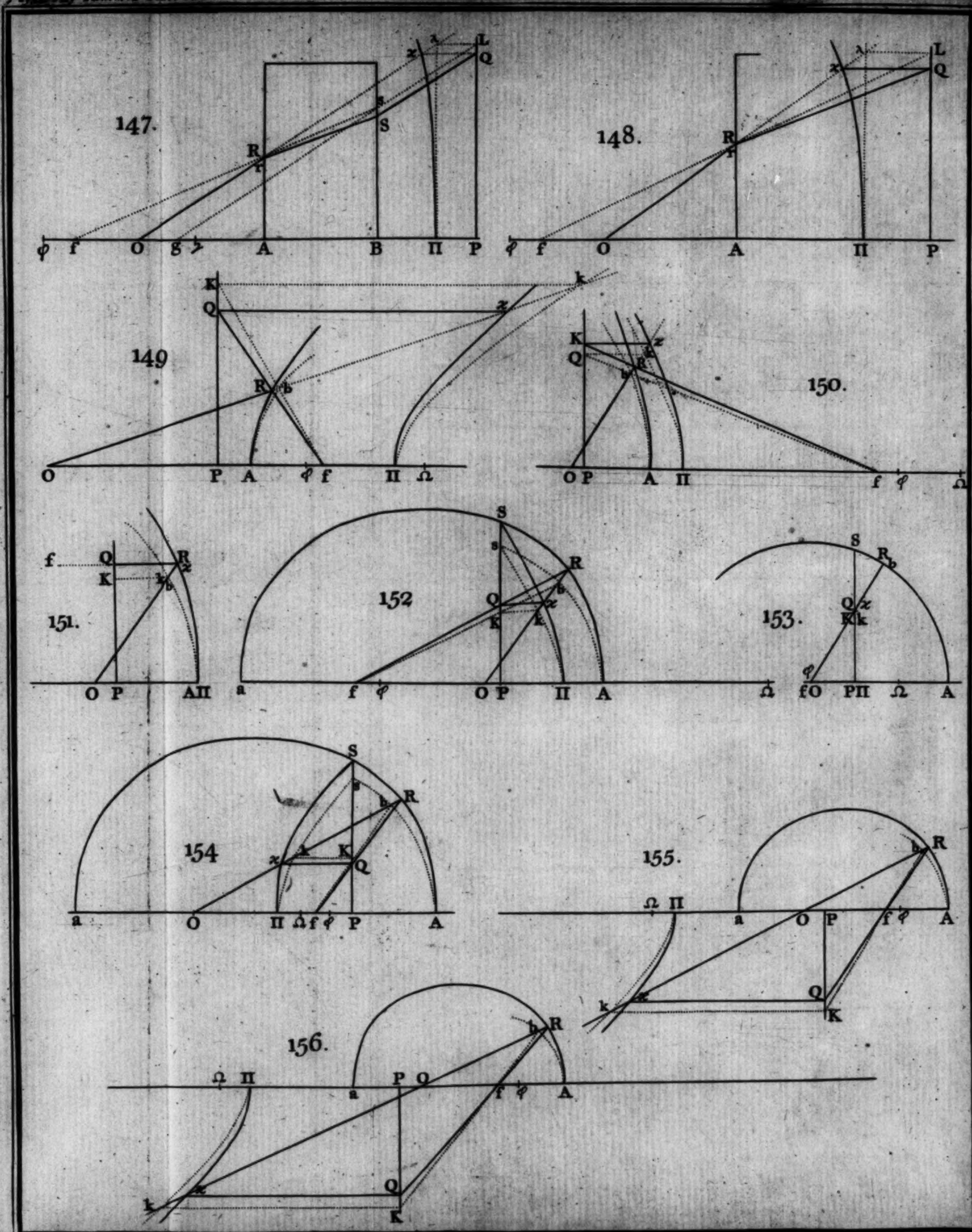
530. In the cases of fig. 159, 160, the object *QS* ought to appear convex to either eye alone and concave to both at once; which change of shape, I was very much surprised with, in viewing a foot ruler held upright between my eyes, with its flat side against my forehead, placed about a foot farther from the speculum than the center of its concavity. In this case the object appears inverted, and accordingly in fig 160, the curves *Πk, Πk* and their sections *kt, kt* are inverted too, and carried along the visual rays *Ob, Ob* as far before the eyes as, by construction, they fell behind them.

Is farther confirmed by experiments.

531. The whole theory may be farther confirmed by forming any two slender equal objects, as two quills stript of their feathers, into two equal curves, to represent the sections *kt, kt*. For by placing them in the several positions

Page 86. of Remarks Plate II.

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tions represented in the figures and by directing the eyes according to the position of the point x, the curvity of the two apparent inner quills, when apparently united, will have the same appearance, as to convexity or concavity and the degree of either, as the straight object QS will have in the speculum; as I have often tryed.

Explains some remarkable phænomena.

532. The 158th figure gives me an opportunity of explaining a phænomenon which Dr. *Barrow* took notice of as somewhat extraordinary*. Viewing your face in a concave speculum held pretty near it, first with your right eye, while the left is shut, observe its apparent place and size. Then do the same with your left eye, while the right is shut, and your face will appear of the same size as before, only moved a little towards the left hand. But upon looking immediately with both eyes, the two apparent faces will seem united in a face much larger, remoter and more concave than before. The reason of which is very evident from the theory above.

* Lect. Opt. Lect. 15. append.

A piece of perspective appears in a concave speculum to great advantage.

533. And for the same reason, a piece of perspective placed a little nearer to a very large concave speculum than its principal focus, as in fig. 158, and viewed by a person standing close behind the piece and looking over the top of it, appears in the glass much larger and more projected, and consequently to much greater advantage than when it is seen by direct rays.

How Dr. Barrow considered this Subject.

534. I do not find that Sir *Isaac Newton* has any where touched upon this subject of apparent shapes, and Mr. *Huygens* has only given us this one observation upon a particular case of it*, "that the apparent curvity of straight "lines often seen about the margin of a lens, "is extremely difficult to be reduced to geometry"; which upon tryal I found so true, that I should scarce have pursued the speculation, but through a desire of having the most rigorous test of the validity and extent of the principle I have all along followed. I will conclude this subject with giving an account of Dr. *Barrow's* manner of considering it*. Let E be the center and $APEpO$ the axis of a large concave speculum AB, PQ a straight object perpendicular to it, QEq the axis of a large pencil of incident rays flowing from Q, qk the caustick formed by the reflected rays, q its cusp, O the pupil of the eye, OkB a tangent to the caustick. Joining QB, the ray which describes it, will be reflected into BkO, and the rays adjoining to it, will enter the pupil diverging from k, which therefore, according to Dr. *Barrow's* Principle*, is the apparent place of the point Q.

* Dioptr. prop. 52. p. 189.

* Lect. Opt. sub finem. Fig. 162.

* Rem. 231.

535. While the axis QEq, of the variable caustick qk, is turning about the center E, let a line OkB turning about O, touch the caustick continually in k; and, in consequence of the said principle, the object PQ will appear in the place and shape of a curve pk described by the motion of the point k.

536. Admiting the principle I see no objection to the consequence drawn from it, but since the principle has no foundation either in reason or experience*, the theory must needs fall with it, even when the image pk is before the eye; and much more when it is behind the eye, or part behind and part before it; which latter case must happen when the object is a little nearer to the speculum than its principal focus. These cases create no difficulty in my theory, but are utterly inexplicable by the Doctor's, and besides that, he gives no account of the appearances to both eyes.

* Rem. 113. &c.

Upon Art. 653.

537. *Hugenius* was the first inventor of the theorem* in this Article, and drew the same consequences from it as in Art. 660 to 665.

* Dioptr. prop. 27.

Upon Art. 741.

A double reflecting microscope of a new invention, theoretically and practically described.

Proposition I.

538. *To compose a microscope with two spherical specula and a convex eye-glass, and to shew its magnifying power.*

539. Between the center E and principal focus T of a concave speculum ABC, whose axis is $EQTC$, place an object PQ; and let the rays flowing from it be reflected from the speculum AB towards an image pq; but before they unite in it, let them be received by a convex speculum abc, and thence be reflected, through a hole BC in the vertex of the concave, to a second image $\pi\kappa$, to be viewed thro' an eye-glass l. Fig. 163.

540. The object may be situated between the specula C, c; or, which is better, between the principal focus t and vertex c of the convex one, a small hole being made in its vertex for the incident rays to pass through.

541. In both cases we have TQ, TE, Tq continual proportionals* in some given ratio, suppose of 1 to n; and also tq, tc, $t\kappa$ continual proportionals in some other given ratio, suppose, of 1 to m. Then if d be the usual distance at which we view minute objects distinctly with the naked eye, and κl the focal distance of the least eye-glass, thro' which the object appears sufficiently bright and distinct, it will be magnified in the ratio of mnd to κl.

* Art. 1075.

542. For the object PQ and its first image pq are terminated on one side by the common axis of the specula, and on the other by a line

line PEp, drawn through the center E of the concave ABC*. Likewise the images pq and $\varpi\varkappa$ are terminated by the common axis and by the line $ep\varpi$, drawn through the center e of the convex abc*. Hence, by the similar triangles $\varpi\varkappa e$, pqe, and also pqE, PQE, we have $\varpi\varkappa : pq :: \varkappa e : qe :: m : 1$*, and $pq : PQ :: qE : QE :: n : 1$*; and consequently $\varpi\varkappa : PQ :: mn : 1$, whence $\varpi\varkappa = mn \times PQ$. Now if $l\varkappa$ be the focal distance of the eye-glass l, the points P, Q, of the object, are seen through it by the rays of two pencils emerging parallel to the lines ϖl, $\varkappa l$ respectively; that is, PQ appears under an angle equal to $\varpi l\varkappa$, which is as $\frac{\varpi\varkappa}{\varkappa l}$* $= \frac{mnPQ}{\varkappa l}$; and to the naked eye at the distance d from PQ, it appears under an angle PoQ which is as $\frac{PQ}{d}$*, and therefore is magnified in the ratio of these angles*, that is of mnd to $\varkappa l$.

* Art. 215.
* Eucl. V. 22.
* Art. 60.
* Art. 60.
* Art. 104.

543. *Corol.* 1. Having the numbers m, n, d, to find an eye-glass which shall cause the microscope to magnify M times in diameter, take $\varkappa l = \frac{mnd}{M}$. For the apparent magnitude is to the true, as $M : 1 :: mnd : \varkappa l$.

544. *Corol.* 2. As soon as the properest numbers m, n and focal distances T, t can be determined, the following rules will give the several parts of the axis of the microscope.

$$t\varkappa = mt$$
$$tq = \frac{1}{m}t$$
$$qc = tc - tq = \frac{m-1}{m}t$$
$$c\varkappa = t\varkappa - tc = \overline{m-1},\, t$$
$$q\varkappa = t\varkappa - tq = \frac{mm-1}{m}t.$$

$$qT = nT$$
$$TQ = \frac{1}{n}T$$
$$qQ = Tq - TQ = \frac{nn-1}{n}T$$
$$qC = qT + TC = \overline{n+1},\, T$$
$$QC = QT + TC = \frac{n+1}{n}T$$
$$Cc = qC - qc = \overline{n+1},\, T - \frac{m-1}{m}t$$
$$C\varkappa = q\varkappa - qC = \frac{mm-1}{m}t - \overline{n+1},\, T.$$

LEMMA.

545. The focus of incident rays upon a spherical speculum being given, to find the exact aberration of any given ray from the conjugate focus.

546. Let E be the center of the speculum AC, T its principal focus, Q and q any two conjugate foci of the nearest rays to the axis $qEQTC$, QA any other incident ray, AV perpendicular to the axis. Towards Q take $EF : EC :: ET : EQ$ and $qR : qE :: VC : VF$, placing qR the contrary way from q to that of VC from C; then will AR be the reflected ray and qR its exact aberration from the focus q.

Fig. 164.

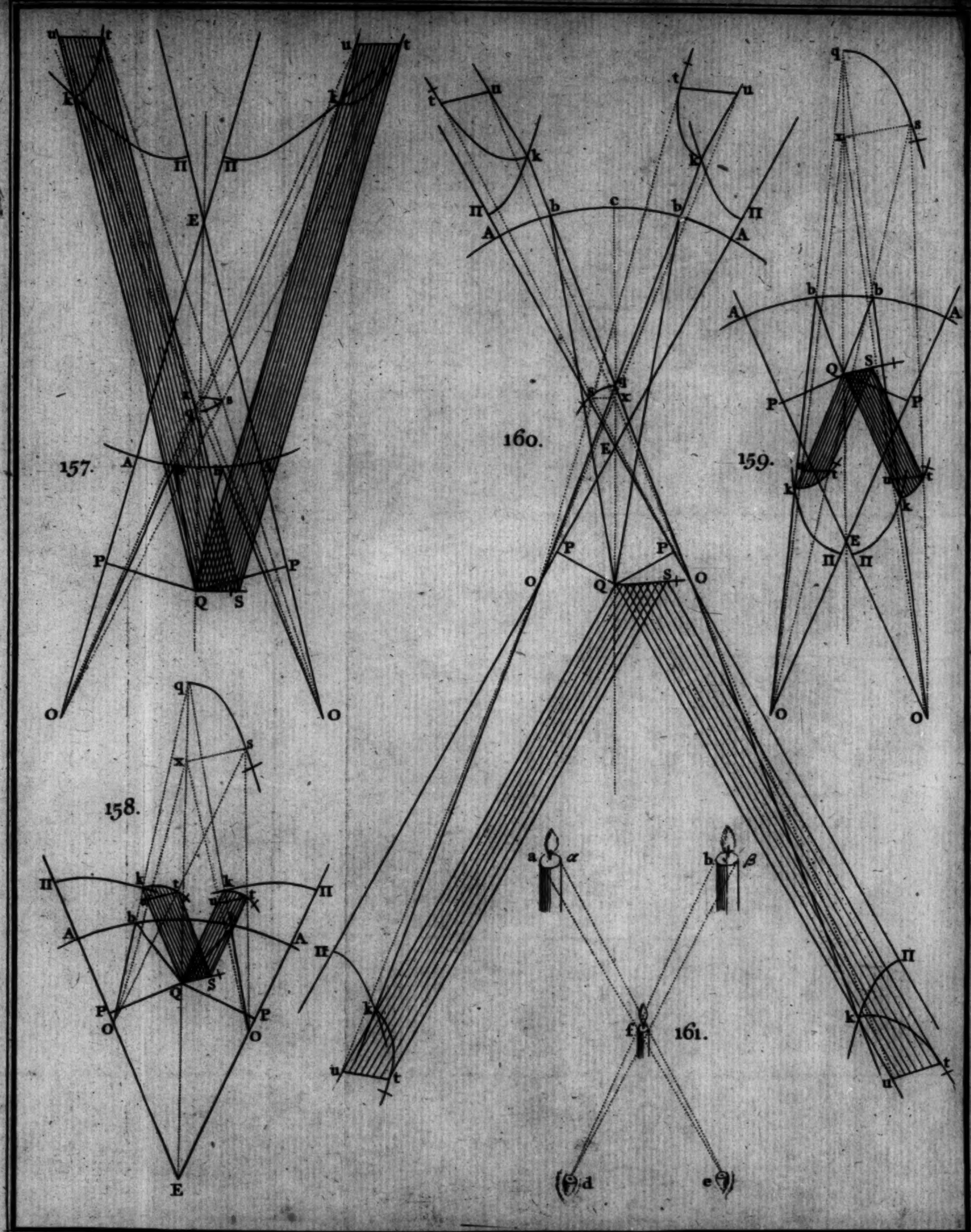
547. For let the incident rays QA, QC, produced, cut the sphere of the speculum again in B and D, and joining EA, EB, let RS drawn parallel to EA, meet QA produced in S. Then we have ER to RQ in a ratio compounded of ER to EQ and EQ to QR, that is, of AS to AQ* and EA to RS, or, because the compound ratio will be the same, of EA to AQ and AS to RS, or EA to AB*. We have then $ER : RQ :: \overline{EA}^2 : (AQ \times AB = AQ \times \overline{AQ + QB} = \overline{AQ}^2 + AQ \times QB = \overline{AQ}^2 + CQ \times QD$* $=$) $\overline{AQ}^2 + \overline{EC}^2 - \overline{EQ}^2$*; and disjointly $ER : EQ :: \overline{EA}^2 : (\overline{AQ}^2 - \overline{EQ}^2$ or) $\overline{EQ}^2 + \overline{EA}^2 - 2EQ \times EV$* $- \overline{EQ}^2 :: \frac{\overline{EA}^2}{2EQ} : \frac{\overline{EA}^2}{2EQ} - EV :: \frac{CE \times ET}{EQ} : \frac{CE \times ET}{EQ} - EV$; that is, by the construction, we have $ER : EQ :: EF : FV$. Which proportion, when the point Q is fixt and A, V are coinciding with C and consequently R with q, becomes $Eq : EQ :: EF : FC$; and the rectangles under the means being the same in both proportions, we have $ER : Eq :: FC : FV$, and disjointly $qR : qE :: VC : VF$; and by the proportion next before this, it appears that ER is less than Eq, and consequently that qR and CV lye contrary ways from q and C, which will easily be found so in all other cases.

* Eucl. VI. 1.
* Eucl. VI. 4.
* Eucl. I. 35.
* Eucl. II. 35.
* Eucl. II. 13.

548. *Corol.* 1. Putting $TQ : TE : Tq \div\!\div 1 : n$, as in prop. 1, TE or $TC = T$, and $CV = V$, in the concave speculum we have the exact aberration $qR = \frac{\overline{n-1}^2,\, T,\, V}{2T + \overline{n-1},\, V}$. For we had $EF : EC :: ET : EQ$*, and disjointly $CF : CE :: TQ : QE :: 1 : n-1$; whence $CF = \frac{2T}{n-1}$,

* Rem. 546.

VF



157.
158.
159.
160.
161.

$VF = \frac{2T}{n-1} + V$ and $qR = \frac{VC}{VF} \times qE = \frac{\overline{n-1}, T, V}{\frac{2T}{n-1} + V}$.

Fig. 165. 549. *Corol.* 2. Likewise supposing κa an incident ray on the convex speculum ac, αar the reflected ray produced, and putting $t\kappa : te : tq \div\div m : 1$, te or $tc = t$ and $cv = v$, we have the exact aberration $qr = \frac{\left.\frac{m+1}{m}\right|^2, t, v.}{2t - \frac{m+1}{m}v}$

For we had $ef : ec :: et : e\kappa$ and disjointly $cf : ce :: t\kappa : \kappa e :: m : m+1$; whence $cf = \frac{2tm}{m+1}$ and $vf = \frac{2tm}{m+1} - v$ and $qr = \frac{vc}{vf} \times qe = \frac{\frac{m+1}{m} t, v}{\frac{2tm}{m+1} - v}$.

550. *Corol.* 3. These aberrations, by dividing an unite by the denominator of each fraction and multiplying the quotients by the numerator of each, may be turned into these series. $qR = \overline{n-1}^2, \frac{V}{2} - \overline{n-1}^3, \frac{VV}{4T} + \overline{n-1}^4, \frac{V^3}{8TT} -$ &c. $qr = \left.\frac{m+1}{m}\right|^2, \frac{v}{2} + \left.\frac{m+1}{m}\right|^3, \frac{vv}{4T} + \left.\frac{m+1}{m}\right|^4, \frac{v^3}{8tt} +$ &c.

Fig. 164. 551. *Corol.* 4. Compleat the rectangle $AVRY$, and while the point A describes the quadrantal arch CAA, the point Y will describe a curve qY, continually convex towards its tangent qX drawn perpendicular to the axis qC. For, we had $qR = \frac{VC}{VF} \times qE$*, in which qE is given, and the ratio $\frac{VC}{VF}$ increases continually. * Rem. 546.

Fig. 165. 552. *Corol.* 5. Likewise if κ be the focus of incident rays, as κa, on the part of the speculum $\kappa ac\kappa$ which is convex towards κ, the curve qy, described by the angular point y of the variable rectangle $avry$, will be continually convex towards its tangent qx, drawn perpendicular to the axis qc. For we had $qr = \frac{vc}{vf} \times qe$*, which increases continually ad infinitum till the point v arrives at f, and then decreases. These two curves belong to the third order. * Rem. 549.

553. *Corol.* 6. When V is very small we have $qR = \overline{n-1}^2 \times \frac{1}{2}V$ very nearly by cor. 3; and consequently the aberration from the principal focus T is $\frac{1}{2}V$ very nearly; and $qR : \frac{1}{2}V :: \overline{n-1}^2 : 1$; that is, if the point of incidence be given and the position of the ray be varied, its aberrations are as the squares of their distances from the center of the speculum very nearly.

PROPOSITION II.

Things remaining as in prop. 1, *if the relation among the focal distances* T, t *and the numbers* m, n *be expressed by this equation* $\mathrm{n}\overline{\mathrm{n}-1}^2, \mathrm{T} = \left.\frac{\mathrm{mm}-1}{\mathrm{mm}}\right|^2, \mathrm{t}$, *the last image of the object will be more distinct than if those quantities* T, t, m, n *were some other ways related.*

554. Any incident ray QA being reflected into the lines Aa, aS, suppose another ray κa flowing from κ, to be reflected into the line $a\alpha$, and let aA and $a\alpha$ be produced backwards till they meet the axis of the specula in R and r; then since the aberrations qR, qr tend the same way from q*, if they were equal to one another, the rays Aa, $a\alpha$, and also aS, $a\kappa$, would coincide; and so the ray QA would be reflected exactly to κ, the focus of the nearest rays to the axis. Fig 166. * Rem. 546.

555. Now the first ratio of the aberrations qR, qr, is expressed by the first terms of the two series found above*, and by supposing those terms equal, we have $\overline{n-1}^2 : \left.\frac{m+1}{m}\right|^2 :: v : V :: \frac{\overline{av}^2}{4t} : \frac{\overline{AV}^2}{4T}$* $:: \frac{\overline{cq}^2}{t} : \frac{\overline{Cq}^2}{T}$* $:: \left.\frac{m-1}{m}\right|^2 t : \overline{n+1}^2, T$*. Whence $n\overline{n-1}^2, T = \left.\frac{mm-1}{mm}\right|^2 t$. Therefore when the relation of m, n, T, t is expressed by that equation, the exact aberrations qR, qr, which set out from q with equal velocities of increase, will continue more nearly equal, during a small increase, than if they had set out with unequal velocities; and since the diminution of Rr (by reason of the equal angles raR, αaA, κaS,) diminishes the aberration κS, the last image $\varpi\kappa$ will be more distinct than if the quantities T, t, m, n were some other ways related. * Rem. 550. ** Art. 264. * Rem. 554.

556. *Corol.* 1. Hence if either of the aberrations qR, qr, belonging to any given semiaperture AV, be made smaller, their difference Rr will also become smaller. For since the first ratio of the subtenses, XY, Xy, of the angles of contact at q, was made the ratio of equality, the

the curves $q\Upsilon$, $c\gamma$ have the same curvature at q; which, if diminished, will diminish any given subtenses $X\Upsilon$, $X\gamma$, and also their difference $\Upsilon\gamma$ or Rr; as is easily understood without farther proof.

557. *Corol.* 2. Consequently, *cæteris paribus*, the appearance of the object will become distincter by diminishing n. For it will become distincter, *cæteris paribus*, by diminishing the aberration $\varkappa S$, which decreases with Rr, which decreases with qR*, which decreases with n, as appears by this expression of $qR = \dfrac{T.V}{\dfrac{2T}{n-1}^2 + \dfrac{V}{n-1}}$*.

* Rem. 556.
* Rem. 548.

Proposition III.

Things remaining as in prop. 2, if the specula have equal focal distances, the use of their innermost parts, which reflect truer than the rest, will be preserved in both.*

* Rem. 546.
Fig. 167.

558. The lines $QAaB\varkappa$ being the course of the outermost ray, let QA produced cut the tangents $cd\alpha$, $C\beta D$ in d and D; join Dq cutting the former tangent in α, and $\alpha\varkappa$ cutting the latter in β, and βq cutting the former in γ.

559. By inspection of the figures it appears that $c\alpha$ and $C\beta$ are somewhat longer than ca and CB, and consequently that a hole at C whose semidiameter is equal to $C\beta$, may be wide enough for collateral pencils to pass thro' to the exterior parts of the image at $\varkappa$.

560. The semidiameter of the hole in the convex speculum must be equal to cd, for cd will differ insensibly from the arch cb cut off by QA, when cQ is small; and since the innermost ray that describes the line $Q\beta$ will be very nearly reflected into $\beta\gamma$, if $c\gamma$ happens to be shorter than cd, this ray will be lost in the hole at c; and so will all those that fell on a zone of the concave, whose breadth $\beta\delta$ is described by the angular motion of the line $\beta\gamma q$, about the point q, till γ coincides with d.

561. On the other hand, if $c\gamma$ happens to be longer than cd, a zone of the convex speculum, whose breadth is γd, will become useless.

562. Therefore to preserve the use of the innermost parts of the specula, which reflect truer than the rest, we must have $c\gamma = cd$; as it will always be if the focal distances of the specula be equal to one another, and consequently $nn - 1 = \dfrac{mm-1}{mm}$*.

* Prop. 2.

563. For the ratio of cd to $c\gamma$, being compounded of cd to CD, CD to $c\alpha$, $c\alpha$ to $C\beta$, $C\beta$ to $c\gamma$, that is, of cQ to CQ, Cq to cq, $c\varkappa$ to $C\varkappa$, Cq to cq, will come out a ratio of equality by putting $T = t = 1$, in the values of these terms*, and $nn - 1 = \dfrac{mm-1}{mm}$, where convenient for the reduction of the compound ratio.

* Rem. 144.

564. *Corol.* 1. When the specula have equal focal distances, the quantity of incident light, is to the part lost in the holes, as 1 to $n - 1 - \dfrac{1}{m}$ nearly. For it is in a duplicate ratio of CD to $C\beta$ nearly, or of $CD \times c\alpha$ to $c\alpha \times C\beta$, that is of $Cq \times c\varkappa$ to $cq \times C\varkappa$, or of $\overline{n+1} \times \overline{m-1}$ to $\dfrac{m-1}{m} \times \overline{\dfrac{mm-1}{m} - n+1}$*, or of 1 to $\dfrac{mm-1}{mm, n+1} - \dfrac{1}{m}$ or $\dfrac{nn-1}{n+1}$* $- \dfrac{1}{m}$ or $n - 1 - \dfrac{1}{m}$.

* Rem. 544.
* Prop. 2.

565. *Corol* 2. Therefore if the angle CQA and consequently the quantity of incident light be given, the part lost in the holes is as $n - 1 - \dfrac{1}{m}$ nearly, and therefore is diminished by diminishing n or m; for they both decrease together; because $nn - 1 = \dfrac{mm-1}{mm} = 1 - \dfrac{1}{mm}$; whence $n = \sqrt{2 - \dfrac{1}{mm}}$.

566. *Corol.* 3. Therefore to save light and improve the apparent distinctness of the object, I have assumed the small numbers 5 and 4 successively for m, and taking $n = \sqrt{2 - \dfrac{1}{mm}}$, have computed the dimensions of the Ist and IId microscopes in the Table, from the rules in cor. 2. prop. 1, taken in the following order: where, putting $T = t = 1$, we have $Cq = n + 1$, $qc = \dfrac{m-1}{m}$, $cC = Cq - qc$; $CQ = \dfrac{1}{n}\, Cq$, $cQ = CQ - Cc$, $c\varkappa = m - 1$, $C\varkappa = c\varkappa - cC$; $lo = \dfrac{c\varkappa + \varkappa l}{c\varkappa} \times \varkappa l$, and the diameter of a hole in a plate next the eye at $o = \dfrac{\varkappa l}{c\varkappa}\, 2c\alpha$.

567. *Corol.* 4. To settle the size of the holes in the metals, it is necessary to assume the ratio of CD to CQ, for which I put 1 to 3 in all the microscopes, judging the resulting angle, CQD, of 18°. 26'. 06", to afford light enough to magnify the object about 300 times, or more, supposing, in prop. 1, the line $d = 8$ inches. Hence by similar triangles we have $c\alpha = \dfrac{cq}{Cq} \times CD$, $C\beta = \dfrac{C\varkappa}{c\varkappa} \times c\alpha$, $c\gamma = \dfrac{cq}{Cq} \times C\beta$ and $cd = \dfrac{cQ}{CQ} \times CD$; the two last, by coming out equal, as they ought*, will verify the calculation.

* Rem. 562.

Reflecting

Reflecting compound microscopes.

Fig. 163. 166. 167.

	I.	II.	III.	IV.
m	5.	4.	5. 7895	5.
n	1. 4	1. 39194	1. 39194	1. 39464
CT	1.	1.	1.	1. 0320
ct	1.	1.	1.	1.
cQ	0. 1143	0. 0765	0. 0626	0. 1007
cC	1. 6	1. 6419	1. 6558	1. 6712
$C\kappa$	2. 4	1. 3581	1. 1337	2. 3288
CD	0. 5714	0. 5725	0. 5728	0. 5906
$c\alpha$	0. 1905	0. 1795	0. 1763	0. 1912
$C\beta$	0. 1143	0. 0813	0. 0716	0. 1113
$c\gamma$	0. 0381	0. 0225	0. 0220	0. 0360
cd	0. 0381	0. 0225	0. 0209	0. 0333
Qg	0. 5143	0. 5763	0. 4545	0. 5104
gh	0. 0343	0. 0222	0. 0189	0. 0323
Mag. power	300 times	300 times	300 times	300 times
κl	0. 1867	0. 1485	0. 1407	0. 1859
lo	0. 1956	0. 1558	0. 1479	0. 1950
diam. hole o	0. 0186	0. 0187	0. 0190	0. 0185
ang. CQD	18°. 26′. 06″	18°. 26′. 06″	18°. 26′. 06″	18°. 26. 06′
ang. $CQ\beta$	3. 48. 53	2. 42. 37	2. 23. 15	2. 35. 40
Incid. light	0. 0513. 180	0. 0513. 180	0. 0513. 180	0. 0513. 180
lost light	22. 155	11. 186	8. 681	19. 671
Residue	0. 0491. 025	0. 0501. 994	0. 0504. 499	0. 0493. 509
ang. AEC	15°. 44′. 41″	15°. 47′. 00″	15°. 47′. 00″	15°. 46′. 13″
ang. aec	5. 13. 57	4. 56. 16	4. 50. 49	5. 15. 12
CV	0. 0750. 396	0. 0754. 058	0. 0754. 058	0. 0752. 819
cv	0. 0083. 348	0. 0074. 226	0. 0071. 526	0. 0084. 008
qR	0. 0059. 144	0. 0057. 076	0. 0057. 076	0. 0057. 791
qr	0. 0060. 312	0. 0058. 259	0. 0057. 388	0. 0060. 792
Rr	0. 0001. 168	0. 0001. 183	0. 0000. 312	0 0003. 001
$\frac{Rr}{nn}$	0. 0000. 596	0. 0000. 611	0. 0000. 161	0. 0001. 543
ang. $\kappa l\lambda$	00°. 02′. 26′	00°. 02′. 29″	00°. 00′. 39″	00° 06′ 15″

568. *Corol.* 5. The quantity of incident light, the part lost in the holes and the residue, are to one another exactly as the versed sines of the angles CQD, $CQ\beta$ and the difference of these sines. For the arches of any given circle, that subtend these angles, being turned about CQ, will describe two segments of a spherical surface, proportional to the incident light and the part lost; and so the residue of light will be as the difference of the segments, or of the versed sines of the generating angles, by *Archimedes*'s Theorems on the Sphere and Cylinder.

PROPOSITION IV.

Having all the dimensions of a reflecting double microscope, to find the angle of aberration at the eye-glass.

569. Let the outermost ray QA, or any other that makes a given angle with the axis, be reflected into Aa and aS, to which last let $\kappa\lambda$ be perpendicular; then joining λl, I call $\kappa l\lambda$ the angle of aberration at the eye-glass, whose sine, to the radius 1, will be $\frac{\kappa\lambda}{\kappa l} = \frac{m}{n} \times \frac{AV}{AQ} \times \frac{Rr}{\kappa l}$ very nearly. Fig. 166.

570. For let ro be perpendicular to RaA, and by the similar triangles $\varkappa\lambda a$, roa, and also roR, AVR, we have $\varkappa\lambda : ro :: a\varkappa : ar ::$
† See below. $e\varkappa : er$† $:: e\varkappa : eq$* $:: m : 1$; and the same ro
* Art. 204. $: rR :: AV : AR$, or in a compound ratio of
* Eucl. VI. 3. $AV : AQ$, and $AQ : AR$ or $EQ : ER$*, or
* Art. 204. $EQ\, Eq$*, or $1 : n$. Therefore $\varkappa\lambda : rR :: m \times AV : n \times AQ$; whence $\varkappa\lambda = \frac{m}{n} \times \frac{AV}{AQ} \times Rr$, and $\frac{\varkappa\lambda}{\varkappa l} = \frac{m}{n} \times \frac{AV}{AQ} \times \frac{Rr}{\varkappa l}$ very nearly.

571. *Corol.* 1. In different microscopes the angles of aberration will be very nearly as $\frac{Rr}{nn}$, if the quantities of light or the angles AQC, and the magnifying powers be the same in all. For then the ratio's of AV to AQ, and of $\varkappa l$ to
* Rem. 543. dnm* and to mn are the same in all.

572. *Corol.* 2. Hence if the residues of light in these microscopes were equal, that would be the best of them in which $\frac{Rr}{nn}$ is the least. For the apparent brightness of the object, being as the residue of light directly and the magnifying power inversely, would be given; and therefore that microscope would be the best which shews the object most distinctly, or in which $\frac{Rr}{nn}$, and consequently the angle of aberration at the eye-glass, is the least; see Art. 344 and 703.

573. *Corol.* 3. Now Rr, being the difference of the aberrations qR, qr, may be thus computed.

Fig. 166. 574. 1. In the triangle QAR, sin. ang. $R = \frac{1}{n}$ sin. ang. Q very nearly, and ang. $\frac{Q+R}{2}$ = ang. E, and CV or $V = \frac{\text{ver. sin. ang. } E}{\text{Rad.}} \times$
* Rem. 548. $2T$, which gives $qR = \frac{\overline{n-1}^2, T, V}{2T + \overline{n-1}, V}$*.

575. 2. In the triangle RaS, sin. ang. $S = \frac{1}{m}$ sin. ang. R very nearly, and ang. $\frac{R-S}{2}$ = ang. e, and cv or $v = \frac{\text{ver. sin. ang } e}{\text{Rad.}} \times 2t$,
* Rem. 549. which gives $qr = \frac{\left.\overline{\frac{m+1}{m}}\right|^2 t, v}{2t - \frac{m+1}{m} v}$*.

576. 1. For in the triangle QAR, sin. ang. R : sin. ang. $Q :: QA : AR$* $:: QE : ER$* $:: QE : Eq$* $:: 1 : n$. * Art. 221. * Eucl. VI. 3.

577. 2. In the triangle RaS, sin. ang. S : sin. ang. R* $:: aR : aS$* $:: eR : eS$* $:: eq : e\varkappa$* $:: 1 : m$, and the ang. Rae = ang. $\frac{R+S}{2}$, and $aeR = (aRa - Rae = R - \frac{R+S}{2} =) \frac{R-S}{2}$. * Art. 204. * Art. 221. * Eucl. VI. 3. * Art. 204.

578. *Corol.* 4. If greater exactness be judged necessary, we may compute Rr by these rules.

579. 1. Put $a = \frac{n-1}{n}$, and θ to 1 for the given ratio of the radius to the tangent of the angle ACQ, then taking $EV = \frac{aT - 2\theta T \sqrt{\overline{1+\theta\theta - \frac{1}{4}aa}}}{1+\theta\theta}$ we have $CE - EV = CV$ or V, which gives qR as above.

580. 2. Put $b = qR$ and $q = \left.\overline{\frac{\overline{n+1}, T - b - V}{V, \overline{4T - V}}}\right|^2$ and $r = \frac{m-1}{m} t - b$, then we have cv or $v = \frac{2qt + r - 2\sqrt{q, \overline{qtt + rt - \frac{1}{4}rr}}}{1+q}$, which gives qr as above.

581. For 1. $TE : EQ :: n : n-1$, whence $EQ = \frac{n-1}{n} T = aT$; put $z = EV$ then $QV = z - aT : VA :: \theta : 1$, whence $VA = \frac{z - aT}{\theta}$. But $\overline{EV}^2 + \overline{VA}^2 = \overline{AE}^2$, that is $\frac{zz - 2aTz + aaTT}{\theta\theta} = 4TT$, which by reduction gives z or EV as above.

582. 2. $RV = qC - qR - CV = \overline{n+1}, T - b - V$, and $Rv = qc - qR - cv = \frac{m-1}{m} t - b - v = r - v$. Again $\overline{AV}^2 = V, \overline{4T - V}$ and $\overline{av}^2 = v, \overline{4t - v}$, and by similar triangles $\frac{\overline{RV}^2}{\overline{AV}^2} = \frac{\overline{Rv}^2}{\overline{av}^2}$, that is, $q = \frac{\overline{r-v}^2}{4v - vv}$, which by reduction gives v as above.

583. *Corol.* 5. Sin. ang. S : sin. ang. $Q :: 1 : mn$, very nearly by cor. 3.

584. *Corol.* 6 Therefore in the IId microscope the semiangle of the pencil at S is but 3°. 15′. 20″, and consequently the eye-glass needs so small an aperture that it cannot sensibly in-

† See Euclid VI. 3. For, by a like demonstration, a line ae, which bisects the supplement of the vertical angle $\varkappa ar$, and cuts the base $\varkappa r$ produced, gives two segments $e\varkappa$, er in the same ratio as the sides $a\varkappa$, ar.

crease-

crease the aberrations caused by the metals; whereas the aperture of the object-glass of a single microscope, to give the same light as ours does, must be sufficient for receiving a radiating angle, of 18°. 26′. 06″, equal to $\mathfrak{Q}$, which is 5, 567 times greater than the angle S. So that here is a great advantage over a single microscope, besides that our image is 5. 567 times greater in diameter than the object and almost 31 times greater in surface.

PROPOSITION V.

To improve these microscopes by diminishing the angle of aberration at the eye-glass.

Fig. 166. 585. The curves qY, qy, according to prop. 2, have the same curvature at q; consequently while CA increases uniformly, tho' Yy or Rr began from q to increase with the least possible velocity, yet since it increases continually with qR, it will become pretty large, and so will the angle of aberration $\kappa l\lambda$, which is as Rr, when the light, the magnifying power, and the number n, are given*. * Rem. 571.

586. But if we retain the curve qY, as given, and alter qy so as to cross qY in any intermediate point z, as represented in Fig. 168, then though Yy will begin at q to increase faster than before, yet it will soon arrive at its *maximum* and then decrease till it vanishes at z, and after that will increase again, but not to so great a magnitude as before, when it increased continually, supposing the semi-aperture AC to be the same in both cases.

Fig. 166. 168. 587. In like manner the angle of aberration $\kappa l\lambda$, which here also is as Rr or Yy, will first increase and then decrease to nothing, and then increase again on the contrary side of the axis κl; and consequently cannot be so great as when it increased continually on the same side of the axis: all which is sufficiently evident from the idea of the curves qY, qy.

588. Let the line $z\alpha$ drawn parallel to the axis, cut the semiaperture AC in α, and compleating the rectangle $z\alpha UR$, join αR, αE, $\alpha\mathfrak{Q}$; then taking the values of n, T and t from the IId microscope as having a little more light in it than the first, and assuming at discretion the value of U or CU, the versed sine of the arch $C\alpha$, we have the corresponding aberration $qR = \frac{\overline{n-1}^2, T, U}{2T + \overline{n-1}, U}$*.

* Rem. 548. Solution. 589. Now to determine a new value of m, which when possible shall make $qr = qR$, put the given $qR = b$, and the given quantity $\frac{\overline{\overline{n+1}, T-b-U}|^2}{U, \overline{4T-U}} = q$; and having found a proper root x of this equation

$q = \frac{\overline{x, \overline{b+xt}, \overline{2t-b}-xt-2bt}|^2}{8bxtt, \overline{b+xt}-4bbtt}$, take $m = \frac{1}{x-1}$ and the business is done.

590. For since $m = \frac{1}{x-1}$ we have $x = \frac{m+1}{m}$ and $qr = \frac{\overline{\frac{m+1}{m}}|^2, t, v}{2t - \frac{m+1}{m}v}$* $= \frac{xxtv}{2t-xv}$ and by * Rem. 549. putting $qr = qR$, that is, $\frac{xxtv}{2t-xv} = b$, we have $v = \frac{2bt}{bx+xxt}$. But $\overline{\alpha U}^2 = 4TU - UU$ and $\overline{av}^2 = 4tv - vv = \frac{8btt}{bx+xxt} - \frac{4bbtt}{\overline{bx+xxt}|^2} = \frac{8btt, \overline{bx+xxt}-4bbtt}{\overline{bx+xxt}|^2}$; and $RU = qC - qR - CU = \overline{n+1}, T - b - U$; and also $Rv = qc - qR - cv = \frac{m-1}{m}t - b - v = 2t - xt - b - \frac{2bt}{bx+xxt} = \frac{\overline{bx+xxt}, \overline{2t-b-xt}-2bt}{bx+xxt}$. Then by similar triangles we have $\frac{\overline{RU}^2}{\overline{\alpha U}^2} = \frac{\overline{Rv}^2}{\overline{av}^2}$, that is $q = \frac{\overline{x, \overline{b+xt}, \overline{2t-b}-xt-2bt}|^2}{8bxtt, \overline{b+xt}-4bbtt}$. Q. E. J.

591. *Corol.* 1. When $T = t = 1$, the equations are somewhat simpler; $qR = \frac{\overline{n-1}^2, U}{2 + \overline{n-1}, U}$, $\frac{\overline{\overline{n+1}-b-U}|^2}{U, \overline{4-U}} = q = \frac{\overline{x, \overline{b+x}, \overline{2-b}-x-2b}|^2}{8bx, \overline{b+x}-4bb}$, $m = \frac{1}{x-1}$.

592. *Example.* If it be desired that the halves of the light of a solid pencil shall fall on opposite sides of the point κ, which probably will produce a very small angle of aberration in the outermost rays; the versed sine of the angle $C\mathfrak{Q}\alpha$ must be half the versed sine of the generating angle $C\mathfrak{Q}A$ of the given pencil, by *Archimedes*'s Theorems abovementioned*. Therefore taking this ang. $C\mathfrak{Q}A = 18°. 26′. 06″$, as before, by a table of versed sines we have the ang. $C\mathfrak{Q}\alpha = 13°. 00′. 26″$, and thence $\frac{1}{n}$ sin. ang. $C\mathfrak{Q}\alpha$ = sin. ang. $CR\alpha$*, which, when $n = 1. 39194$, gives $CR\alpha = 9°. 18′. 21″$, and Fig. 168. * Rem. 568. * Art. 322.

the

the ang. $\frac{Q+R}{2}$ = ang. $CE\alpha = 11°.09'.23''\frac{1}{2}$, whose versed sine, to the radius $CE=2$, is $0.0377948 = U$. Hence $b = 0.0028811$ and $q = 29.340$.

593. Now because the difference Rr, of the aberrations belonging to the outermost ray QA, is very small, and that which belongs to the ray $Q\alpha$ is smaller; in order to find such a value of x as shall cause the ray $Q\alpha$ to have no aberration from κ, at first we may take $m=4$ as in the given microscope, and consequently $x = \frac{m+1}{m} = 1.25$ and thence approach towards the required value of x by correcting the assumed one as follows.

594. Putting $y = x, \overline{b+x}, \overline{2-b-x} - 2b$ and $z = 8bx, \overline{b+x} - 4bb$ the equation to be resolved has this form $q = \frac{yy}{z}$* or $yy - qz = 0$. Suppose the computed $yy - qz = p$, and if p comes out very small, the value of x was nearly right; if not, compute $\dot{y} = \overline{2-b-x}, \overline{b+2x} - x, \overline{b+x}$ and $\dot{z} = 8b, \overline{b+2x}$, and $2y\dot{y} - q\dot{z} = \dot{p}$; then $x - \frac{p}{\dot{p}}$ will be a correcter value of x, to be used in repeating the work over again till p comes out as small as you please. This method of resolving the equation I take for granted, and thus I found $x - \frac{p}{\dot{p}} = 1.56444$ = the new x', and therefore $m' = \frac{1}{0.56444} = 1.7717$; but this value of m being so small as to produce a negative value of $cQ = qc - qQ = 1 - \frac{1}{m} - n - \frac{1}{n}$* $= -0.098$, I therefore re-assumed at discretion a smaller value of $U = 0.03230$ and by repeating the same work I found another new $x'' = 1.263891$ and $m'' = 3.7895$, from which I computed the dimensions of the IIId microscope.

* Rem. 591.

* Rem. 544.

595. Now this latter value of $U = 0,03230$ is the versed sine $10°.19'$ to the radius $CE = 2$; and so the conical periphery of incident rays (described by the ray $Q\alpha$ turned about the axis QC,) that will be collected accurately to κ, lyes nearer to the axis than the periphery which bisects the whole pencil by $50'.23''\frac{1}{2}$*. In this I fully satisfied my self by calculating, from the 2d theorem in cor. 4. prop. 4, the corresponding values of $v' = 0,0030.805$, $qR' = 0,0024.6531$ and $qr = 0,0024.6524$, whose difference Rr', $= 0,0000.0007$, is quite inconsiderable.

* Rem. 592.

596. The semi-angle of the radiating pencil, $CQA = 18°.26'.06'$, will afford light enough for magnifying the object about 300 times in diameter, as I collected from an estimate taken from the best dioptrick microscopes, and was afterwards confirmed in it by an actual essay of the IVth double reflecting microscope, hereafter described. An eye glass whose focal distance κl is $= 0.1407$ of an inch, will give the IIId microscope that magnifying power*; and hence the angle of aberration $\kappa l \lambda = 39''$, which is far too small to cause a perceptible indistinctness, especially in that degree of brightness with which the object will appear. For in broad day-light, scarce one person in a hundred can discern an object, with the naked eye, that subtends a smaller angle than a minute*. Besides, it is well known that the best dioptrick microscopes do usually bear an angle of aberration at the eye-glass of 15 or 20 minutes without much inconvenience.

* Rem. 543.

* Art. 97.

The result of the whole theory.

597. Therefore if the IIId microscope be well executed, we have great reason to expect it will bear a much larger angle of aberration than $39''$ without inconvenience, and consequently a larger aperture or angle of the radiating pencil, in order to give light enough for magnifying much more than 300 times: whereas I find that the best dioptrick double microscopes now in use, do not magnify much above 200 times with sufficient light and distinctness.

Proposition VI.

The place of the object and the length of the microscope being given, to find its other dimensions agreeably to prop. 2.

598. What I call the length of the microscope is $c\kappa = \overline{m-1} \times t$, which, by assuming t, gives m. The given place of the object gives the ratio of tQ to tc, for which putting r to 1 we have the number r. Then putting s for $\overline{\frac{mm-1}{mm}}\big|^2$ find the root n of this equation $n^3 - n - \frac{s}{r - \frac{1}{m}} = 0$; and taking $T = \frac{s \times t}{\overline{nn-1}\big|^2}$ compute all the other dimensions by the rules above*.

* Rem. 544.

599. For by prop. 2, we have $T = \frac{s \times t}{\overline{nn-1}\big|^2}$ and since $tQ = tq + qQ$, that is, $r \times t = \frac{1}{m}t + \frac{nn-1}{n}T$*, by substituting for T its value and

* Rem. 544.

and by reduction, we have the equation $n^3 - n - \frac{s}{r - \frac{1}{m}} = o$.

The reflecting microscopes preferable to refracting ones.

600. I computed the dimensions of the IVth microscope, by the rule in this proposition, some years before I had thoroughly considered the foregoing method by equal speculums; and having caused an essay to be made of it, which I have now by me, I found it performed very nearly as well in all respects as the very best refracting microscopes, and do not doubt but it might have excelled them had it been more exactly executed according to the proposed dimensions in the IVth column; where the angle of aberration, being but 6′. 15″, is above three times less than the like angle in the best refracting microscopes*: and since the angle of aberration of 39″ in the IIId column is near ten times less than that of the IV, we have reason to expect that a microscope exactly executed according to those dimensions, will far excel any other yet invented; for the essay-microscope wants distinctness rather than light. I will therefore conclude with some practical directions to the workman.

* Hugenii Dioptr. prop. 64.

Some directions for making and managing this microscope. Fig. 169.

601. Supposing the construction described in prop. 1, let the object be included between two little round plates of moscovy-glass, fixt as usual in a hole of an oblong brass plate mn, intended to slide close to the backside of the convex speculum; which therefore must be ground flat on that side, and so thin that the object may come precisely to its computed distance from the vertex of the speculum. The exactest way to determine this small distance is, first to fix the speculums and the eye-glass precisely at their computed distances from one another; and then having made the sliding plate full thick enough at first, to file it gradually thinner and thinner, till, upon its application to the metal, you find the object appears perfectly distinct. The slider must be kept tight to the back of the metal by a gentle spring. The distance of the object being thus determined once for all, at other times distinct vision to different sorts of eyes, and thro' different eye-glasses, must be procured by a gentle motion of the little tubes that contain these glasses. These tubes must be made in the usual form of those that belong to Sir *Isaac Newton*'s reflecting telescope, having a small hole in the middle of each plate, at the ends of the tube, situated exactly in each focus of the glass; the use of these holes and plates is to limit the visible area and hinder any stragling rays from entering the eye.

602. One way of procuring distinct vision by a screw-motion of the eye-glass, while the object is viewed, may be easily understood by the figure. The tube of the eye-glass is screwed into a ring or collar pq, having a handle qr, in which there is a hole at r; the end of the rod rs turns round in this hole without sliding endways, and its other end turns and slides endways in a hole s, while the middle part t works in a hollow screw t, made in the handle tu of another collar ux fixt to the tube of the microscope. The knob for turning the rod is at y. Different eye-glasses must have different tubes.

603. The rays which flow from the object directly through the hole in the concave speculum and through the eye-glass, by mixing with the reflected rays, would dilute the image upon the retina, and therefore must be intercepted as follows. Let the little hole in the convex speculum be ground conical, with a conical tool or bit whose semi-angle is rather less than the angle $C\mathfrak{Q}D$, till the diameter of the larger orifice, on the polished side, be exactly equal to its computed measure $2cd$, and then the narrower orifice, on the backside will be rather wider than a section of the principal pencil, made by the plane of the backside; an exceeding small excess above that section will give room enough for the passage of collateral pencils equal to the principal one. Now let the first and last parts $\mathfrak{Q}\beta$, $\gamma\varkappa$ of the innermost ray $\mathfrak{Q}\beta\gamma\varkappa$, cross each other in b, and let bg, drawn perpendicular to the axis, be the semidiameter of the base of a conical solid; this base being larger than the orifice in the back of the convex speculum, will intercept all direct rays from the eye-glass. All the tubes must be strongly blacked on their insides, and so must the conical solid to hinder its reflection of rays upon the convex speculum; its little base may be made concave, having the object $\mathfrak{Q}$ for the center of its cavity, that whatever light it may still reflect, may be thrown back upon the object, and its backside being conical and blacked all over, will either absorbe or laterally disperse any stragling rays which the concave speculum may scatter upon it, and so prevent their coming to the eye-glass. This conical solid may be held in its proper place by a very thin arm, like a narrow blade of a knife, whose edge is turned towards the object.

Fig. 167, 169.

604. Notwithstanding the interposition of this conical solid, yet when the eye glass is taken out, you may see distant objects distinctly through the microscope, by rays reflected from the metals and diverging upon the eye from an image behind the convex speculum. But this mixture of foreign rays with those of the object is common to all kinds of microscopes, in viewing transparent objects, and is usually prevented by placing before the object a thick double convex lens, to collect the sky-light, exactly upon the object. This lens should be just so broad as to subtend the opposite angle to that which Fig. 169.

the

which the concave speculum subtends at the object. The annular frame of the lens must be very narrow, and connected to the microscope by two or three slender wires or blades whose planes produced may pass through the object, and intercept from it as little sky-light as possible.

605. A whitish grey sky-light is the best for illuminating microscopical objects, and the due quantity of it may be found by holding the microscope at different distances from the window, or if this light be too faint, by going without doors and giving the microscope different elevations, that more or less of the sky may be exposed to the object.

Fig. 167.

606. The rule for computing the place and breadth of the conical solid is this. $\mathfrak{Q}g = \frac{\mathfrak{Q}\kappa}{1+mn}$, $gb = \frac{\text{sin. ang. } C\mathfrak{Q}\beta}{\text{Rad.}} \times \mathfrak{Q}g$. For we had $1 : mn ::$ sin. ang. κ : sin. ang. $\mathfrak{Q}$* :: $\mathfrak{Q}b : b\kappa$*, and conjointly $1+mn : 1 :: \mathfrak{Q}\kappa : \mathfrak{Q}g$ when the angles are vanishing. Then in the triangle $\mathfrak{Q}gb$, we have $gb : g\mathfrak{Q}$ :: tang. ang. $g\mathfrak{Q}b$: Radius. The light stopt by this solid is an inconsiderable part of the whole pencil. For if it could possibly be conveyed to the eye, in the IVth microscope it would only suffice to increase the apparent diameter of the object in the ratio of 51 to 52. In the third microscope the light lost is not half so much as in the IV, and therefore would not serve to increase the apparent magnitude in the ratio of 100 to 101 with the same degree of brightness. And Mr. *Huygens*, speaking of Sir *Isaac Newton*'s telescope, says *, "that by the meer reflection of the metalline speculum, there are not so many rays lost as in glasses; which reflect a considerable quantity by each of their surfaces, and besides intercept many of them by the obscurity of the matter."

* Rem. 583.
* Art. 221.

Phil. Transf. Abr. Vol. I. p. 199.

Rule for the measures upon the axis

607. Extraordinary care must be taken to get two speculums exactly spherical and exactly of the same sphere. The least deflection of their figures, even towards those of the conick sections, though advantageous in telescopes, would here have a contrary effect, as being contrary to the design of the present theory. That the angle of aberration may be the same as in the Table, the focal distance of each speculum should be an inch, but since it is difficult to figure the specula so exactly to an inch focal distance, as to suit precisely with the other measures calculated from it; when several concave and convex specula are finished by the same tools, first examine their several focal distances, by analogy of the methods described in Art 63 and 64, if no better occurs, and measure them by a diagonal scale of inches and decimals, and couple each concave and convex whose focal distances come the nearest to equality: then, by an arithmetical mean between the two focal distances multiply the measures in the axis of the IIId microscope, *viz.* 0.0626, 1.6585 1.1337, 0.4545 for obtaining the measures of $c\mathfrak{Q}$, cC, $C\kappa$, $\mathfrak{Q}g$ suitable to the present specula. The rule is eivdent by cor. 2. prop. 1.

Rule for the apertures and holes.

608. The sum of these new measures of $c\mathfrak{Q}$ and cC gives the new $C\mathfrak{Q}$, and $\frac{1}{3}$ of it the new CD, which being divided by the given $CD = 0.5728$, gives a quotient $\mathfrak{Q}$, which being multiplied into the given measures of $c\alpha$, $C\beta$, $c\gamma$, cd, gb, in the IIId column, gives the new ones: So that the new microscope, being similar to the old one in all its parts, will have exactly the same magnifying power* and light, and nearly the same distinctness* as before, when used with the same eye-glass.

* Rem. 541.
* Rem. 571.

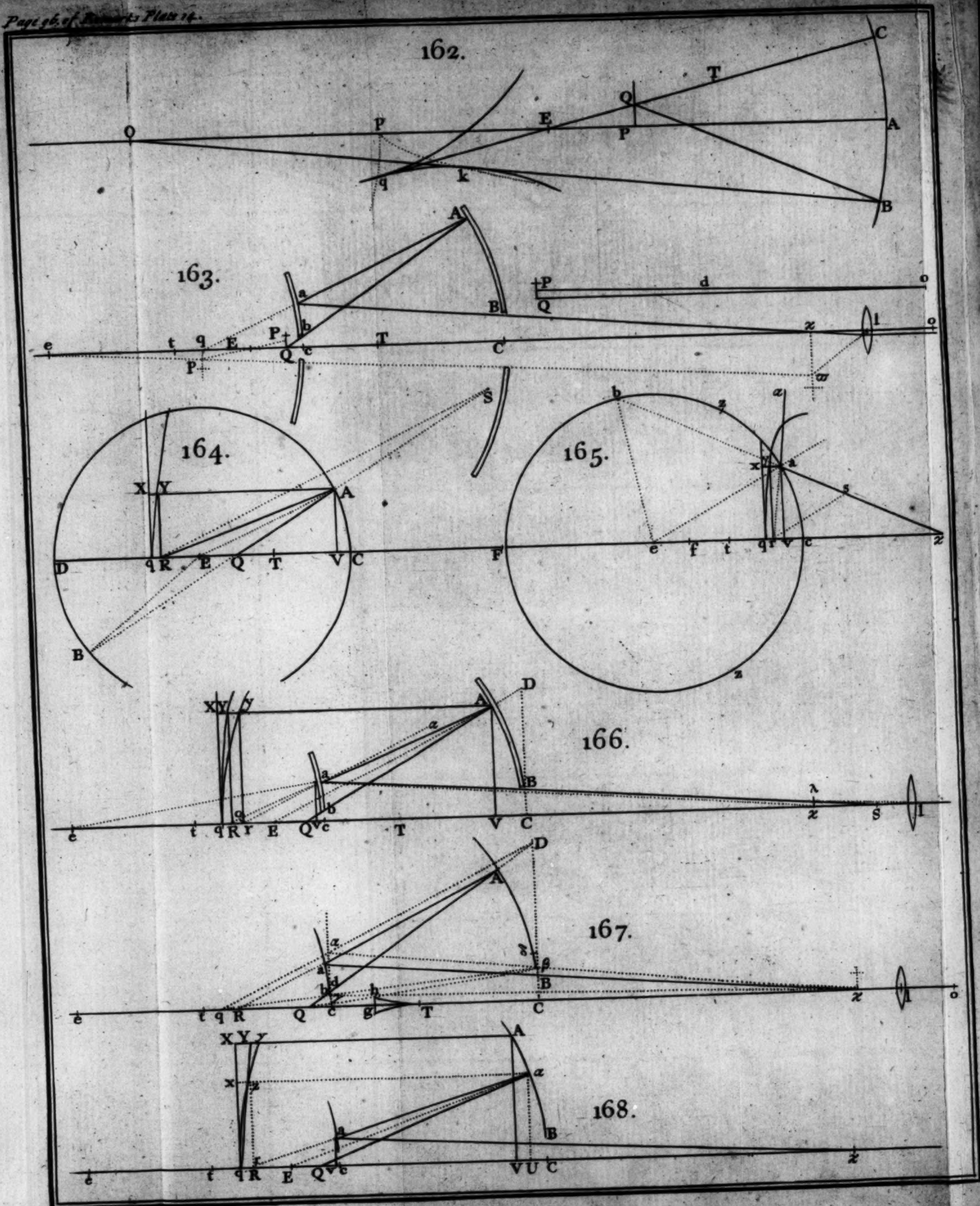
609. The concave speculum must have an annular aperture of thin brass blacked all over; the semidiameter AV, of the hole, may be found by numb. 1. cor 3. prop. 3; or with less trouble by drawing the figure by a scale.

610. If in magnifying much the object should appear very distinct but not bright enough, multiply all the the new transverse measures CD, $c\alpha$, $C\beta$, $c\gamma$, cd, gb, by some small number as 1.05 or 1 1, and enlarge the apertures, the base of the conical solid and the holes in the metals according to the resulting numbers. And if upon tryal the microscope will bear a greater enlargement of those measures, repeat the same operation. Because, if the holes in the metals be made too large at first, some of the innermost and best rays will be lost in them. It may not be improper to have an aperture upon the convex speculum, lest the uncovered annulus should reflect any scattered rays to the eye,

Sir *Isaac Newton*'s thoughts upon telescopes and microscopes.

611. In looking over Sir *Isaac Newton*'s and Mr. *Ja. Gregorie*'s letters, published * soon after I had invented the foregoing microscope, I had the pleasure to find its construction answered more perfectly to Sir *Isaac*'s thoughts upon the improvement of microscopick instruments than any other construction yet extant. Mr *Gregorie*, in arguing for the preference of his own to Sir *Isaac Newton*'s reflecting telescope; proposes the following question *. "But above "all things I desire to know this; that seeing "the image made by the great speculum may "be esteemed a small visible; and seeing Mr. "*Newton*, in the *Transactions* pag. 3080, thinketh it fitter to mak an microscope, or tube "to hold an smal visible, of one concave speculum and one eye-glasse, rather than with "one single eye-glasse, and much rather than "with one plane speculum and with one eye-"glasse: wherefor also to look to this smal visible the first also should not be preferred to "the last." To this Sir *Isaac* answered as follows.

* Append. to *Gregorie*'s Elem. of opticks.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lows *. "Why I assign a small concave with "an eye-glass to magnifie small objects (in *Transactions* pag. 3080.) and yet an eye-glass without such a concave to magnifie the image of "the great concave, is, because that image doth "not require to be magnified so much as an "object by a microscope; and further because "the angle of the penicil of rays which flow from "any point of the small object, that the object "may appear sufficiently luminous, ought to "be as great as possible; and a concave will, "with equal distinctness, reflect the rays with "a greater angle of the penicil than a lens; but "in the telescope the angles of those penicils "are not so great as to transcend the limits at "which an eye-glass may with sufficient distinctness refract them: and therefore in these "instrumenss I chuse to lay all the stress of "magnifying upon the eye-glasses. In microscopes also I would lay as much stress of magnifying upon the eye-glass as it is well capable of, and the excess only upon the concave." Since the design of this double microscope is, to magnify the minutest objects more than usual, it answers the end though it serves only for transparent objects; because the minute parts of all kinds of objects are transparent, as Sir *Isaac Newton* and others have observed.

* Ibid. pag. 277.

Application to Mr. *Cassegrain*'s telescope. Fig. 170.

612. Imagine TQ to increase till it vastly exceeds TE, and our microscope will be changed into Mr. *Cassegrain*'s telescope, whose magnifying power will be as $m \times TC$ to κl.

613. For the apparent magnitude is to the true as the angle $\varpi l \kappa$ to PEQ or pEq, or in a ratio compounded of $\varpi l \kappa$ to $\varpi e \kappa$ and of $\varpi e \kappa$ or peq to pEq, that is, of $e\kappa$ to κl* and Eq to eq, or because the rectangles will be the same, of Eq to κl and $e\kappa$ to eq or $t\kappa$ to tc or m to 1; which make the ratio of $m \times Eq$ to κl, or, when the object is very remote, of $m \times ET$ to κl.

* Art. 60.

614. Here the nascent aberrations from the intermediate focus q, will become equal to one another by putting $ct : CT :: m^4 : \overline{mm-1}^2$ *. For the number n, being infinitely diminished, gives $\overline{nn-1}^2 = 1$. Hence I collect that the speculum ac will intercept the least light from AC, when $m = 3$ and consequently when the semiaperture ac is to AC, that is, qc to qC or TC as 27 to 32 *; the duplicate of which being 729 to 1024, shews that almost three quarters of the incident light will be intercepted; which renders the design of a perfect image at κ impracticable. Nevertheless though we cannot apply a convex ac of so large a sphere as is necessary for that purpose (for $ct = \frac{81}{64} CT$) yet any smaller convex, *cæteris paribus*, is preferable to a concave of the same size; because the imperfection of the image κ will result only from the difference of the aberrations from q in the former case and from their sum in the latter.

* Rem. prop. 2.

* Rem. 544.

615. Hence by making the convex ac exactly spherical, and by inclining the concave AC towards a parabolick figure *, the aberrations in the image $\varpi\kappa$ will be diminished. For when both speculums are spherical, it is easy to collect * that the aberration qR is to qr as CT to $\left|\overline{\frac{mm-1}{mm}}\right|^2 \times ct$ nearly, and since to gain light we are obliged to make ct smaller than is requisite for producing qR equal to qr, it follows that qR is bigger than qr, and therefore ought to be diminished by the method proposed.

* Art. 804

* Rem. prop. 2.

616. The late admirable improvements in the performance of Mr. *Gregorie*'s telescope, even beyond what its theory seemed to promise *, has encouraged me to consider it more minutely than before *; and to calculate tables of its dimensions, and also of Mr. *Cassegrain*'s, in order to farther improvements of both.

Gregorie's and *Cassegrain*'s telescope farther considered.

* Phil. Trans. Abr. Vol. 1. p. 204.

* Rem. 137.

LEMMA I.

617. The aperture of the larger speculum being given, if the area of the lesser be just sufficient to receive all the rays of the principal pencil, and the hole in the larger be not greater than this area, the brightness of the last image will be as great as possible at its center; but will go decreasing outwards though very slowly and perhaps imperceptibly in telescopes that magnify much.

618. Let T be the focus and TC the focal distance of the larger speculum $ABCBA$, CA its semiaperture, CB the semidiameter of the hole in its vertex, through which the last image $\varpi\kappa$ of a remote object PQ, is projected from the lesser speculum aca. And let the outermost rays QA, QA, which come parallel to the axis cC and after the first reflection belong to the focus T, fall upon the lesser speculum at a and a. Then the area whose breadth is aca will be just sufficient to receive all the rays of the principal pencil and to reflect them to κ, the center of the last image.

Fig. 171. 172.

619. Now if the lesser speculum were narrower than aa, some rays of this pencil, after their first reflection would pass by it and be lost; and if it were broader than aa, it would intercept more of the incident rays, which would also be lost.

620. As to the breadth BB of the hole in the larger speculum, if it were greater than aa, some of the innermost incident rays of the principal pencil would be lost in it; and if it were less than aa, whose shadow is rather larger than it self, no more of the principal pencil could fall on the ambient annulus, whose breadth is AB, than if BB were as large as aa. Therefore

fore the point κ, to which these rays are reflected, is as bright as possible when the breadth aa is just sufficient to receive the principal pencil, and BB not greater than aa, which was the first thing to be proved.

621. The half breadth ST of the first image of the half object PQ, is terminated by the line PES drawn through the center E of the larger speculum AC*; and likewise the half breadth $\varpi\kappa$ of the last image, is terminated by the line $Se\varpi$ drawn through the center e of the lesser speculum ac*.

* Art. 115.

* Art. 115.

622. Now to determine the ratio of the quantities of light at ϖ and κ, let the circle $AGAG$ be the section of the sphere of the larger speculum cut by a plane passing through the circumference of its aperture AA, and $BgBg$ another section of it through the circumference of the hole BB, and K the common center of the sections. In the radius KA take $KF : ST :: Cc : cT$ and with the center F and semidiameter FD, equal to KA, describe the circle $DGHG$. Again upon KF take $Kf : ST :: Cc : CT$, and with the center f and semidiameter fb equal to KB, describe the circle $bgig$. Then, I say, the brightness of the point κ will be to that of ϖ, as the annular difference of the circles $AGAG$, $BgBg$, to the same annulus diminished by the lunar spaces $AGDGA$, $BgbgB$ whose greatest breadths are AD, Bb.

623. For, drawing the lines aS, aS and producing them, the one will fall at D within the aperture AA and the other at H as much without it very nearly, because the angles SaT, SaT are very nearly equal. Consequently if the speculum AA were inlarged, and a circular section of it of the breadth DH nearly equal to AA, were filled with rays that come from P, parallel to PES, these rays after their first reflection would just fill the whole area aca of the lesser speculum, as before, and be thence reflected to ϖ, which would now be as bright as κ. But the rays which fall on the lunar part $AGHGA$, without the given aperture AA, are wanting at ϖ while κ has those of the principal pencil which fall on the equal lunula $AGDGA$. Now by the similar* figures ADa, STa, we have $AD : ST :: (Aa : aT ::)\ Cc : cT$; therefore KF by construction is equal to AD, and consequently the center F of the circle $DGHG$ was rightly determined.

* Art. 204.

624. Again the smaller circle $bgig$, answers to the shadow of the lesser speculum aca, cast upon the section of the larger by the rays that come from P parallel, as it were, to ES. For the oblique ray ab, being parallel to ES, and the line aB to ET, the angle Bab is equal to SET. Whence, by the similar triangles Bab, SET, we have $Bb : ST :: (Ba : TE ::)\ Cc : CT$; therefore by construction Kf is equal to Bb, and consequently the center f of the said shadow was rightly determined. Now κ has all the rays which fall upon the lunar part $BgbgB$, and ϖ wants an equal quantity intercepted from that part. Hence the proportion of their brightness is very plain.

625. Lastly by diminishing the angle PEQ or SET, the images ST, $\varpi\kappa$ and the breadths AD, Bb of those lunar areas are diminished, so that the brightness of ϖ increases towards κ, and differs less from that of κ as the angle PEQ is smaller, that is as the telescope magnifies more.

626. For the magnifying power of the telescope is measured by the ratio of the angles $\varpi l\kappa$, PEQ, the former of which, must be nearly the same in every telescope as being limited by the breadth of the eye-glass, which breadth must bear a certain given ratio to its focal distance. For then the refracting angle at its edge will be of a given magnitude, and consequently the margin of the visible area will appear with a given degree of distinctness. And this is the reason, by the by, that the more a telescope magnifies, the less it takes in at one view.

627. *Corol.* 1. Hence the lunar space $AGDGA$ is equal to the rectangle $AD \times AA$ very nearly when AD is small. Perpendicular to AKA draw the semidiameter KL, cutting the arch DG in M, and FN cutting the arch AGA in O; then from the quadrants AKL, DFN, taking away the common area DKM, we have the space $ADML$ equal to $MNFK$, and by adding the space LMG to both, we have the semilunar space $ADGA$ equal to $KLGNF$ equal to $KL \times KF$ or $KA \times AD$ very nearly when AD is small. The same is true of the lunar space adjoining to the hole $BgBg$.

628. *Corol.* 2. Hence it is easy to collect, that the circle $AGAG$ is to the lunar part $AGDGA$, as the quadrantal arch AL to AD very nearly.

629. *Corol.* 3. If the lesser speculum be convex and the hole in the larger be equal to it, the last image may also be equal to the hole, but not exceed it without suffering a farther diminution of its brightness towards the margin. For, the outermost ray PD will be reflected into the lines Da, $a\varpi$; because all the rays that come from P belong to the points S, ϖ; and if $ca = CB = \varpi\kappa$, the line $aB\varpi$ is parallel to the axis cC. But if $\varpi\kappa$ exceeds CB, the outermost ray as $a\varpi$ will be stopt by the speculum at B.

630. *Corol.* 4. If the lesser speculum be concave and the hole in the larger be equal to it, the last image may be bigger than the hole, but not much, unless it be projected far beyond the hole, which would increase the length of the telescope.

telescope. For, the outermost ray PA will be reflected into the lines ASb, $b\varpi$; and cb being less than ca or CB, the ray $b\varpi$ has room to diverge a little from the axis cC.

LEMMA II.

631. The hole in the larger speculum being equal to the lesser speculum as before, if this speculum be increased by a small zone whose breadth is to half the breadth of the first image, as the distance between the speculum's is to the focal distance of the larger, the last image will become uniformly bright; but somewhat less bright than its center was before, by the loss of as much light as this zone intercepts.

Fig. 171. 172. 632. For, drawing the lines AS, AS, the arch aca will cut the one in b, and if produced will touch the other in d; and then the rays flowing from P upon the arch AA, and belonging to S after their first reflection, will all be received upon the arch bcd, and be thence reflected to ϖ; and by turning the arch cad about the axis cT, the speculum aca will be increased by a zone of the breadth ad, and will receive all the rays which flowed from a circular object described by PQ turned about the same axis QC. Now by the similar figures Aad, ATS, we have $ad : TS :: (Aa : AT ::) Cc : CT$.

633. A circle $bkmk$ described with the center K and semidiameter Kb equal to the sine of the arch cd, is now equal to the shadow of the lesser speculum, cast upon the larger by the principal pencil; and when cast by the collateral pencil flowing from P, its edge m will touch the edge of the hole BB. For Bb is nearly equal to ad, because the angles Bab, QAP, aAd are equal, and also the distances Ba, aA very nearly. Hence every point of the last image is equally deprived of as many rays as the lesser speculum intercepts and therefore is uniformly bright. But every point is less bright than the center $\varkappa$ was, before the addition of the said zone, by the deduction of as many rays as the zone intercepts.

634. *Corol.* 1. Hence the former brightness of the center $\varkappa$ is to the latter, as $\overline{CA}^2 - \overline{ca}^2$ to $\overline{CA}^2 - \overline{cd}^2$; I mean as the differences of the squares of the sines of these arches.

635. *Corol.* 2. The lesser speculum dad being convex, if the semidiameter of the last image $\varpi\varkappa$ be not rather less than ca or that of the former hole CB, some of the outermost rays of the most oblique pencil, now reflected from ad, will be stopt at B.

636. *Corol.* 3. But if the lesser speculum dad be concave, the last image $\varpi\varkappa$ may continue equal to, or rather bigger than the said hole BC; because the same ray $b\varpi$ is still the outermost of the most oblique pencil; the additional rays reflected from ad to ϖ being here on the inside of that pencil.

637. *Corol.* 4. If the breadth of the hole be made equal to the increased breadth dd of the lesser speculum, whether concave or convex, the last image tho' less than this hole, will be rather brightest in its center; because those rays of the oblique pencils which fall on the lunar parts of the hole, explained above, will be lost in them. But this loss is almost inconsiderable, in comparison to what was lost before by the larger lunar spaces $AGDGA$.

LEMMA III.

638. Instead of a single eye-glass, adapted to these telescopes, to substitute two, which, without altering the apparent magnitude of objects, shall somewhat enlarge the visible area. Fig. 173.

639. Supposing the focal distance $l\varkappa$ of the single eye-glass lk to be given, if towards the speculums we take $lm = 2\,l\varkappa$ and $ln = \frac{1}{3}\,lm$, and instead of the glass lk we place two other convex ones at m and n, whose focal distances are lm and ln respectively, the telescope will magnify just as much as before; but the visible area will be somewhat distincter and freer from colours about its margin, and therefore may be somewhat enlarged if it were sufficiently distinct before.

640. For bisecting mn in q, we have $qn = nl$ by construction, and taking $mf = ml$, we have $\varkappa f$ to $\varkappa m$ and $\varkappa m$ to $\varkappa q$ as 3 to 1. Therefore the rays of the principal pencil, which by reflection tended towards $\varkappa$, will be refracted through the glass m to q*, and passing on to the glass n, will emerge from it in parallel lines as requisite for distinct vision. * Art. 239.

641. Hence by the glass m the image $\varpi\varkappa$ will be contracted to pq, terminated at p by the line $m\varpi$. Therefore joining pn, we have a triangle mpn isosceles and similar to $m\varpi l$, and therefore pn parallel to ϖl. Consequently, to the eye at any place o, the object will appear through the glasses m, n under an angle equal to pnq or $\varpi n\varkappa$, that is, of the same magnitude as usually through the single glass l.

642. Bisecting ln in o, the eye at o will see the largest area at one view. For each pencil of rays reflected from the larger speculum to the lesser, takes up nearly its whole breadth aca and therefore a ray of each, reflected from its middle point c, is nearly the middlemost ray of each pencil; consequently the eye will receive all these middle rays, if it be placed at their focus after refractions through both glasses, which focus might easily be found. But supposing ag to be that ray of any oblique pencil, which falls on the glass m in a line parallel to its

its axis; after refraction it will tend towards l, the principal focus of that glass, till meeting the glass n, it will emerge from it in a line bo parallel to pn, and will bisect ln in o; and since all the rays of that pencil will emerge parallel to bo and exceedingly near to it, we may take this point o for the place of the eye, or rather of a little hole in a thin plate hereafter to be mentioned.

643. Now supposing the glasses m, n were taken away, the said parallel ray $ag\varpi$ would fall on the single eye-glass kl at k, and be refracted into ki parallel to ϖl, to which all the rest of that pencil would also go parallel.

644. But the vision caused by the same rays is distincter to the former eye at o than to this at i, because the ratio's of the focal distances to the respective apertures of the glasses m, n, that is, of lm to mg and ln to nb, are severally double* the ratio of the focal distance to the aperture of the glass l, that is, of li or $l\varkappa$ to lk.

* Ratio dupli.

645. The distinctness will be farther promoted by making the glasses m, n plano-convex and placing their plane sides towards the eye; so that their second refractions of the rays, into air, which contribute much more to the production of colours than their first, may be less than they would have been in a contrary position of the glasses. For in the common experiment with a prism*, if it be turned about its axis that way which makes the rays emerge more obliquely out of the second refracting surface, the sun's coloured image on the wall will soon become an inch or two longer, or more; and if it be turned the contrary way, so as to make the rays fall more obliquely on the first surface the image will soon become an inch or two shorter; the reason of which might easily be shewn by theory*. The proportions and places of the glasses m, n might be variously altered but with little difference in their effect.

* Rem. 392.

*See Newt. Opt. Lect. P. I. sec. 2. n. 29.

Hole in the plate next the eye.

646 *Corol.* 1. Let the outermost ray $arqs$, of the principal pencil, cut the glasses m, n at r and s, and since $qn = qm$ we have $ns = (mr =) \frac{\varkappa m}{\varkappa c} \times ac$. Now it is easy to understand, that ns is the semidiameter of the little hole to be placed at o, through which the principal pencil and all others, as having originally the same breadth or base at the speculum aca, will just pass; and consequently, that all foreign rays which pass by the circumference of the speculum aca, considered as belonging to pencils of a broader base than aca, will fall wide of this hole and be stopt by the plate from coming to the eye. But to receive all the rays of the pencils made to diverge a little upon the eye, especially of a short-sighted person, the hole at o must be somewhat enlarged tho' very little.

Hole in the plate that limits the angle of vision.

647. *Corol.* 2. By placing the center of a larger hole at q, whose semidiameter is pq, the ambient plate will intercept some stragling rays reflected from the imperfect margins of the speculums; and by circumscribing the last image, will limit the visible area, and angle of vision, to such a size as the glasses will permit, without shewing it too much coloured about the margin.

648. *Corol.* 3. Hence in any given telescope we may find the half angle of vision pnq (in a gross manner) by measuring the diameter of the hole which circumscribes the image at q, and the distance between the glasses, m, n, whose half is qn, or, rather by finding the focal distance qn of the glass n, by the sun's rays. Then is $\frac{pq}{qn}$ the tangent of the angle pnq to the radius 1. But this angle may be found more accurately by the time of a star's transit over the telescope when fixt.

Application of the theory to a given telescope.

649. *Corol.* 4. Hence also we have the place l and focal distance $l\varkappa$ of a single eye-glass which will magnify just as much as the two given glasses m, n. For $\varpi\varkappa$ being equal to ac is given by measuring ac, and $\varkappa l : \varkappa\varpi :: qn : qp$, that is, in the ratio of the radius to the tangent of the half angle of vision found before, and $ml = 2\varkappa l$ by construction. But this single glass may be found more exactly by the magnifying power as shewn hereafter.

650. *Corol.* 5. Hence also we have $T\varkappa$ the distance of the image $\varpi\varkappa$ from the focus of the object-metal, by measuring TC and Cm: because $T\varkappa = TC + Cm + m\varkappa$.

LEMMA IV.

651. In a given telescope of Mr. *Gregorie's* or *Cassegrain's* form to find the angle of aberration.

652. Put d for the focal distance CT of the object-metal, s for the sine AS of its semiaperture AC, e for the focal distance of the single eye-glass l, n to 1 for the ratio of $T\varkappa$ to TC, r to 1 for that of $c\varkappa$ to cT; then supposing the outermost ray $QAqar\chi$, that comes parallel to the axis, to cross it in q and r and the last image $\varpi\varkappa$ in χ, and a be the tangent of the required angle of aberration $\varkappa l\chi$, we shall have, Fig. 174. 175.

in *Gregorie's* telescope, $a = \frac{s^3}{8dde} \times \overline{r + n - \frac{n}{rr}}$,

and in *Cassegrain's*, $a = \frac{s^3}{8dde} \times \overline{r - n + \frac{n}{rr}}$.

653. For since Tq and $\varkappa r$ are the successive aberrations of the outermost ray from the first and second focus's T, $\varkappa$ of the innermost rays, taking k for the conjugate focus to q of other rays reflected from the innermost points of the lesser speculum ac, we have $tk : tc :: tc : tq$ and also $t\varkappa : tc :: tc : tT$*, and consequently* $tk : t\varkappa :: tT : tq$, and disjointly $tk : k\varkappa :: tT : Tq$, Fig. 174.

* Art. 207.
* Eucl. VI. 14.

$: \mathcal{T}q$, and alternately $k\varkappa : \mathcal{T}q :: (tk : t\mathcal{T} :: t\varkappa$
* Art. 204. $: t\mathcal{T}$* $:: \overline{t\varkappa}^2 : \overline{tc}^2 ::)\ rr : 1$; because $t\varkappa : tc$ $:: tc : t\mathcal{T} :: (t\varkappa + tc$ or $c\varkappa : tc + t\mathcal{T}$ or $c\mathcal{T} ::)$ $r : 1$ by construction. Therefore $k\varkappa = rr \times \mathcal{T}q$.

654. Let a ray za come parallel to the axis, and let tu be its aberration from the focus t, and since kr is the aberration of the ray qar
* Rem. 553. from the focus k, we have $kr : tu :: \overline{r-1}^2 : 1$*; because we had $t\varkappa : tc$ or $te :: r : 1$, and disjointly $e\varkappa$ or $ek : et :: r-1 : 1$. Hence $kr = \overline{r-1}^2 \times tu$.

655. Therefore since kr and es, the versed
* Rem. 546. sines of ca, tend contrary ways from k and c*, in *Gregorie's* telescope we have the whole aberration $\varkappa r = k\varkappa + kr = rr \times \mathcal{T}q + \overline{r-1}^2 \times tu$

656. In the construction we put $\mathcal{T}\varkappa : \mathcal{T}C$ or $d :: n : 1$, whence $\mathcal{T}\varkappa = nd$; we also put $c\varkappa : c\mathcal{T} :: r : 1$ and disjointly $\mathcal{T}\varkappa : c\mathcal{T} :: r-1 : 1$, whence $c\mathcal{T} = \frac{nd}{r-1}$ and $c\varkappa = \frac{ndr}{r-1}$, and by the
* Art. 204. similar figures qac, qAC*, we have $ac : AC$ or $s :: (cq : Cq ::)\ tc : c\mathcal{T} :: r : r+1$, whence $ac = \frac{ns}{r-1}$. We had also $tc : t\mathcal{T} :: r : 1$ and conjointly $tc : c\mathcal{T}$ or $d :: r : r+1$. Whence $tc = \frac{r}{r+1} \times c\mathcal{T} = \frac{ndr}{\overline{r-1} \times \overline{r+1}}$.

* Rem. 553. 657. Now the aberration $\mathcal{T}q = \frac{\overline{CA}^2}{8C\mathcal{T}}$* $= \frac{ss}{8d}$ and likewise $tu = \left(\frac{\overline{ca}^2}{8ct} = \frac{nnss}{\overline{r-1}^2} \times \frac{\overline{r-1} \times \overline{r+1}}{8ndr}\right) = \frac{nss}{8dr} \times \frac{r+1}{r-1}$. Therefore the longitudinal aberration $\varkappa r = (rr \times \mathcal{T}q + \overline{r-1}^2 \times tu = \frac{ss}{8d} rr + \frac{nss}{8dr} \times \overline{rr-1} =)\ \frac{ss}{8d} \times \overline{rr + nr - \frac{n}{r}}$.

658. Lastly, by the similar figures $\varkappa\chi r$, car
* Art. 204. we have $\varkappa\chi : \varkappa r :: (ca : cr :: ca : c\varkappa$*, that is, $:: \frac{ns}{r-1} : \frac{ndr}{r-1} :: s : dr$; whence the lateral aberration $\varkappa\chi = \left(\frac{s}{dr} \times \varkappa r =\right) \frac{s^3}{8dd} \times \overline{r + n - \frac{n}{rr}}$ and $a = \left(\frac{\varkappa\chi}{\varkappa l} =\right) \frac{s^3}{8dde} \times \overline{r + n - \frac{n}{rr}}$. Q. E. J.

Fig. 175. 659. By revising the computation with regard to the other figure and changing a sign or two, it will appear, in *Cassegrain's* telescope, that $a = \frac{s^3}{8dde} \times \overline{r - n + \frac{n}{rr}}$. For now $\varkappa r = k\varkappa - kr$, because the versed sine cs and consequently the aberration kr* are moved to the other sides of c and k. Q. E. J. * Rem. 546.

660. *Corol.* For the magnifying power of the telescope put m to 1, then $m = \frac{rd}{e}$*, and in * Rem. 612.
Gregorie's, $a = \frac{ms^3}{8d^3} \times \overline{1 + \frac{n}{r} - \frac{n}{r^3}}$ and in *Cassegrain's*, $a = \frac{ms^3}{8d^3} \times \overline{1 - \frac{n}{r} + \frac{n}{r^3}}$.

PROBLEM.

To compose a telescope of Mr. Gregorie's *or* Cassegrain's *form, which being of a given length, shall have a given angle of vision, and shew objects with a given degree of brightness and distinctness, and magnified as much as these given conditions can permit.*

661. The algebraick expressions of the given degrees of apparent brightness and distinctness may be taken from a given telescope of Sir *Isaac Newton's* form, or rather of either of the forms proposed. In the given telescope let A be the half breadth of the larger speculum, B that of the lesser, that is, half the least breadth of the oval plane if *Newton's* telescope be made choice of, and M to 1 the ratio of its magnifying power; and for the quantity $\frac{\overline{A+B} \times \overline{A-B}}{MM}$ put b, which being as the apparent brightness in the given telescope*, may be retained in the tele- * Art. 348.
scope required, or be increased or diminished in the same ratio as this given degree of apparent brightness is intended to be increased or diminished. In the telescope required, to the radius Fig. 171. 172.
1 let a be the given tangent of the proposed angle of aberration, which likewise may be the same and consequently the distinctness the same as in the given telescope, or different at discretion; and v the tangent of half the proposed angle of vision; d the given focal distance $C\mathcal{T}$ of the larger speculum ABC, $\varkappa$ the given place of the last image formed by the lesser speculum ac, and n to 1 the given ratio of $\mathcal{T}\varkappa$ to $\mathcal{T}C$.

662. *Cas.* 1. In *Gregorie's* telescope having Solution.
computed a number $c = \frac{64n^4aa}{bv^4} dd$, find the greatest affirmative root r of this equation,
$rr \times \overline{r + n - \frac{n}{rr}}\Big|^2 \times \overline{r-1}^4 - c \times \overline{1 + \frac{n}{r-1}}$ Fig. 171.
$\times \overline{1 - \frac{n}{r-1}} = 0$, and in $C\mathcal{T}$ produced taking $\mathcal{T}c = \frac{nd}{r-1}$, and back again $ct = \frac{r}{r+1} \times \mathcal{T}c$, the point c will be the vertex and t the principal

pal focus of the ſmaller concave ſpeculum ac. The ſemiaperture of the larger concave will be

$$d\sqrt{\frac{8na}{v \times \overline{r-1} \times \overline{r+n-\frac{n}{rr}}}} = CA,$$ and that

of the leſſer will be $\frac{n}{r-1} \times CA = ca = CB = \varpi\kappa$, and the focal diſtance of the eye-glaſs will be $\frac{1}{v} \times \varpi\kappa = \kappa l$. This teleſcope will ſhew objects as required and magnified in diameter in the ratio of $r \times d$ to κl.

663. *Caſ.* 2. In *Caſſegrain*'s teleſcope having computed the number $c = \frac{64n^4aa}{bv^4}dd$, find the greateſt affirmative root r of this equation,

$$rr \times \overline{r-n+\frac{n}{rr}}\Big|^2 \times \overline{r+1}^4 - c \times \overline{1+\frac{n}{r+1}}$$

Fig. 172. $\times \overline{1-\frac{n}{r+1}} = 0$, and in TC taking $Tc = \frac{nd}{r+1}$

and back again $ct = \frac{r}{r-1} \times Tc$, the point c will be the vertex and t the principal focus of the convex ſpeculum ca. The ſemiaperture of the concave ſpeculum will be

$$d\sqrt{\frac{8na}{v \times \overline{r+1} \times \overline{r-n+\frac{n}{rr}}}} = CA,$$ and that

of the convex will be $\frac{n}{r+1} \times CA = ca = CB = \varpi\kappa$ and the focal diſtance of the eye-glaſs will be $\frac{1}{v} \times \varpi\kappa = \kappa l$. This teleſcope will ſhew objects as required and magnified in the ratio of $r \times d$ to κl.

Analyſis. Fig. 171. 664. *Caſ.* 1. For ſince $1 : n :: TC$ or $d : T\kappa$, we have $T\kappa = nd$; we have alſo $t\kappa$, tc, tT conti-

* Eucl. V. 12. nual proportionals in ſome unknown ratio, ſuppoſe of r to 1. Hence $c\kappa : cT :: r : 1$*, and disjointly $T\kappa : Tc :: r-1 : 1$, whence $Tc = \frac{nd}{r-1}$. Again, ſince $tc : tT :: r : 1$, conjointly we have $tc : Tc :: r : r+1$, whence $tc = \frac{r}{r+1} \times Tc$.

665. Let the required ſemiaperture $CA = s$,

* Art. 204. then by the ſimilar* figures acT, ACT, made by the reflection of the outermoſt ray ATa of the principal pencil, we have $ac : cT :: (AC : CT ::) s : d$, whence $ac = \frac{ns}{r-1}$. Taking this for the half breadth of this ſpeculum, and the hole's half breadth CB equal to it, the center of the viſible area will be the brighteſt poſſible*, and the quantity of light in it will be * Rem. 617.

as $\overline{AC}^2 - \overline{ac}^2 = (\overline{AC+ac} \times \overline{AC-ac} = \overline{s+\frac{ns}{r-1}} \times \overline{s-\frac{ns}{r-1}} =) \overline{1+\frac{n}{r-1}} \times \overline{1-\frac{n}{r-1}} \times ss$.

666. Suppoſe $\kappa l = e$ and m to 1 as the apparent magnitude to the true, that is, in a ratio compounded of TC to κl and $t\kappa$ to tc*; then * Rem. 144. or 611.
we have $m : 1 :: r \times d : 1 \times e$, whence $m = \frac{rd}{e}$.

667. Now the apparent brightneſs of the center of the viſible area, is as $\frac{\overline{AC}^2 - \overline{ac}^2}{mm}$* $=$ * Art. 348.
$\overline{1+\frac{n}{r-1}} \times \overline{1-\frac{n}{r-1}} \times \frac{eess}{rrdd}$. For the rule here quoted, being not confined to any certain magnitude of the viſible area, holds good of its center conſidered as an area diminiſhed to a phyſical point. Therefore putting this value of the apparent brightneſs equal to b, as ſettled in the *data*, we have $b = \overline{1+\frac{n}{r-1}} \times \overline{1-\frac{n}{r-1}} \times \frac{eess}{rrdd}$, which I ſhall call the firſt equation.

668. For ſhortneſs of notation putting $t = r+n-\frac{n}{rr}$, we have the tangent of the propoſed angle of aberration, that is, $a = \frac{s^3t}{8dde}$*, * Rem. 652.
our ſecond equation.

669. By the *data* we have $\varpi\kappa : \kappa l$ or $e :: v : 1$, whence $\varpi\kappa = ev$. Now ſince we took the hole's half breadth $CB = ca$, we may chuſe any magnitude for the image $\varpi\kappa$ not much exceeding CB*. For the angle $\varpi l \kappa$ being equal * Rem. 630.
to the given angle of viſion, and the breadths of ſimilar eye-glaſſes being as their focal diſtances, the refractions of the rays flowing from ϖ upon the margin of theſe glaſſes will alſo be ſimilar and equal, and conſequently the indiſtinctneſs and colours about the margin of the viſible area will be nearly the ſame whatever ſize we pitch upon for the image $\varpi\kappa$; we may therefore put $\varpi\kappa = ca$, that is, $ev = \frac{ns}{r-1}$, which is our third equation.

670. Hence $e = \frac{ns}{v, \overline{r-1}}$ and $ee = \frac{nnss}{vv, \overline{r-1}^2}$, which values being ſubſtituted in

the

the first and second equations, give $b = \overline{1 + \frac{n}{r-1}} \times \overline{1 - \frac{n}{r-1}} \times \frac{nns^4}{ddvvrr, \overline{r-1}^2}$ and $a = \frac{sstv, \overline{r-1}}{8ndd}$, whence $ss = \frac{8andd}{tv, \overline{r-1}}$ and $s^4 = \frac{64aannd^4}{ttvv, \overline{r-1}^2}$; therefore $b = \overline{1 + \frac{n}{r-1}} \times \overline{1 - \frac{n}{r-1}} \times \frac{64n^4ddaa}{v^4ttrr, \overline{r-1}^4}$. Lastly by restoring $r - n + \frac{n}{rr}$ for t and putting $c = \frac{64n^4ddaa}{bv^4}$, we have $rr \times \overline{r - n + \frac{n}{rr}}\big|^2 \times \overline{r-1}^4 = \overline{1 + \frac{n}{r-1}} \times \overline{1 - \frac{n}{r-1}} \times c$. We must have the greatest affirmative root of this equation in order to magnify the more*, which root being found, the solution of the problem will appear very evident by looking back upon the analysis. *Q. E. J.*

* Rem. 666.

671. *Cas.* 2. By revising the analysis of the first case with regard to the other figure, the analogous changes will appear so plain as to need no repetition. *Q. E. J.*

672. I observed above that in this solution the visible area will have the proposed degree of brightness at its center only. But its diminution at the circumference, when computed by the corollaries to the lemma's, will be found so inconsiderable that I see no sort of inconvenience in it, but rather some small advantages. First the telescope can magnify somewhat more than it would do if the zone abovementioned* were added to the area of the lesser speculum, because there is more light left. Secondly this zone would intercept an annulus of the innermost rays that are reflected the most accurately to every part of the image; instead of which we should take in that lunar figure* of the exterior rays which are reflected the most inaccurately, and that only to the exterior parts of the last image. Thirdly the interior parts of the image, being distincter than the rest, will bear and therefore deserve a stronger light than the less distinct and tinctured parts about the margin, which in a stronger light would only become more offensive.

* Rem. 631.

* Rem. 631, 632.

673. This is the result of the theory; but in practice some small addition should be made to the breadth *aa* of this little speculum, by reason of the difficulty of placing and moving it so exactly along the axis of the telescope as to receive all the reflected rays; none of which, as being very much condensed by their first reflection, should be suffered to pass by the little speculum. But no addition should be made to the breadth of the hole in the large concave.

Resolution of the equation.

674. By multiplying the factors in the equation found above, and ranging the terms of the product in the order of the dimensions of r, the result will be a compleat equation of twelve dimensions, whose resolution by the common rules would be so tedious, that I chose to perform in the following manner. The equation of Cas. 1. being put into this fractional form

$$\frac{rr \times \overline{r + n - \frac{n}{rr}}\big|^2 \times \overline{r-1}^4}{c \times \overline{1 - \frac{nn}{\overline{r-1}^2}}} = 1,$$

at first I reject the small fractions $\frac{n}{rr}$ and $\frac{nn}{\overline{r-1}^2}$ and seek the root of this simpler equation

$$\frac{rr \times \overline{r+n}^2 \times \overline{r-1}^4}{c} = 1,$$

as follows.

675. Having computed the logarithm of the denominator c, I first assume $\frac{1}{8}$ of it for the log. of r, and thence compute the log. of the numerator $rr \times \overline{r+n}^2 \times \overline{r-1}^4$. Then I take the difference of the logarithms of the numerator and denominator, and, if the former exceeds the latter, I substract $\frac{1}{8}$ of their difference from the log of r, otherwise I add it. With this corrected log. of r I repeat the same operation two or three times, till I find a log. of the numerator agreeing with that of the denominator in 3 or 4 of their first figures exclusive of the index; then the corrected log. of r gives me 3 or 4 figures of the root of this latter equation; and this root will be the greatest of all if being increased a little, it considerably increases the value of the numerator, which will generally be evident enough by revising their variations in the several operations.

676. Lastly with the log. of r last found I compute the logarithms of the denominator and numerator of the equation

$$\frac{rr \times \overline{r + n - \frac{n}{rr}}\big|^2 \times \overline{r-1}^4}{c \times \overline{1 + \frac{n}{r-1}} \times \overline{1 - \frac{n}{r-1}}} = 1,$$

as first proposed, and by $\frac{1}{8}$ of their difference I correct the log. of r as before, which after two or three operations gives me the root r sufficiently exact. A little attention will easily discover the reason of this method, whose success is chiefly owing to the largeness of c and the smalness of n.

Examples of the solution of the problem.

677. In order to give some examples of this Problem, I procured from Mr. *Short* of *Edinburgh*

burgh the following dimensions of his best telescope of Mr. *Gregorie*'s form, to serve as a model for calculating others of any given lengths.

	Inches
The focal distance of the larger speculum	9. 6
Its breadth or aperture	2. 3
Focal distance of the lesser speculum	1. 5
Its breadth	0. 6
Breadth of the hole in the larger speculum	0. 5
Distance between the lesser speculum and the next eye-glass	14. 2
Distance between the two eye glasses	2. 4
Focal distance of the eye-glass next the metals	3. 8
Focal distance of the eye-glass next the eye	1. 1

678. By experiment the telescope was found to magnify 60 times in diameter *, and by the time of the transits of stars over the visible area, to take in an angle of 19 minutes at the naked eye: therefore the magnified angle of vision was $60 \times 19' = 19$ degrees. * Rem. 485.

679. Before we can calculate the dimensions of other telescopes, the given one must first be reduced to the form proposed in the problem, by finding a single eye-glass with which it shall magnify just as much as with the two given eye-glasses; and by calculating the mathematical aperture aa of the lesser speculum, as possessed by the principal pencil, whose breadth will be somewhat less than the breadth of this speculum; then we may deduce the logarithms of the numbers b, a, v, n abovementioned, which (excepting n) shall be kept unaltered in all the other telescopes in the following table.

Fig. 173. 680. First then, we have $mq = mn - nq = 1.3$, whence I collect * $mx = \frac{mq \times mf}{mf - mq} = 1.9760$, and $cx = cm + mx = 16.1760$ and $tx = cx - ct = 14.6760$. Now putting r to 1 for the ratio of the continual proportionals tx, tc, tT, we have $r = \frac{tx}{tc} = 9.7840$, $tT = \frac{tc}{r} = 0.15331$, $Tx = tx - tT = 14.5227$ and $n = \frac{Tx}{TC} = 1.51278$, whose log. is $0.17977.58$. * Rem. 639, &c.

681. Secondly, we have $cT = ct + tT = 1.6533$ and $ca = \frac{cT}{TC} \times CA = 0.19805 = CB = \varpi x$ according to the solution of the problem; and since $M = 60$ we have $xl = \frac{r \times CT}{M}$ * $= 1.5654$, whence log. $v = \text{l}: \frac{\varpi x}{xl} = \bar{1}.10213.36$ and therefore $9.10213.36$ is the log. tangent of $7^\circ.12'.40''$. This is the semiangle of vision in the telescope reduced to the form proposed in the problem, and this we shall retain in all the telescopes in the following table, till we enlarge it to $9^\circ.30'$, by two eye-glasses, as in Mr. *Short*'s. * Rem. 666.

682. Thirdly, we have the log. $a = \text{l}: \overline{\frac{CA^3}{8\,\overline{CT}^2 \times xl} \times r + n - \frac{n^3}{rr}} = \bar{2}.17217.14$ and therefore $8.17217.14$ is the log. tangent of $51'.05''$, which angle of aberration, being two or three times greater than usual in catoptrick telescopes of Sir *Isaac Newton*'s form, would cause an intolerable degree of indistinctness, were it not diminished by correcting the spherical figure of the large speculum, and inclining it towards a parabolick *, which Mr. *Short* takes constant care to do. * Rem. 652. * Art. 804.

683. Fourthly, we have the log. $b = \text{l}: \frac{\overline{CA + ca} \times \overline{CA - ca}}{MM} = \bar{4}.55201.77$.

684. From these *data*, according to the solutions in Cas. 1 and 2, I computed the following Tables, where the measures of the given telescope are also inserted; and have added the value of the root r belonging to each telescope for the satisfaction of such as shall please to examine the calculations.

A Table

A Table of the dimenſions and magnifying powers of ſome teleſcopes of Mr. Gregorie's *form.*

Fig. 171.

CT	Cx	Tc	ct	CA	$ca=CB$	xl	mag. pow.	r
5. 65	2. 987	1. 131	1. 106	0. 773	0. 155	1. 223	39. 69	8. 5589
9. 60	4. 923	1. 653	1. 5	1. 15	0. 198	1. 565	60.	9. 7840
15. 50	7. 948	2. 343	2. 148	1. 652	0. 250	1. 973	86. 46	11. 0090
36.	4.	3. 724	3. 432	3. 132	0. 324	2. 561	165. 02	11. 7408
60.	6.	5. 391	5. 012	4. 605	0. 414	3. 271	242. 94	13. 2426

A Table of the dimenſions and magnifying powers of ſome teleſcopes of Mr. Caſſegrain's *form.*

Fig. 172.

CT	Cx	Tc	ct	CA	$ca=CB$	xl	mag. pow.	r
15. 5	7. 948	1. 992	2. 196	1. 769	0. 227	1. 797	92. 91	10. 7720
15. 5	3.	1. 766	1. 974	1. 761	0. 201	1. 585	92. 65	9. 477
36.	4.	3. 253	3. 569	3. 286	0. 297	2. 347	173. 28	11. 2966
60.	6.	4. 786	3. 173	4. 804	0. 383	3. 028	253. 44	12. 7913½

685. I have calculated for the focal diſtances of 5. 65 and 15. 5 inches as belonging to the ſizes of Mr. *Short*'s tools, who tells me that the teleſcopes made with them perform very nearly according to this calculation. What tryals have been made of Mr. *Caſſegrain*'s form I know not, but by the tables it appears to have the advantage of Mr. *Gregorie*'s, (at leaſt for aſtronomical uſes where the inverſion of the object is not regarded,) as being ſhorter by twice the focal diſtance of the leſſer ſpeculum, and yet magnifying more. The two calculations for the 15½ inch teleſcope are founded on different lengths of Cx, to ſhew that the magnifying power is ſcarce ſenſibly altered thereby. The length of the eye-piece, as being troubleſome in high obſervations, ſhould therefore be no longer than is neceſſary for holding the eye-glaſſes, unleſs, as Mr. *Short* imagines, it may contribute to ſecure the eye-glaſſes from foreign light which paſſes by the ſides of the leſſer ſpeculum through the hole in the larger; for which reaſon he alſo chuſes to make that ſpeculum ſomewhat broader than the hole. But this point had better be ſecured if poſſible, by taking due care of the ſize and place of the little hole in the plate next the eye.

686. The following tables give the poſitions and focal diſtances ml, nq of the two eye-glaſſes m, n, together with the place and ſemidiameter pq of the hole in the plate that limits the viſible area and magnified angle of viſion to 19 degrees, as in Mr. *Short*'s teleſcope. Fig. 173.

For Gregorie's *teleſcopes.*

CT	Cm	mn	ml	nq	no	pq
5. 65	1. 764	1. 631	2. 446	0. 815	0. 408	0. 136
9. 6	3. 358	2. 087	3. 130	1. 043	0. 522	0. 174
15. 5	5. 975	2. 631	3. 946	1. 315	0. 658	0. 220
36.	1. 439	3. 415	5. 122	1. 707	0. 854	0. 286
60.	2. 783	4. 289	6. 434	2. 144	1. 072	0. 359

Fig. 173.

For Caſſegrain's *teleſcopes.*

CT	Cm	mn	ml	nq	no	pq
15. 5	6. 151	2. 396	3. 594	1. 198	0. 598	0. 200
15. 5	1. 415	2. 113	3. 170	1. 057	0. 528	0. 177
36.	1. 653	3. 029	4. 694	1. 565	0. 782	0. 262
60.	2. 972	4. 037	6. 056	2. 019	1. 010	0. 338

687. The breadth of the hole at o in the plate next the eye must be $\frac{1}{50}$ inch in the telescopes of Mr. *Gregorie*'s form and $\frac{1}{60}$ in Mr. *Cassegrain*'s.

Compared with Sir *Isaac Newton*'s telescope. 688. A 60 inch telescope of Sir *Isaac Newton*'s form, made in as great perfection as Mr. *Hauksbee*'s standard *, will magnify 313 times, and therefore excels Mr. *Cassegrain*'s, of the same length, in the ratio of 6 to 5 nearly, and is more commodious for high observations.

* Rem. 631.

689. If according to Lemma 2, we would have the visible area uniformly bright, to the half breadth ac of *Gregorie*'s little speculum we must add $ad = \frac{dv}{m} \times \overline{1 + \frac{n}{r-1}}$ which is scarce 0.022 of an inch when $d = 5.65$ inches, and but 0,034 when $d = 60$ inches. To *Cassegrain*'s ac we must add $ad = \frac{dv}{m} \times \overline{1 - \frac{n}{r+1}}$, which is but 0.032 of an inch when $d = 60$ inches. These small additions will alter the apparent brightness scarce sensibly, and the apparent distinctness not at all.

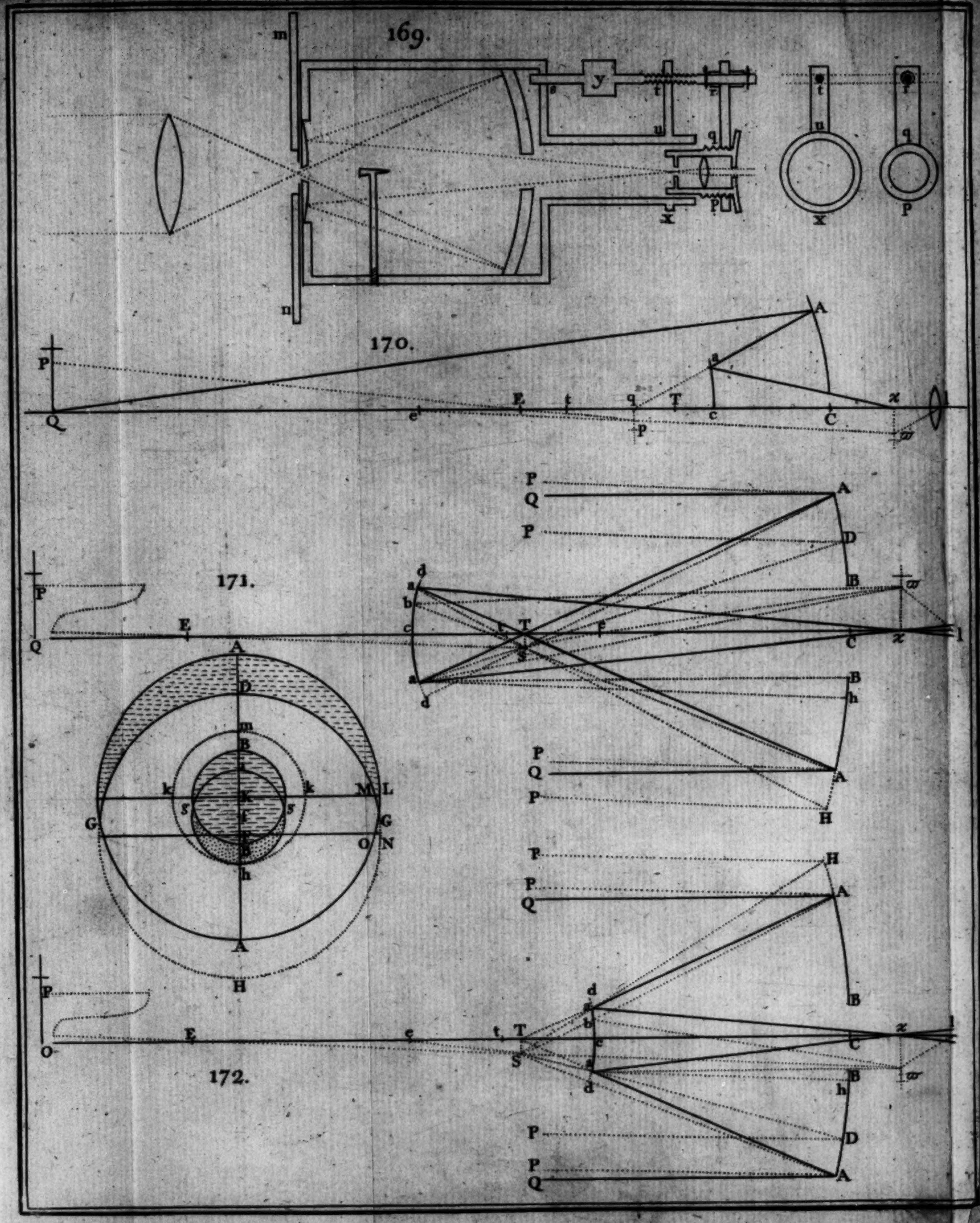
Fig. 171. 690. The reason of the rule is this. In the analysis of the problem we had $m = \frac{rd}{e}$ and $\varpi\varkappa = ev = \frac{rdv}{m}$, and by the similar triangles $ST e$, $\varpi e\varkappa$ we have $ST : \varpi\varkappa :: Te : e\varkappa :: 1 : r$, whence $ST = \frac{dv}{m}$; and by the similar triangles adA, STA we have $ad : ST :: CT + Tc : CT :: d + \frac{nd}{r-1} : d :: 1 + \frac{n}{r-1} : 1$, whence $ad = \frac{dv}{m} \times \overline{1 + \frac{n}{r-1}}$ in *Gregorie*'s. But in *Cassegrain*'s, $ad : ST :: CT - Tc : CT :: d - \frac{nd}{r+1} : d :: 1 - \frac{n}{r+1} : 1$, whence $ad = \frac{dv}{m} \times \overline{1 - \frac{n}{r+1}}$. Fig. 172.

691. If d, s, m, v and n were all given, we might find the other dimensions of *Gregorie*'s telescope by taking $r = 1 + \sqrt{n + \frac{nms}{dv}}$, $Tc = \frac{nd}{r-1}$, $ct = \frac{r}{r+1} \times Tc$, $\varkappa l = \frac{rd}{m}$, $cd = CB = \varpi\varkappa = v \times \varkappa l$. For by putting $\varpi\varkappa = cd = ca + ad$, that is, $\frac{rdv}{m} = \frac{ns}{r-1} + \frac{dv}{m} \times \overline{1 + \frac{n}{r-1}}$, and by algebraick reduction, the value of r will come out as above.

692. These theorems, by substituting for r its value and putting $n = 1$, will come to the same as Mr. *Hadley*'s *, who has given some very useful tables of the dimensions of *Gregorie*'s telescopes of different lengths from those in mine.

* Appendix to *Gregorie*'s Elements of opticks pub. by Dr. *Desagulier* pag. 286.

THE


169.
170.
171.
172.

THE AUTHOR'S REMARKS UPON BOOK III.

Upon Art. 806.

How to clean a tarnished speculum.

MR. *Hearne's* way of cleaning a tarnished speculum is this. Get a little of the strongest sope-lye from the sope-maker; and having laid the speculum on a table with its face upwards, put on as much of the lye as it will hold and let it remain about an hour; then rub it softly with a silk or muslin till the lye is all gone; then put on some spirit of wine and rub it dry with another part of the silk or muslin. If after this the speculum will not perform well, it must be new polished. A few spots of faint tarnish may be rubbed off with spirit of wine only.

Upon Art. 825.

693. To these methods of rectifying telescopick sights Mr. *Molyneux* has added an account of the controversy about them between *Hevelius* and *Hook* *, and shews that *Hevelius's* opposition to the use of those sights arose chiefly from his misapprehension of the nature of them, in taking the line of collimation to be no longer than from the cross-hairs to the eye, whereas its real length is from the hairs to the object-glass; and that Dr. *Hook* should have demonstrated this to him in order to induce him to use these sights instead of his plain ones.

* Dioptr. Part. II. chap. 5.

Upon Art. 915.

Aberrations of rays by a bad polish. Fig. 176.

694. Mr. *Molyneux* means, that the angular aberration of a ray from its true course, when reflected from an irregular particle of a surface, is about five or six times greater than it would be, if the same ray were refracted through that particle. Let AC represent the regular surface, E its center, $QGRqCFS$ its axis, QA an incident ray, EA the perpendicular of incidence, Aq the regular reflection, and AF the regular refraction of that ray. Then imagine the particle of the surface about A to be so changed, that AG becomes perpendicular to it; and let AR and AS be the irregular reflection and refraction of the same ray QA, making the angular aberrations qAR by reflection and FAS by refraction. Through A produce FA towards f and SA towards s. Then by reflection we have the angle $QAq = 2QAE$ and also $QAR = 2QAG$ and consequently their difference $qAR = 2EAG$. Likewise in small refractions from the perpendicular, the angle $QAf = \frac{1}{2}QAE$ and also $QAs = \frac{1}{2}QAG$ nearly, (the ratio of the sines being 3 to 2) and consequently the difference fAs or $FAS = \frac{1}{2}EAG$. Therefore the former difference qAR, is to the latter FAS, as ($2EAG$ to $\frac{1}{2}EAG$, as) 4 to 1. But in refractions towards the perpendicular the like ratio is 6 to 1.

695. Hence if the angle EAG, which measures the irregularity of the particle at A, be given in different surfaces, the aberration of the ray in the image at the focus of a reflecting telescope, will be to the like aberration in a refracting one, in a ratio compounded of their focal distances directly and of 5 to 1 taking one refraction with another at the two surfaces.

Upon Art. 974 —— 978.

Binocular telescope.

696. A binocular telescope of the dioptrick kind being heavy and cumbersome, a more commodious one might be made with a couple of small reflecting telescopes of Mr. *Gregorie's* form. The phænomenon of the enlarged circle of the visible area, explained in these articles, may also be seen very plainly in looking at distant objects through a pair of spectacles, removed from the eyes about as far as the length of the hold-fasts (made for fixing them to the temples) and held steady at that distance. The two innermost of the four apparent rings, that hold the glasses, will then appear united in one larger and more distant ring, than the two outermost, which will hardly be visible unless the spectacles be farther removed.

Dr. *Jurin's* experiments, to find how much brighter an object appears to both eyes than to one alone.

"In order to find, whether an object seen with both eyes appears brighter, or stronger, or larger than when seen with one alone, I made the following experiments.

697. I laid a slip of clean, white paper directly before me upon a table, and applying the side of a book close to my right temple, so as that the book advanced considerably more forward than my face, I held it in such manner, as to hide from my right eye that half of the paper, which lay to my right hand, while the left half of the paper was seen by both eyes without any impediment.

Then looking at the paper with both eyes, I observed it to be divided from top to bottom by a dark line, and the half of the paper

which lay to the right hand of this line, to appear considerably darker than the half which lay to the left hand.

698. In looking at other objects in the same manner, as at the wainscot, or the cieling, I constantly found that part which was seen by one eye only, to appear manifestly darker, than that which was seen by both eyes.

When the book was applied to my left temple instead of the right, the same difference was observed, which shewed my two eyes to be of equal goodness.

699. When I looked in this manner upon a page of a book divided into two columns, I found the column, that was seen with both eyes to be much plainer and more legible than that which was seen with one only.

This difference was more conspicuous, when, in making the experiment by candle-light, the book was at such a distance from the candles, as that there was scarce light enough to read with both eyes. For then the column, which was seen by one eye only, was not at all legible: but I could read the other, though with some difficulty.

700. Being now fully satisfied, that an object seen with both eyes, appeared brighter and stronger than when viewed with one, I next endeavoured to find to what degree this excess of brightness amounted: particularly, whether there was as much difference in the brightness of an object when seen with one eye, and with both, as when illuminated by one candle, and by two.

To this end, I set upon a table two candles of equal height, and burning to appearance with equal light, at about a foot distance beyond a slip of white paper lying before me, and about four inches from one another, so as that the distance between the candles was parallel to the slip of paper.

Then I set a book upon one end, between the right hand candle and the paper, so as to cast a shade from that candle upon the right hand half of the paper: Thus the left half of the paper was illuminated by the two candles, and the right by one only: consequently, the left half was twice as luminous as the right; and the boundary between those two halves was pretty well defined by the edge of the shade.

I then took another book, and applyed it to my left temple, in such manner as to hide the left and brighter half of the paper from my left eye; so that the left half was seen by my right eye only, while the other half was seen by both eyes: Now I expected, that the right half of the paper, having the light of one candle only thrown upon it, but being seen by both eyes, would appear as luminous as the left half, which had twice as much light cast upon it, but was seen by one eye only. In which I found my self mistaken. For the left half appeared much whiter and brighter than the right. Consequently, an object seen with both eyes is nothing near twice as luminous, as when seen with one only.

701. Being desirous to know the quantity of this excess of brightness more exactly, I fixed a slip of white paper flat against the wainscot by the help of pins, and at a yard distance I set a candle, so as that the flame was about the same height with the paper, and nearly opposite to the middle of it, but rather inclining to the right side. At two yards distance from the paper I placed another candle with its flame at the same height, and opposite to the middle of the left half of the paper. Then I set up a book so as to cut off the light of this second candle from the left half of the paper; which half therefore being illuminated by one candle only, appeared considerably darker than the right half of the paper, on which both candles shone without interruption.

The difference in brightness of the two halves of the paper is easily estimated, by considering that the second candle, being at twice the distance, must throw upon the right half of the paper just a quarter of the light, that was cast upon the same half by the nearer candle*; and consequently that the luminousness of the right half of the paper was to that of the left half, as five to four.

* Art. 58

Things being thus disposed, and the candles burning with equal brightness, I applyed a book to my right temple, so as to hide the right half of the paper from the right eye. Then looking at the paper with both eyes, the right half of it, which had five degrees of light, and was seen by the left eye only, appeared manifestly whiter than the left half, which had four degrees of light, and was seen by both eyes. Consequently, an object seen with both eyes, is not a quarter part more luminous than when seen with one only.

702. After the same manner, by setting the second candle at three yards distance from the paper, I found, that the right half of the paper, which then had ten portions of light thrown upon it, and was seen by one eye, appeared something whiter than the left half, which was illuminated by nine portions of light, and was seen by both eyes.

703. When I removed the same candle one foot farther, so that the distances of the two candles from the paper were respectively as 3 and 10; and consequently, the quantities of light they threw upon the right half of the paper were as 100 and 9 respectively, *i.e.* nearly as 11 and 1; the right half of the paper seen with

with one eye only, seemed still a little whiter than the left half seen with both.

704. When the second candle was set at the distance of four yards from the paper, the right half of the paper seen with one eye, appeared a little darker than the left half seen with both eyes.

705. Hence it follows, that, when the second candle is about 11 foot distant from the paper, the right half seen with one eye, and the left half seen with both eyes, must appear of an equal whiteness.

Consequently, an object seen with both eyes, appears brighter than when seen with one only, by about a thirteenth part. But it would be difficult to make the experiment exactly.

If a person, who squints, should go about to make these experiments, I suppose he would find none of them to succeed, but that in the first experiment particularly, both halves of the paper would appear equally luminous. If so, this would prove, that he saw the paper with one eye only, in confirmation of what was laid down in Remark 178 &c.

And whether larger to both eyes than to one alone.

706. I proceeded next to enquire, whether an object seen with both eyes, appeared any thing bigger than when seen with one alone.

In order to which, applying the side of a book to one of my temples as before, I looked at two prints of equal size hanging beside one another, at two panes in a window, two small pannels in a door, and several other such objects; but could perceive no difference in the apparent magnitude, when one of them was seen with both eyes, and the other with one.

707. Not content with these tryals, I proceeded to make the experiment with smaller objects, as the letters of a printed book, and found, that though they appeared brighter and stronger, yet they did not seem at all larger, when viewed with both eyes, than when seen with one only. But the tryal which gave me the fullest satisfaction, was that which follows.

708. Upon a narrow rectangular piece of paper I drew two lines with ink, parallel to one another, and likewise to the long sides of the paper. Then laying the paper directly before me, so as that my eyes were equidistant from the black lines and from the edges of the paper, I applyed a book to one of my temples, so as that one half of the paper and of the parallel lines upon it, was seen with one eye only, and the other half was seen with both eyes.

I then carefully considered the parallel lines, particularly at the joining of the two several halves: but could observe no difference between those halves, except that the half seen by one eye only, did not make so strong an appearance as the other. The lines and the edges of the paper appeared perfectly straight; which they could not have done, if that half of the paper, or of the space intercepted between the black lines, which was seen by both eyes, had appeared at all larger than that which was seen by one eye only.

From which I conclude, that an object does not appear larger to both eyes than to one, unless by reason of some particular circumstances, as in the observation of *Leonardo da Vinci* *, or in the case of the binocular telescope Art. 976, or of concave specula Rem. 520.

* Rem. 244, 245.

Some surprising phænomena in double vision.

709. Among the several experiments I made in order to find the difference between the brightness and largeness of objects, when viewed with both eyes, and when seen with one eye only, there was one, which, though no way necessary to be mentioned for that purpose, after those I have already related, yet was attended with some phænomena so singular and surprising, as on that account to deserve a particular recital.

Before three prints, equal in size, and hung up against a wall in the situation represented by Fig. 177, I set a table with two candles upon it, at about four foot distance, so as to throw a strong light upon the prints. Then standing about four foot from the candles, with my face towards them and the prints beyond them, I opened a thin quarto book to an angle of about 40 degrees, and held it directly before me, at such a distance, as that I could but just see the whole book though very indistinctly with each eye, the back of the book inclining obliquely downwards and towards my breast, by which position the direct light of the candles was kept off both from my eyes, and from the inside of the book. Then directing my eyes towards the prints, I held the book so, as that the left hand print *A* was hid from my left eye by the left side of the book, and the right hand print *B* was hid from my right eye by the right side of the book: while the print *A* was seen by my right eye, the print *B* by my left eye, and the print *C* by both eyes. Fig. 177.

Things being in this posture, I observed the prints *A*, *B*, and *C* to appear, as near as I could judge, exactly of the same size. Nor could I well satisfy my self, whether the print *C*, which was seen with both eyes, appeared clearer or stronger than *A* or *B*, which were severally seen by one eye only; this experiment not being so well adapted to that purpose, as some of those above related.

710. But observing the wainscot to appear unequally luminous in different parts, I turned my self about still holding the book in the same posture with regard to my face, and directed my eyes first to another part of the wainscot free from prints, where the appearance I am going to describe was more perfect; and then

to

to the cieling, where the surface being white and plain, the phænomenon was still more conspicuous.

Fig. 178. In Fig. 178 part of the cieling is represented by the space *LMABmrlRL*, wherein the line *AL* represents the projection upon the cieling, of the upper right hand edge of the book from my left eye; *BCR* the projection of the same edge from my right eye; *ACl* the projection upon the cieling of the upper left hand edge of the book from my left eye; *Br* the projection of the same edge from my right eye; *RM* the projection of part of the right hand fore edge of the book from the right eye; *lm* the projection of part of the left hand fore edge from the left eye.

By these lines the visible part of the cieling was divided into six different portions, *LMR*, *RCl*, *lmr*, *AMRC*, *BmlC*, and *ACB*. Of these, the three first portions *LMR*, *RCl*, and *lmr*, were the brightest, and were equally bright one with another: the portions *AMRC* and *BmlC* were equal in brightness to one another, but considerably darker than the three first, and the portion *ACB* was the darkest of all.

711. The cause of this phænomenon is easily seen, being no other than what follows. By the description given above it appears, that the portion of the cieling *AMRCB* is entirely hid from the right eye by the interposition of the right side of the book; and that the portion *BmlCA* is entirely hid from the left eye by the interposition of the left side of the book: that is, the portion *ACB* is hid from both eyes, and consequently exhibits the darkest appearance from the dark inside of the book projected on it by both eyes; *AMRC* is seen by the left eye only, *BmlC* by the right eye only; and *LMR*, *RCl*, and *lmr* are seen by both eyes, which therefore appear the brightest of all.

712. But the most extraordinary part of this appearance is yet to come. For the dark part *ACB* does not appear like a plane, as might be expected on considering it as a portion of the cieling; but is seen as a pyramid extending it self from the book to the point *C* in the cieling. And if, instead of the cieling, the eyes be directed to any other object how remote soever, either upwards or downwards, or horizontally; the apex *C* of this pyramid will appear to be extended to, and to touch the very remotest object in view; provided that object be situated in the intersection of two planes, one drawn through the line *BCR* and the right eye, the other through *ACl* and the left eye. In which respect this pyramid is analogous to the middle point of the compasses, in the appearance described in Art. 977.

713. It is farther observable, that there appears another dark body *ABbcaA*, in form of a wedge, from which the pyramid *ACB* seems to arise and to be a continuation of it: but this wedge-like dark body appears within the book it self, between the binding and the eyes.

714. In order to explain these appearances, it must in the first place be remembered, that the axes of my eyes were at that time not directed to the book it self, in which case all these phænomena would immediately have vanished; but that they were pointed at the cieling, as I said before. Now, in this case, it is obvious, that the book and every thing within it must appear double; because the images formed in the two eyes will not fall upon corresponding parts of the retina*. We are therefore to consider the several portions of those images, and to see how they contribute to the production of the phænomena abovementioned.

* Art. 137.

715. First then, the juncture of the leaves, or the line in which they meet at the binding, appearing double, the image of this juncture formed in the right eye will appear to the left hand, as the line *bB* Fig. 178, and the image of the same juncture formed in the left eye will appear to the right hand, as the line *aA*.

Consequently, the image of the left hand leaf formed in the right eye, as *bBrd*, must appear adjoining to the left hand of the juncture *bB* seen by the right eye: And the image of the right hand leaf formed in the left eye, as *aALS*, must appear adjoining to the right hand of the juncture *aA* seen by the left eye. These two images will therefore be intirely distinct one from the other, being separated by the space *bBAa*.

716. But the image of the left hand leaf seen by the left eye, as *aAls*, must appear adjoining to the left hand of the juncture *aA* seen by the left eye.

And the image of the right hand leaf seen by the right eye, as *bBRD*, must appear adjoining to the right hand of the juncture *bB* seen by the right eye.

Consequently, these two images will coincide with one another in the space *aACBbca*; and will coincide with the two former images, (Rem. 715.) in the spaces *bBmn*, and *aAMN*, respectively. But in order to see more particularly, how these two last images contribute to the production of the phænomena above related, it will be necessary to distinguish them into several parts.

717. The image *aAls* of the left hand leaf seen by the left eye may be conceived to be divided into the four parts *bBmn*, *BClm*, *bcsn*, and *bBCAacb*.

718. Of these four parts, the first *bBmn* lying to the left hand of the juncture *bB* seen by the right eye, will consequently coincide with part of the image of the same leaf *bBrd* formed

ed in the right eye, and will be obliterated by that image. For, as the two images coincide, that formed by the right eye, which is stronger and more distinct as being seen more directly and from a greater distance, will swallow up and efface that formed by the left eye, as being seen more obliquely, and much nearer.

719. The second and third parts of this image, as *BClm*, and *bcsn*, will fall without the image of the same leaf formed in the right eye; and will be spread like a faint shade over part of the outward objects, as the cieling, the walls, or the floor seen by the right eye.

720. The fourth part of the image, as *bBCAacb*, having been already observed to coincide with part of the opposite image, it will be proper to consider these two parts together by and by.

721. In like manner, the image of the right hand leaf *bBRD*, seen by the right eye, may be conceived to be divided into the four parts *aAMN*, *ACRM*, *acDN*, and *aACBbca*.

722. The first of these *aAMN* will coincide with and be obliterated by part of the image of the same leaf *aALS*, formed in the left eye, for the reason already given in the like case, in Rem. 718.

723. The second and third parts *ACRM*, and *acDN*, will fall without the image of the same leaf formed by the left eye, and will be spread like a faint shade over part of the outward objects seen by the left eye.

724. The fourth part of this image, namely *aACBbca*, will coincide with a like part of the opposite image, which two opposite parts we come now more particularly to consider; in order whereto we shall distinguish this fourth part *aACBbca* into the two portions *bBCc*, and *CcaA*; and shall distinguish *bBCAacb* the fourth part of the opposite image into the two portions *aACc*, and *CcbB*.

725. Since the book was held directly before me, not inclining to one side more than to the other, and was opened to an angle of about 40 degrees, it follows, that the image of the two leaves seen by the right eye, as *bBRD* and *bBrd*, must appear to contain nearly the same angle of 40 degrees; and likewise the image of the two leaves seen by the left eye, as *aAls*, *aALS*, must appear to contain nearly the same angle of 40 degrees. That is, *bBCc* being part of the image of the right hand leaf seen by the right eye, must appear as making an angle of about 40 degrees with *bBrd* the image of the left hand leaf seen by the same right eye; and likewise *aACc* being part of the image of the left hand leaf seen by the left eye, must appear as making an angle of about 40 degrees with *aALS* the image of the right hand leaf seen by the same left eye.

Consequently, these two first portions *bBCc* and *aACc* must appear inclined to each other in an angle of about 40°, or must appear as two sides of a wedge containing such an angle.

For let *LPr*, Fig. 179, represent the angle of about 40°, to which the book was opened; and let *LACl*, and *RCBr* represent the double image of a section made by a plane passing perpendicularly through both leaves. Then will *Br* stand for the image of the left hand leaf seen by the right eye; *BC* for that portion of the image of the right hand leaf, which in Fig. 178 is exprest by *bBCc*, seen by the same right eye; *AL* for the image of the right hand leaf seen by the left eye; and *AC* for that portion of the image of the left hand leaf seen by the same left eye, which is expressed in Fig. 178, by *aACc*. Fig. 179.

Now, as we have already taken notice, that the angle *LPr* is about 40°; and that the two angles *rBC*, *CAL* are each of them nearly of 40°; it follows, that the lines *Br*, *AC*, and likewise the lines *BC*, *AL* are nearly parallel. Consequently, the angle *ACB* must also be nearly of 40°: that is, the portions *aACc*, and *bBCc* of the images abovementioned, must appear to make two sides of a wedge, containing an angle of about 40°, which is the wedge-like appearance we were to explain.

726. It remains for us to shew what becomes of *CcaA* the second portion of the fourth part of the image of the right hand leaf seen by the right eye; as likewise of *CcbB* the second portion of the fourth part of the image of the left hand leaf seen by the left eye; neither of which we have as yet accounted for.

In order to which we must observe.

1. That the first of these, *CcaA*, coincides with *aACc* the first portion of the fourth part of the image of the left hand leaf seen by the left eye, and constituting the right hand side of the wedge; and that the second, *CcbB* coincides with *bBCc* the first portion of the fourth part of the image of the right hand leaf seen by the right eye, and constituting the left side of the wedge.

2. That since the book was too near the eyes to have any of these images distinct, that part of each leaf which was farther from the eye it was seen by, must form a distincter, and therefore a stronger image than that part which was nearer to the eye.

Hence it follows, that of the two coinciding images *CcaA* and *aACc*, the latter, which is nearer to the juncture *aA*, and consequently farther from the left eye by which it was seen, must be distincter and therefore stronger than the first, which is farther from the juncture *bB*, and consequently nearer to the right eye by which it was seen.

Therefore

Therefore the first of these *CcaA* must be entirely obliterated and effaced by the latter.

727. In like manner it may be shewn, that *CcbB* is entirely obliterated and swallowed up by *bBCc*.

728. The truth of this explication is confirmed by the following observation.

If one of the leaves, *ex. gr.* the left leaf be blank and the right be print, then *CcaA*, seen by the right eye, though more remote from the juncture *aA*, and consequently nearer to the eye it is seen by, and therefore less distinct, yet, as it makes a stronger appearance on account of the printing, will obliterate and swallow up *aACc*, seen by the left eye, though nearer to the juncture, and consequently more remote from the eye it is seen by, and therefore more distinct, but making a weaker impression: And the letters of the right hand page will appear to be continued from the left hand side of the wedge *bBCc* across the left side *cCAa* of the same wedge. This is easily seen to be effected by the projection of *Cv* Fig. 179 upon *CA*.

729. Likewise, when neither page is blank paper, both sides of the wedge will appear imprinted with the letters or figures of one and the same page, provided they make a stronger impression than those of the other, either by reason of their greater strength or magnitude, or, even in case of equality, by our attending more particularly to them. In which latter case we can vary the phænomenon at pleasure, by fixing our attention, even without changing the posture of our eyes, first upon one page, and then upon the other. From which consideration some other phænomena of coinciding images may be explained, which seem to be otherwise inexplicable.

Fig. 178. 730. It remains for us to shew, why the farthest part of the wedge, as *ABC*, instead of appearing within the book, as the rest of the wedge, is seen without it, and extended to a considerable distance beyond it; also, why it appears in form of a pyramid; and lastly, why its extremity *C* seems to touch the most distant object, towards which the book and the eyes are directed.

Here it is to be observed, that the distinctest and strongest image of the right hand leaf being that seen by the left eye, or *ALSa*, and the distinctest and strongest image of the left hand leaf being that seen by the right eye, as *Brdb*, we look upon the edges of these images, *AL* and *Br*, as the edges of the book.

731. Consequently, *BR* the image of the farther edge of the right hand leaf seen by the right eye, and *Al* the image of the farther edge of the left hand leaf seen by the left eye, must both appear without the book: that is, the lines *BC* and *AC*, part of those edges, and such part of the wedge as lies between these lines, as *ABC*, must appear without the book.

732. And as the objects, upon which the edges *BR*, *Al* are projected in looking horizontally or obliquely downward, are much more remote than those objects upon which the edges *AL* and *Br* are projected; it is manifest, that the edges *BR*, *Al*, and the parts of them *BC*, *AC*, or the part of the wedge *ACB* must appear to be projected to a considerable distance beyond the edges *AL*, *Br*, i. e. beyond the book. And the point *C* appearing on this account to be farther extended beyond the book than any other part of the wedge, the part *ACB* must necessarily put on the appearance of a pyramid having *C* for its vertex.

733. Lastly, as the point *C* is seated in the plane drawn through the right eye and the line *BR*, and likewise in the plane drawn through the left eye and the line *Al*; i. e. in the intersection of those two planes; it must necessarily appear in the same line with any remote object likewise situated in the same intersection.

Therefore the image of *C*, and that of the object, must fall upon contiguous parts of the retina in each eye, and consequently must appear in the same place, or the point *C* must appear to touch the object.

These experiments will succeed by daylight, and with a book of any size, and opened to any angle, *mutatis mutandis*." So far Dr. *Jurin*.

Upon Art. 1016.

734. A comparison of different ways of illuminating microscopical objects, pictures in a magick lantern &c. and of the burning powers of glasses and speculums.

Proposition.

Of a luminous object QR *let* qr *be the image formed by reflection from a concave surface, or by refraction through a convex lens, or sphere* AC; *whose center is* E, *principal focus* F, *axis* QEFC *and semiaperture* AC; *and let a perpendicular* FG, *to the axis, cut the outermost ray* QA *in* G; *I say the brightness of the several pictures* qr, *will be very nearly as* $\overline{FG}^2$ *directly and* $\overline{FE}^2$ *inversely.* Fig. 180. 181. 182.

735. For, not regarding the small losses of light by the several reflections and refractions, the quantity collected to the point *q* is very nearly as $\frac{\overline{AC}^2}{\overline{CQ}^2}$*, and consequently the quantity in the area of the whole picture *qr*, as $\frac{\overline{AC}^2}{\overline{CQ}^2} \times \overline{QR}^2$ or $\frac{\overline{FG}^2}{\overline{FQ}^2} \times \overline{QR}^2$. But the area of the picture

* Art. 58.

picture is as $qr^2 = \frac{\overline{Eq}^2}{\overline{EQ}^2} \times \overline{QR}^2 = \frac{\overline{FE}^2}{\overline{FQ}^2} \times \overline{QR}^2$. Because, in the reflecting concave, we
* Art. 207. have $Fq : FE :: FE : FQ$ * and consequently $Eq : EQ :: FE :: FQ$; and in the lens and
* Art. 239. sphere we have $Qq : QE :: QE : QF$ * and consequently $Eq : EQ :: FE : FQ$. Therefore the brightness of the picture, or the density of the rays in its area, being as their quantity directly and the area inversely, is as $\frac{\overline{FG}^2}{\overline{FE}^2}$ very nearly; and the more exactly as the aperture is smaller and the object farther off.

736. *Corol.* 1. In a given speculum, lens or sphere, the brightness of the picture of a given object, is as $\overline{FG}^2$; and therefore increases continually with the distance of the luminous object from the focus F.

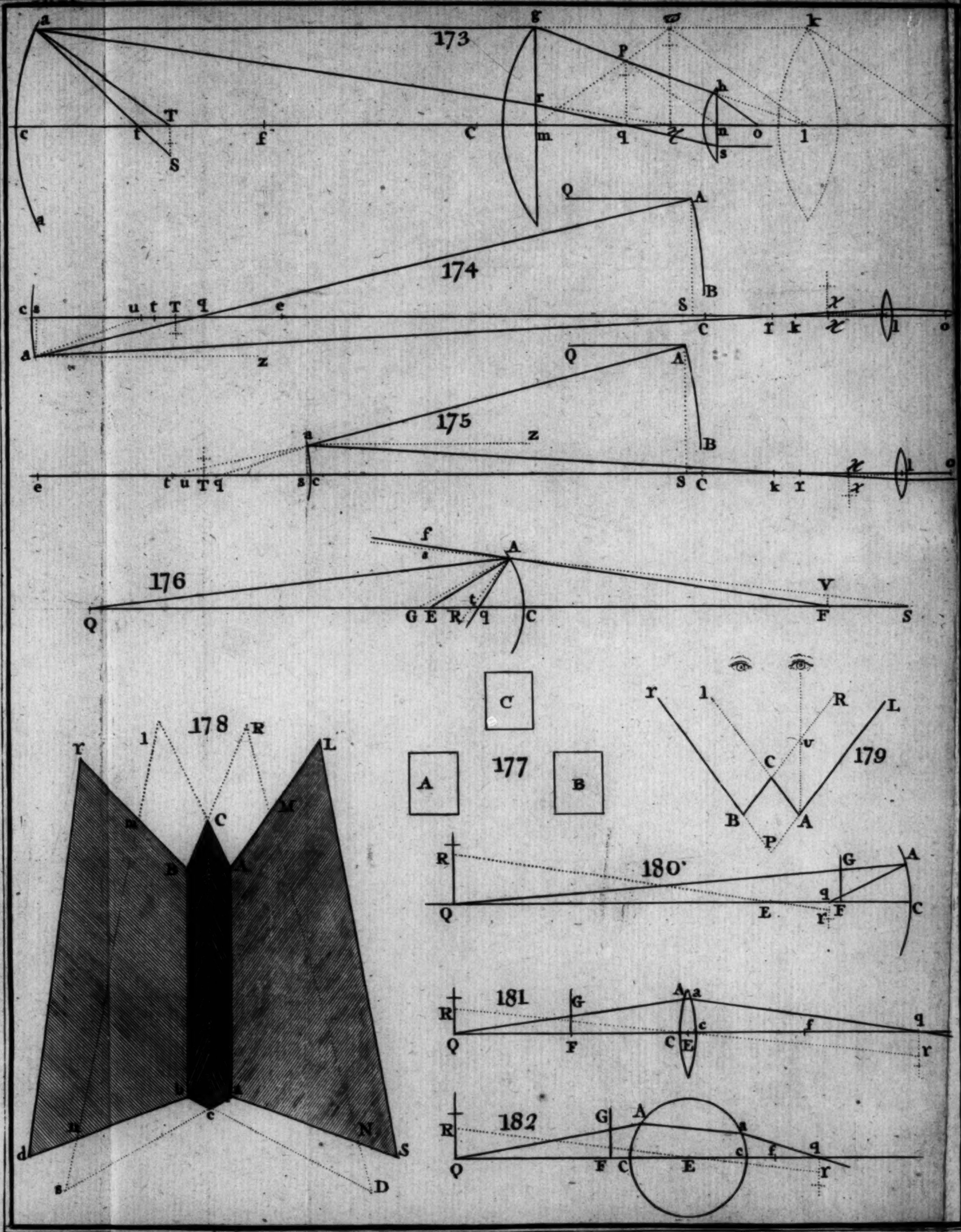
737. *Corol.* 2. If the luminous object be very remote, and the apertures of several specula, lens's and spheres be equal to one another, the degrees of brightness of the several pictures formed by them are reciprocally as the squares of their respective focal distances very nearly.

738. *Corol.* 3. Consequently if the several apertures be equal portions of equal spheres, the degrees of brightness of the several pictures formed by a concave speculum, a double-convex glass, a glass sphere and a plano-convex glass, are respectively as the squares of the decreasing musical progression 12, 6, 4, 3. Because the respective focal distances are $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ of the diameter of the given sphere, by Art. 205, 235, 227: and the reciprocals of an arithmetical progression are called a musical progression.

739. *Corol.* 4. Therefore the concave speculum has greatly the advantage of the sphere and lens's, for illuminating microscopical objects, and also for burning things in the sun-shine, tho' not in so great a proportion, as will appear by the next corollary.

740. *Corol.* 5. Though the rays in two pictures of the sun formed by similar specula, be equally dense *, yet the picture formed by the larger speculum, being proportionably larger, * Rem. 375.
will burn things more vehemently than the smaller; because the burning particles of matter communicate and propagate their heat to one another. And when the specula are similar, the aberrations of the rays, from the peripheries of the pictures, are also similar *. * Art. 337.

End of the REMARKS.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AN

ESSAY

UPON

DISTINCT AND INDISTINCT VISION.

TO Dr. SMITH.

SIR,

As my first entrance upon the following speculation was principally owing to the hints and observations contained in your Book, the greatest part of which I have had by me in print for several years, and I was encouraged to pursue it thus far by the admirable facility of those methods of computation which you have therein delivered, I look upon whatever may be of any value in these papers, as more yours than mine. To you therefore I send them, to make what use of them you think proper.

I might have added a great many other uncommon and hitherto unobserved appearances, particularly in looking thro' small perforations in cards or thin plates, but that I found it would take up too much time to digest them into order and insert them in their proper places; and to retard the publication of your long expected work for any thing of mine, would be doing a sensible injury to the publick. I am,

SIR,

your most affectionate friend,

and very humble servant,

JAMES JURIN.

Objects when seen distinctly.

1. AN object is said to be seen distinctly, when its outlines appear clear and well defined, and the several parts of it, if not too small, are plainly distinguishable, so as that we can easily compare them one with another, in respect to their figure, size and colour. For instance, the words of this book are distinctly seen, when the letters appear well defined, and their shape and the intervals between them are plainly perceived and

and diſtinguiſhed, ſo as that the book may be read with eaſe. A ſingle letter alſo is diſtinctly ſeen, when the ſeveral parts of the letter, the connexion of thoſe parts, and the intervals between them are clearly perceived and diſtinguiſhed.

2. In order to ſuch diſtinct Viſion, it has hitherto been commonly thought, that all the rays of a pencil flowing from a phyſical point of an object, muſt be exactly united in a phyſical, or at leaſt in a ſenſible point of the *Retina*.

Exact union of rays not neceſſary.

3. But that ſuch an exact union of the rays is not neceſſary to diſtinct viſion, will manifeſtly appear upon making the following tryals.

Experiments to prove it.

4. Take a title-page of a book, in which there is print of three or four different ſizes: And firſt, place the book at ſuch a diſtance, as that every ſort of print may without any ſtraining of the eye, appear perfectly diſtinct. In this caſe it may reaſonably be preſumed, the rays of every pencil flowing from the letters are accurately collected into ſo many ſeveral phyſical, or at leaſt, ſenſible points upon the *Retina*.

5. Afterwards bring the book by degrees ſo near, as that the letters of the ſmalleſt print may now begin to appear a little confuſed, and cannot by any endeavour or ſtraining of the eyes be rendred ſo diſtinct as they were before. Then, keeping the book at that ſame diſtance, look at a print ſomewhat larger than the former, and that larger print ſhall ſeem perfectly diſtinct without any the leaſt appearance of confuſion.

Here, it is manifeſt from the leſs diſtinct appearance of the ſmaller print, that at this diſtance the rays of each pencil are not accurately united in a ſenſible point of the *Retina*, notwithſtanding which the larger print appears diſtinct.

6. If the book be brought ſtill nearer, the ſmalleſt print will now be quite confuſed, and the larger print will begin to appear indiſtinct: But keeping the book at this ſame nearer diſtance, a print ſtill larger will appear diſtinct.

In this caſe the rays are ſtill leſs accurately collected into points; and yet the largeſt of theſe prints appears as diſtinct as the two ſmaller prints had done in the former tryals.

7. Theſe experiments may be made the contrary way likewiſe, by putting on a pair of ſpectacles of a ſufficient convexity, or by a ſhortſighted perſon without any ſpectacles. In this caſe the book muſt firſt be held at ſuch a diſtance, as that every ſort of print may appear as diſtinct as poſſible: And muſt afterwards be removed ſucceſſively to ſuch greater diſtances from the eye, as that the ſmaller prints may one after another begin to appear confuſed, while the larger preſerve their diſtinctneſs.

8. To thoſe, who ſhall make theſe experiments with ſufficient care, it will manifeſtly appear, that ſometimes we have diſtinct viſion, when the rays of each pencil iſſuing from the object are not accurately collected by the eye into a ſenſible point upon the *Retina*; and that this ſort of diſtinct viſion, without an exact union of the rays, is not to be diſtinguiſhed, even by an attentive eye, from that other ſort of diſtinct viſion where the rays are the moſt accurately collected into ſenſible, or even into phyſical points.

Two ſorts of diſtinct Viſion.

9. Diſtinct Viſion may therefore not unfitly be divided into the two following ſorts, or ſpecies, namely, Viſion perfectly diſtinct, or *Perfect Viſion*, and Viſion imperfectly diſtinct, which I ſhall uſually call, ſimply, by the name of *Diſtinct Viſion*.

Perfect Viſion.

10. Viſion perfectly diſtinct, or *Perfect Viſion*, is that, in which the rays of a ſingle pencil are collected into a ſingle phyſical, or ſenſible point of the *Retina*, as in ART. 4.

Diſtinct Viſion.

11. Viſion imperfectly diſtinct, or ſimply, *Diſtinct Viſion*, is that, in which the rays of each pencil are not collected into a ſenſible point, but occupy ſome larger ſpace upon the *Retina*, yet ſo as that the object is diſtinctly perceived, as the larger point in Art. 5 and 6.

12. *Perfect Viſion*, in a given eye, and a given diſpoſition of that eye, depends only upon the diſtance of the object. It has no dependance upon the magnitude of the object, except only that it is requiſite the object ſhould not be ſo very ſmall as on that account not to be perceivable. This excepted, at whatever diſtance any one object is ſeen perfectly diſtinct, all other objects do likewiſe appear perfectly diſtinct.

13. *Diſtinct Viſion*, in a given eye, and a given diſpoſition of that eye, depends upon the diſtance and magnitude of the object jointly. In Art. 5. the leſſer print appears confuſed, becauſe it is too near: But, at a greater diſtance, as in Art. 4. it appeared diſtinct.

And

And the larger print in Art. 5. appears diſtinct becauſe of its magnitude: But it will appear confuſed at a leſs diſtance, as in Art. 6.

14. It appears therefore, that there is a real difference between *Perfect Viſion*, and what we call *Diſtinct Viſion*, and we deſign to enquire particularly into the reaſon, Why an object may be ſeen diſtinctly without *Perfect Viſion*: But in order thereto, it will firſt be neceſſary to take notice of the principal *Phænomena* when objects are confuſedly or indiſtinctly ſeen, and to examine into the cauſes of thoſe appearances.

In doing which, we ſhall generally ſpeak of the object we conſider, as being white or luminous upon a dark ground, though the *Phænomena* would be the ſame, and from the ſame cauſe, if the object were dark or black upon a white ground.

15. If a circular object be viewed at a diſtance proper for *Perfect Viſion*; its picture upon the *Retina* will be circular, and proportional in diameter to the angle which the object ſubtends at the eye; its limb will be well defined, and all parts of the circular picture will be equally ſtrong. Conſequently, the idea thereby excited in us, will be that of a circle equally ſtrong in all parts, and well defined.

Phænomena of indiſtinct Viſion in a circular object.

16. If the ſame circular object be viewed at a diſtance much too ſmall for *Perfect Viſion*; its picture upon the *Retina* will be circular, but the diameter will be greater than in proportion to the angle which the object ſubtends at the eye: Nor will the picture be equally ſtrong in all its parts, but the middle part will generally be the ſtrongeſt, and will be ſurrounded with a *penumbra* growing gradually fainter towards the outer edge, whereby the limb will appear indiſtinct and ill defined. Conſequently, the idea thereby excited in the mind will be that of a circle too large, and too faint and indiſtinct towards the limb. That is, inſtead of the appearance *A* Fig. 1. we ſhall have the appearance, *B*, or *C*, or *D*.

17. The cauſe of this is not difficult to diſcover. For ſince, by our ſuppoſition, the diſtance of the circular object from the eye is too ſmall for *Perfect* or *Diſtinct Viſion*, it is obvious that the rays of each pencil iſſuing from the object cannot be united but at a point beyond the *Retina*; conſequently, the rays of each pencil will occupy a circular ſpace upon the *Retina*.

Let therefore, the circle *A B D C*, Fig. 2. repreſent that circular ſpace upon the *Retina*, which the image of the object would take up, if that image were perfectly diſtinct: Or, which comes to the ſame thing, let *A B D C* repreſent that circular ſpace upon the *Retina*, which is occupied by the centers of all the pencils of rays belonging to the indiſtinct image of the circular object. This circular ſpace upon the *Retina*, repreſented by *ABDC*, we ſhall call the image of the object, and ſometimes, to diſtinguiſh it from other appearances, we ſhall call it the ***true image***. Alſo let the circle *f g h c*, having its center *c* in the circumference of the former circle *A B D C*, repreſent that circular ſpace upon the *Retina*, which is taken up by one of the extreme pencils of rays iſſuing from the object. This circle *fghc* we ſhall call the ***circle of diſſipation***, becauſe the rays of a pencil, inſtead of being collected into the central point *c*, are diſſipated all over this circle: And the radius of this circle, *cf*, or *cg*, we ſhall for the ſame reaſon call the ***radius of diſſipation***. And let the circle *abdfC*, concentrick with the firſt circle, or the ***true image***, *A B D C*, touch the circle *fghc*, in the point *f*.

True image what.

Circle of diſſipation what.

Radius of diſſipation what.

18. Then, I ſay, 1. that part of the image of the circular object, which is repreſented by the circle *abdfC*, will be equally ſtrong in all parts; and will be of the ſame ſtrength, as if the image of the object had been perfectly diſtinct.

To prove this, let the circles *ABDC*, *abdC*, Fig. 3. repreſent the ſame things as before, and taking any point at pleaſure, as *c*, within the circle *abdC*, from that point as a center, with the ***radius of diſſipation cf***, draw the circle *fghc*. Then it is manifeſt, that, as the pencil of rays whoſe center is the point *c*, is diſſipated or ſcattered over the whoſe circle *fghc*, and thereby helps to illuminate all the points or centers of pencils ſituated within that circle; ſo on the other hand, the point *c* muſt receive light from every pencil whoſe center is ſituated within the ſame circle: Or, the point *c* beſtows a part of its light upon every other point of the circle *fghc*, and receives an equal portion of light from each of thoſe points; ſo that it is juſt as much illuminated, as if the rays of its pencil had never been diſſipated, but had been all collected into that one point *c*, as they would have been, if the image had

had been perfectly distinct. And as this is true of every point within the circle *abdC*, it is manifest, that this whole circle must be as strongly illuminated, as if the image had been perfectly distinct, and must be equally strong in all its parts.

False image what.

This part of the *true image*, represented by the circle *Cabd*, which loses none of its light by dissipation, but is as strongly illuminated as if it were seen by *Perfect Vision*, and is equally luminous in all parts of it; we shall, for distinction sake, call the *false image*.

19. I say, 2. The circular ring *ABDdba*, comprehended between the circumferences of the two circles *ABD*, *abd*, Fig. 2, 3, whose breadth is equal to the *radius of dissipation*, or that part of the *true image ABDC* which lies without the *false image abdC*, will not be so strongly illuminated as the *false image abdC*, and will gradually grow fainter towards its extremity.

For, let the circles *ABDC*, *abdC*, Fig. 4. represent the same things as before, and taking two points within the circular ring, *ABD dba*, one more inward as *c*, and the other more outward as *m*, from the centers *c* and *m*, with the radii *cf*, *mn*, each equal to the *radius of dissipation*, draw the two circles *ch fg*, *mno*, cutting the circumference *ABD* in the points *h* and *f*, *n* and *o*, respectively.

Then is it plain, that the pencil, whose center is the point *c*, will dissipate its rays into the whole circle *chfg*; but that it will not receive light from every point in that circle, but only from those points of it, that are likewise situated within the circle *ABDC*. All those points therefore, that are comprehended within the *Lunula hf*, return no light to the point *c* in recompence of what they receive from it. The point *c* therefore gives away more light than it receives, and will consequently appear darker than any point within the circle *abdC*; and this excess of darkness will be measured by the area of the *Lunula hf*.

In like manner it will be found, that the point *m* must appear darker than any point in the circle *abdC*, and that this excess of darkness is measured by the *Lunula no*. But the *Lunula no* is greater than the *Lunula hf*, and consequently the point *m* which is nearer to the outside of the ring, is darker than the point *c* which is more inwardly situated. The whole ring therefore is darker than any part of the circle *abdC*, and grows gradually darker towards its outer edge. And at the very extremity it has not half the light of any part of the circle *abdC* as is manifest by the inspection of the circle *gfhc*, Fig. 2.

20. I say, 3. Besides the ring last described, which is darker than the *false image*, or the circle *abdC*, there is another ring of equal breadth, situated without the *true image*, or the circle *ABDC*, which is still darker, and whose light gradually diminishes towards the outside, till it becomes insensible, and at last vanishes away into nothing.

For, let *ABDC*, Fig. 5. represent the same thing as in Fig. 2, and from the center *c* taken any where in the circumference *ABD*, with the *radius of dissipation cg*, draw the *circle of dissipation cghf*: and from the center *C* describe the circle *GFHC*, touching the *circle of dissipation* in the point *g*. Then, between the two circumferences *ABD* and *GFH* will be comprehended a new ring *ABDHFG*, of the same breadth with the former ring *ABD dba*, Fig. 2, 3, 4. This new ring will receive light from the pencils, whose centers lie within the former ring; but it will be darker than any part of the former, and its light towards the outer edge will gradually diminish till it vanishes into nothing. This is manifest from the inspection of the figure, where the light received by the point *c* situated on the inner edge of this ring is measured by the circular segment *hf*, and the light received by the point *m* situated near the outer edge is measured by the much smaller circular segment *no*.

This second ring, which lies without the *true image*, and gradually decreases in light towards its outside, we shall take leave to call for distinction sake, the *annular penumbra*.

Annular penumbra what.

From what has been said it appears, that if the radius *CA* of the circle *ABDC*, or of the *true image*, be called *r*, and the radius of the *circle of dissipation cfgh* be called ϱ, the radius of the *false image*, or of the circle *abdC*, will be the difference of those two radii, or $r-\varrho$, the breadth of each of the rings *ABDdba*, Fig. 2, and *FGHDBA*, Fig. 5, will be ϱ, and the radius of the whole appearance or of the circle *CFGH* will be the sum of the radii of the *true image* and of the *circle of dissipation*, or $r+\varrho$.

Having shown that a circular object, when viewed at too small a distance for *Perfect* or

Distinct

Diſtinct Viſion, muſt appear more ſtrongly illuminated in the middle and fainter towards the edge, I proceed now to conſider ſome particular cauſes of this phœnomenon, which I ſhall have occaſion to apply hereafter to other purpoſes.

21. If in Fig. 2, the *radius of diſſipation* be increaſed, the *falſe image*, or the circle *abdC*, which preſerves its full quantity of light as much as if the image were perfectly diſtinct, will grow leſs, and the ring *ABD dba* and the *annular penumbra* without it, *ABDHFG*, will both grow proportionably broader, as in Fig. 6, and 7, ſo that the circular object, from appearing as *B*, Fig. 1, will come to the appearance *C*, or to ſome other where the *nucleus*, which is fully illuminated, will be ſtill ſmaller, and the ſhade ariſing from the two rings abovementioned, will be broader.

22. If the *radius of diſſipation* be equal to the radius of the *true image*, as in Fig. 8, the *falſe image*, or the circle *Cabd*, whoſe radius is $r-\varrho$, will now vaniſh, and the ring *ABD dba*, Fig. 6, 7, will degenerate into and take up the whole circle *ABDC*. Conſequently, there will now be no part of the image, that will have its full quantity of light as if the image were perfectly diſtinct, except the central point alone which will receive light from every other point; nor will there be any two points at different diſtances from the center, that will be equally illuminated: but the light decreaſing all the way from the center will grow gradually fainter towards the extremity, as is manifeſt from Fig. 9, 10, in each of which the meaſure of light received by the point *c*, nearer to the center in the one, and more remote in the other is expreſſed by the gibbous ſegment *gh*. For in the firſt of theſe Figures, where the point *c* is nearer to the center C of the image, the gibbous ſegment *gh* and conſequently the light thrown upon the point *c*, is greater than in Fig. 10, where the point *c* is more remote from the middle of the image.

23. If the *radius of diſſipation* exceed the radius of the *true image*, as in Fig. 11, we ſhall again have a portion of the *true image*, which will be more ſtrongly illuminated than the reſt, though not ſo ſtrongly as by *Perfect Viſion*, and which will be equally luminous in all parts. And this portion will be ſeated in the middle of the *true image*, and will be ſurrounded by an appearance leſs luminous and decreaſing gradually in light towards the outer edge.

For in Fig. 11. let the circle *CABD* repreſent, as before, the *true image* of a circular object upon the *Retina*: And taking a point as *c* in the circumference *ABD*, from the center *c*, with the *radius of diſſipation cb*, greater than *Cc* the radius of the *true image*, draw the *circle of diſſipation cfgh*. Then from the center *C*, with the radius *Cb*, equal to the difference between the *radius of diſſipation bc*, and the radius of the *true image Cc*, deſcribe the circle *Cabd*, touching on the inſide the *circle of diſſipation* in the point *b*. Alſo, from the ſame center *C*, with the radius *Cg* equal to the ſum of the *radius of diſſipation cg*, and the radius of the *true image Cc* added together, draw the circle *CFGH*, touching the *circle of diſſipation* on the outſide in the point *g*.

24. Then, I ſay, 1. that part of the *true image*, which is repreſented by the circle *Cabd*, will be equally luminous in all parts. For the point *b* in the circumference of this circle will be diſtant from *c*, the moſt remote point of the *true image*, by juſt the *radius of diſſipation bc*, and will therefore receive light from that point *c*. Much more will the point *b* receive light from every other point of the *true image* leſs remote from it than the point *c*. And much more ſtill will any other point within the circumference *abd*, receive light from every point of the *true image*.

Since therefore each point of this circle *Cabd* receives light from every point of the *true image*, this circle muſt be equally illuminated in all parts.

25. I ſay, 2. this circle *Cabd* will not be ſo ſtrongly illuminated, as if it were ſeen by *Perfect Viſion*.

For, ſince any point in this circle, as *C*, ſcatters its light into a circle as large as the *circle of diſſipation*, and receives light only from the circle *CABD*, leſs than the *circle of diſſipation*, it is manifeſt, that this point gives away more light than it receives back, and conſequently muſt be leſs luminous than if it were ſeen by *Perfect Viſion*.

26. I ſay, 3. that this circle *Cabd* will be ſurrounded by an appearance leſs luminous, and gradually decreaſing in light towards its outer extremity.

For around this circle will appear the ring *abdDBA*; and without that ring will appear the

the *annular penumbra ABDFGH*, in the same manner and for the same reasons, as in Art. 19, 20.

Faint false image what. 27. This circle *Cabd*, which is equally luminous in all parts, and brighter than all the rest of the appearance, though not so bright as if seen by *Perfect Vision*, we shall for that reason call the *faint false image*.

From what has been said, it is manifest, that if, as before, we call the radius of the *true image*, r, and the *radius of dissipation*, ϱ, the radius of the *faint false image* must be $\varrho - r$, the breadth of the first ring without the *faint false image* will be $\varrho - 2r$, the breadth of the second ring or *annular penumbra* will be ϱ, and the radius of the whole appearance, or of the circle *CFGH*, will be $r + \varrho$, these two last the same as in Art. 20.

And if the *radius of dissipation cb* be encreased, the *faint false image Cabd* will increase and approach nearer to the magnitude of the *true image CABD*; and thereby the ring *abdDBA* will grow narrower.

28. If the *radius of dissipation* be equal to the diameter of the *true image*, as in Fig. 12, 13, the *faint false image*, or the circle *Cabd*, whose radius is $\varrho - r$, or $2r - r = r$, will now be equal to and coincide with the *true image CABD*. And this image will be equally illuminated in all parts, but each point will have but a quarter part of the light, and the whole image will be but a quarter part as strong, as if it had been seen by *Perfect Vision*.

For any point, as c, taken in the extremity of the image *CABD*, Fig. 12, will dissipate its light into the circle *gfbc*, and will receive light only from the circle *ABDC*, which is but a quarter part of the *circle of dissipation*. And another point as c taken in the middle of the image *CABD*, Fig. 13, will likewise dissipate its light into the circle *gfbc*, and will receive light only from the circle *ABDC*, which is just a quarter part of the *circle of dissipation*. The middle therefore and the extremities of the image *ABDC* will be equally illuminated, having each a quarter of the light they would have had, had the image been perfectly distinct.

29. The ring *abdDBA*, Fig. 11, whose breadth was $\varrho - 2r$, which was comprehended by the circumferences of the *faint false image* and the *true image*, will now vanish by the coincidence of those circumferences.

30. But there will still appear the *annular penumbra*, incompassing the image, which will be fainter than the image, and will gradually grow weaker towards the extremity, till it vanish and decline into nothing. And the breadth of this *annular penumbra* will be equal to the *radius of dissipation*.

For, in Fig. 14, 15, 16, let the circle *ABDC* represent, as before, the *true image* of the circular object upon the *Retina*, and with the radius *CG*, composed of *CA*, the radius of the *true image*, and the *radius of dissipation* added together, draw the circle *GFHC* concentrick with the circle *ABDC*: Then will the ring *GFHDBA* comprehended between the two circumferences *GFH* and *ABD*, be equal in breadth to the *radius of dissipation*.

Likewise, from any point of this ring as c, with the radius *cg* equal to the *radius of dissipation*, draw the *circle of dissipation cfgb*.

31. Then, I say, 1. this ring will form a *penumbra* fainter than the image.

For the point c will receive light only from the pencils whose centers are situated in the circular segment *fg*. Therefore this point c, and consequently every point in the ring, will be less illuminated than every point within the image; since every point within the image, does, as we have already seen, (Art. 28.) receive light from every other point within the image.

32. I say, 2. this *penumbra* will grow gradually fainter towards its outer edge, and at the very extremity will dwindle into nothing.

For, if the point c be gradually removed farther towards the outer edge, as in Fig. 14, 15, it is manifest from the inspection of the figures, that the circular segment *fg*, and consequently the light derived from that segment to the point c, will gradually diminish: And when the point c arrives at the very extremity of the ring, as in Fig. 16, the segment *fg* and the light derived from it must vanish into nothing, the *circle of dissipation* then only touching the circle *ABD*.

33. If the *radius of dissipation* exceed the diameter of the *true image*; then to the *true image* will be joined a ring of scattered light, which ring will be illuminated equally with the *true image*, so that the *true image* itself and this ring joined together will form one appearance, or *faint false image*, equally strong in all parts, but of a far less strength than if the image were perfectly distinct.

Also

Also round this appearance, or *faint false image*, will be formed an *annular penumbra*, the light of which will gradually diminish towards its outer edge, and at the very extremity will dwindle away into nothing.

For in Fig. 17, 18, 19, let the circle *ABDC*, as before, represent the *true image* of the circular object upon the *Retina*, and with the radius *CG*, composed of *CA* the radius of the *true image* and the *radius of dissipation* added together, describe a new circle *FGHC*, concentrick with the *true image* *ABDC*. Also, with the radius *Cb*, equal to the difference between the *radius of dissipation* and the radius of the *true image*, describe a third circle *Cabd* concentrick with the two former.

34. Then, I say, first, the ring *ABDdba*, comprehended between *ABD* the circumference of the *true image* and the circumference *abd*, will be equally illuminated with the *true image* *ABDC*.

For let any point as *c* be taken in the outer edge of this ring, Fig. 17, and with the radius *co*, equal to the *radius of dissipation*, draw the circle *cofgh*. This circle must touch the circumference of the *true image* in the point *o* opposite to *c*, because *co* is the *radius of dissipation*, and *cC*, by the construction, was equal to the difference between that radius and the radius *Co*. The point *c* therefore must receive light from every pencil whose center lies in the *true image* *ABDC*, and must consequently be illuminated equally with any point within the *true image*. Much more must any other point of the ring, situated more inwardly than *c*, be equally illuminated with any point within the *true image*. Consequently, the whole ring will be equally illuminated with the *true image*, and will form one uniform appearance with it, without any distinction, that is, a *faint false image*, like that described in Art 23, &c.

35. I say, secondly, this whole appearance, or *faint false image*, composed of the ring *ABDdba* and the *true image* *CABD* will be of far less strength than the *true image* alone would have been, were it formed by vision perfectly distinct. For the light of all the pencils belonging to the *true image* would in the case of *Perfect Vision* be confined to that image only, whereas now it is scattered thro' the whole appearance, or *faint false image*, composed of the ring and *true image*, and consequently must be much weaker. Besides, a part of this light is farther scattered beyond the ring. For,

36. I say, thirdly, the ring *abdHGF*, comprehended between the two circumferences *abdc* and *HGF*, will form a *penumbra* round the *faint false image* above mentioned, which *annular penumbra* will gradually decrease in strength towards its outer edge, till at the very extremity it vanishes to nothing.

For the point *c*, Fig. 17, at the inner edge of this ring, will be equally luminous with any point in the *true image*, as has been already shown, in Art. 34. And at the point *e*, Fig. 18, situated in the outer edge of this ring, the light will quite vanish, the circle of dissipation *efgho* only touching the *true image* in the point *o*. Therefore all the intermediate points of the ring, as *c*, Fig. 19, must be illuminated with intermediate degrees of light decreasing towards the outer edge of the ring; that is, the ring will form a *penumbra* decreasing gradually in strength towards the outer edge, and vanishing away at the outer edge itself.

37. From this construction it appears, that if, as before, we call the radius of the *true image* r, and the *radius of dissipation* ϱ, the radius of the *faint false image* will be $\varrho - r$, the radius of the whole appearance will be $\varrho + r$, and consequently the breadth of the *annular penumbra* will be $2r$.

38. If the radius of the *true image* be exceeding small in respect of the *radius of dissipation*, the *faint false image*, or the circle *Cabd*, whose radius is $\varrho - r$, Fig. 17, 18, 19, will be very nearly equal to the *circle of dissipation*. Also the breadth of the *annular penumbra* *abdHGF*, whose breadth is $2r$, by Art. 37, will be exceedingly small, so that the whole *penumbra* will be utterly insensible.

39. It may not here be amiss to make one or two general remarks with regard to these *annular penumbra* of circular objects, when the *radius of dissipation* is given. Remarks upon *annular penumbra*.

1. The *annular penumbra* will be sensible nearer to its outer edge, when the object is brighter, than when it is less luminous.

For the light in a given point of the *annular penumbra* will, *cæteris paribus*, be proportional to the luminousness of the object. Therefore, if at a given distance from the

outer

outer extremity of the *annular penumbra* of a brighter object, the light is but just strong enough to affect the sense; it will not at all affect us at that same distance, when the object is less luminous.

2. The *annular penumbra* will be sensible nearer to its outer edge, when the object is larger, than when it is smaller.

For the light in a given point of the *annular penumbra* will, *cæteris paribus*, be proportional to the circular segment cut off from the *true image* by the *circle of dissipation*, of which the given point is the center. Let therefore the point *m*, in Fig. 5, be taken so near the outer extremity of the *annular penumbra*, as that the light in that point may be but just strong enough to affect the sense. Then will the *circle of dissipation* belonging to this point *m*, cut off from the *true image* *ABDC*, a circular segment *no*, of a given depth; and the light in the point *m*, will be proportional to that segment, *no*. But if the circular object, or the *true image*, *ABDC* be diminished, the circular segment *no*, whose depth is given, will have a less breadth, and consequently a less magnitude. Therefore the light in the point *m* will not now be strong enough to affect the eye.

40. As the reason of all these *phænomena* is only deduced from this one principle, that a pencil of rays issuing from the object is not collected into a point upon the *Retina*, but occupies a circular space thereon, it is manifest that in all cases where a pencil takes up a circular space upon the *Retina*, the *phænomena* will be the same.

But when an object is too remote for *Perfect Vision*, the rays of a pencil issuing from it will converge to and unite in a point, before they arrive at the *Retina*, and by diverging from that point will take up a circular space upon the *Retina*.

Therefore all the above recited *phænomena* may arise from a circular object placed at too great a distance for *Perfect Vision*.

41. Now, the human eye, as we shall have occasion anon particularly to shew, when not too much flatted by age, is only fitted to see distinctly at moderate distances from our bodies, and is not at all accomodated to see objects very remote, tho' at much less distances than those of the Planets or Fixed Stars, with perfect distinctness. Therefore a Planet or Fixed Star must to our eyes exhibit some of the *phænomena* abovementioned, and it will not be amiss to consider the principal of them.

42. The full Moon will appear broader than a circular object subtending an equal angle seen by *Perfect Vision*. Appearance of the full Moon.

For, by Art. 20, if the circle *ABDC*, Fig. 5, represent the *true image* of the full Moon upon the *Retina*, or the image of a circular object seen by *Perfect Vision*, and subtending an equal angle with the full Moon; without this circle there will appear a *penumbra* represented by the ring *ABDHFG*, which being added to the circle *ABDC* will constitute the whole appearance *GFHC*, which is greater than *ABDC*.

43. If the Moon instead of being a globe, were only a plane disk, something scabrous, so as to reflect the light equally every way from every part of it; it would then exhibit all the *phænomena* described in Art. 18, 19, 20.

For instance, if this plane disk were to subtend at the eye an angle of 32′, and consequently its *true image* *ABDC*, Fig. 2, upon the *Retina* were such, as corresponds to an angle of 32′ subtended by this disk at the eye; and the *radius of dissipation* in a human eye be such as corresponds to an angle of 2′ subtended at the eye by a very distant object, as I think it generally is, in what are reckoned good eyes, upon viewing celestial bodies; then by Art. 18, a part of the *true image*, represented by the circle *abdC*, would be brighter than the rest, and the diameter of this part would be 28′: And by Art. 19. another part of the *true image*, represented by the ring *abdDBA*, 2′ in breadth, would gradually diminish in brightness till at its outer extremity *ABD* it had less than half the brightness of the part *abdC*: And this ring by Art. 20, would be surrounded by the *annular penumbra* *ABDHFG*, Fig 5, likewise 2 minutes in breadth, which from having an equal brightness with the outer edge of the ring within it, would gradually grow weaker towards its outer edge, and there would vanish to nothing. So that the breadth of the whole appearance composed of the *true image* and this *annular penumbra* would be 36′, except that a little of the outer part of the *annular penumbra* might be too faint to be discerned, and consequently the breadth of the whole appearance might be something less than 36′.

44. But

44. And farther, If the surface of the moon, although spherical, yet were neither polished, nor considerably uneven, but a little scabrous all over, so as to reflect the light equally every way from all parts of it; then this surface ought to reflect light to the eye from every part of it, just in the same manner as the corresponding parts of a plane surface, understood to be likewise a little scabrous, into which the parts of the spherical surface are projected by the eye. Consequently, this scabrous spherical Moon ought to exhibit all the same *phænomena* as the scabrous plane disk of the preceding article.

45. But, in fact, the Moon does not exhibit all these *phænomena*. The middle part does not appear brighter than the limb. On the contrary, the limb of the Moon for a digit or two in breadth appears as bright, if not brighter than the middle. What should be the cause of this appearance, which from our reasoning above seems so little to be expected?

46. This appearance proceeds, as I apprehend, from the two following causes.

1. The middle part of the Moon is in great measure taken up with the faint resemblance of the human face, which is darker than the other parts, and being likewise more depressed than the rest, is therefore by some Philosophers taken for seas in the Moon: But the outer part of the Moon seems to have a much less proportion of those dark parts. This is one reason, why the outer part should appear brighter than the middle.

2. Although the outer part of the Moon had an equal proportion of what is taken for sea, with the middle part; yet it must appear brighter than the middle part. For the sea in the middle part lies directly exposed to the eye, and consequently is seen of its full magnitude in proportion to the land: But the seas towards the limb must be either wholly, or in great part hid from the eye by the prominence of the land, by reason of the convexity of the spherical figure; just as the cliffs at *Dover*, to an eye raised to a great height in the air and viewing them obliquely, might be so projected, as to appear adjoining to those at *Calais*, and by that means to hide the intervening sea; or as the water of a river is hid from an eye placed upon a hill, by the seeming union of its opposite banks.

47. These two causes may have such an effect as to render the limb of the Moon, notwithstanding the dissipation of its light by indistinct vision, yet still considerably stronger than the middle part, where no light is lost by dissipation.

For instance, if the limb by distinct vision be above twice as bright as the middle part, it may notwithstanding it loses half its light by the dissipation, be still brighter than the middle of the Moon.

48. The second of these causes may give us the reason, why the *annular penumbra*, which by our theory ought to encompass the Moon, may decrease very slowly in light towards its outer edge, so as that the inequality of brightness between its inner and outer part may be less sensible than it would otherwise be.

For, if a point as *c*, Fig. 5, be taken near the inner edge of this *penumbra*, and another point as *m*, be taken near its outer edge, the light thrown upon the first point *c* will bear a much less proportion to the light thrown upon the second point *m*, than the proportion between the circular segments *fh*, *no*, which are cut off from the *true image* ABDC by their respective *circles of dissipation* *cfgh*, *mno*, by reason that the segment *no* does in proportion to its magnitude give a greater quantity of light than the segment *fh*.

49. And if to this consideration we add, that the outer edge of the *annular penumbra* will have the advantage of having its light heightned by comparison with the adjoining darkness of the sky, it will appear upon the whole, that the degrees of light towards the outer edge of this *penumbra* ought hardly to be sensible.

50. But now another difficulty presents itself. If the Moon appears to our eyes 36′ in breadth, when in fact its *true image* is but 32′, the ancient astronomers, and those of the moderns likewise, who observed by plain sights, as the noble *Hevelius*, must have made the apparent mean diameter of the Moon much larger than it is found to be by telescopick sights, that is, by *Perfect* or at least by *Distinct Vision*.

51. I answer, that the observations, which are handed down to us from antiquity, are probably those of the most skilful and most experienced observers. But those persons, who have been much accustomed to contemplate very remote objects, as must be the case of the most experienced astronomers, have

have thereby acquired a facility, or habit rather, of so altering the conformation of their eyes, as to see those remote objects much more distinctly than the rest of mankind, who have not been so accustomed; just as those, who are much employed in viewing small objects very near, as engravers, painters in miniature, *&c.* do thereby acquire the facility and habit of seeing those near and small objects much more distinctly than other persons. We must therefore suppose, that though the *radius of dissipation* in viewing the stars or planets be 2′ in the eyes of the generality of Mankind, yet to practical astronomers it may be less than one minute, possibly not above half a minute, and then the diameter of the Moon observed by plain sights will exceed the diameter taken by telescopick sights by about 1′ only.

52. But farther, it is possible, that the observations of the ancient astronomers which are handed down to us, are not the product of their younger days, but made when they were grown famous and were advanced in years, at which time, no doubt, some of them might correct the observations they had made when younger. Consequently, their eyes being now somewhat flatted by age, might be thereby the better fitted for seeing distinctly at a great distance; and by this means the *radius of dissipation* would be rendered still less, and their observations would come nearer to the truth. At least, that this was the case of *Hevelius*, I shall have occasion anon to shew.

Appearance of the New Moon.

53. In the new Moon of three or four days old, the illuminated part must appear too broad in proportion to the obscure part, and likewise must seem to extend more outwards, or to have a greater diameter than the obscure part.

For, in Fig. 20. let *CABDE* represent the *true image* of the whole Moon, *ABDIA* the *true image* of the enlightned part, *AIDEA* the *true image* of the obscure part, *abd* the outer edge of the *false image* of the obscure part, *FGH* the outer edge of the *annular penumbra* of the whole Moon, *LGMKL*, the whole appearance of the enlightned part consisting of the *true image* and the *penumbra* added to it.

54. Then this whole appearance of the illuminated part is manifestly too broad in proportion to the obscure part, for as much as to the *true image* of the illuminated part is added not only the *annular penumbra LGMDBAL* on the outside, whose breadth *BG* is equal to the *radius of dissipation*, but also a like *penumbra* on the inside of the same breadth, *LKMDIAL*; whereas the obscure part is lessened by the space which this last *penumbra* takes up.

55. Likewise, as by Art. 19, 20, the light of the obscure part diminishes outwards from the edge of the *false image abd*, it can hardly be visible much more outwardly than that edge, especially where it is contiguous to the stronger light of the cusps. Consequently, it must appear of a less diameter than the illuminated part, the stronger light of which is sensible farther outward.

56. For the same reason as the illuminated part of the New Moon appears too broad in proportion to the obscure part; in an eclipse of the Sun or Moon the bright part must appear too broad in proportion to the dark part, or the eclipse must appear less than it really is. Conformable to this is the observation of our famous countryman, Mr [a] HORROX. *Nudi oculi defectum semper justo minorem exhibent: at Telescopium veram exhibet tum defectus tum diametri lunaris quantitatem.*

That the effect we have been speaking of, arises from the cause here assigned, and not from the principle that has sometimes been made use of to account for such like *phænomena*, viz. that a light object affects the *Retina* to a greater distance than a dark one, will plainly appear by the following experiment.

Let a representation of the new Moon, Fig. 21, or the circle in Fig. 22, half black, half white, be viewed at a distance proper for *Perfect Vision*, and these two figures will appear in their just proportion. Then let them be viewed at a distance too small for *Perfect Vision*, and the light part will appear to intrench upon the dark part, and likewise to extend outwards beyond the dark part. Afterwards let them be removed to a distance too great for *Perfect Vision*, which is easily done by a shortsighted person, or by one longsighted, by applying a convex-glass to his eye, and the white part will again intrench upon the dark part, and will extend outwards beyond the dark part.

[a] Venus in sole visa. Cap. xvi.

N. B. To make this experiment succeed clearly, the figure must not be bounded with a black line, as is here represented, but the whole circle must be cut in white paper, the dark part blackened with ink, or a black-lead pencil, and the paper laid upon a blacker ground.

Appearance of the lesser Planets.

57. The other Planets, which by reason of their greater distance appear much smaller than the Moon, will appear fainter but much larger by indistinct, than by *Perfect Vision*: and their diameters will appear enlarged in a much greater proportion than that of the Moon.

1. For instance, if the apparent diameter of *Jupiter*, at his mean distance from the earth, be about 38″ when seen by *Perfect Vision*; and in seeing him indistinctly the *radius of dissipation* be 2′, or 120″, as was before supposed; then by Art. 37, the circle *ABDC*, Fig. 17, which represents the *true image* of *Jupiter*, or the image of *Jupiter* upon the *Retina* when he is seen by *Perfect Vision*, will be 38″ in diameter; and the diameter of the *faint false image*, *Cabd*, equally bright in all parts, will be 202″, that is, above five times the diameter of the image of *Jupiter* seen by *Perfect Vision*. And also, around this *faint false image* of *Jupiter* there will be an *annular penumbra* represented by the *annulus abdHGF*, of 38″ in breadth, which *annulus* being added to the *faint false image* already mentioned, will give 278″, or 4′, 38″ for the breadth of the whole phænomenon, that is, above seven times the diameter of the image of *Jupiter* seen by *Perfect Vision*. But of this something must be abated, because the *annular penumbra* cannot be sensible to its very extremity, by Art 39, the image of *Jupiter* being much smaller than that of the Moon.

2. If the apparent diameter of *Mars*, at his mean distance from the earth, be 6″ when seen by *Perfect Vision*, the breadth of the *faint false image*, equally bright in all parts, will be 234″, that is 39 times the diameter of *Mars* seen by *Perfect Vision*.

3. If the apparent diameter of *Venus*, at her mean distance from the earth be 18′, the breadth of the *faint false image*, equally bright in all parts, will be 222″, that is above 12 times the diameter of *Venus* seen by *Perfect Vision*.

By this means therefore we may account for the *radii adventitii*, which Mr. [a] HORROX says imposed so far upon all the astronomers before him, as to make them think the apparent diameters of the planets 9 or 10 times bigger than they really are.

58. When any of these lesser planets is at the greatest distance from the earth, and consequently has the least apparent diameter, seen by *Perfect Vision*; its *faint false image* is the largest. For, by Art. 37, the diameter of the *faint false image* is $2\rho-2r$: But when the apparent diameter is the least, the image of that diameter, or $2r$, is the least, and 2ρ is a constant quantity: Therefore $2\rho-2r$ is then greatest, or the diameter of the *faint false image* is then greatest.

59. When the apparent diameter of a planet seen by *Perfect Vision* is greatly altered in magnitude between the least and mean distance of the planet from the earth, the change in its apparent magnitude seen by indistinct vision may be inconsiderable.

1. For instance, let us suppose the mean distance of *Jupiter* from the earth, to bear to his least distance the proportion of 5 to 4. This will make a change in the apparent diameter of *Jupiter* seen by *Perfect Vision*, from 38″ to 48″ nearly; and this addition of 10″ to 38″ is very considerable, being a quarter part. But the diameter of the *faint false image* of *Jupiter*, by Art. 37, at his mean distance is 202″, and at his least distance is 192″, the difference being $\frac{1}{19}$ part only. And the diameters of the whole appearance of *Jupiter*, taking in the *annular penumbra*, are at those distances, 278″, and 288″, respectively, the difference between which is only $\frac{1}{28}$ part.

2. If the mean distance of *Mars* from the earth bear to his least distance the proportion of 3 to one, and the apparent diameter of *Mars* at his mean distance be 6″, his apparent diameter at his nearest distance will be three times as much, or 18″. But the diameter of the *faint false image* of *Mars* at his mean distance will, by Art. 37, be 234″, and at the least distance will be 222″, the difference between them being only 12″ in 222″, which is less than $\frac{1}{18}$ part.

Why Mars and Venus appear round, when gibbous or horned.

60. When *Mars by Perfect Vision* would appear gibbous, and *Venus* by *Perfect Vision* would appear gibbous, or dichotomised, or horned, the appearance of these planets by indistinct vision will be much the same as if their whole disks were illuminated.

For

[a] Venus in sole visa. Cap. XVI.

For instance, when *Venus* at her mean distance from the earth is dichotomised, her *faint false image* will be an oval tranverse to the line of ditchotomy,[a] whose shortest diameter will be $2\rho-2r$, that is $240''-18''$, or $222''$: And the longest diameter will be $2\rho-2r$, that is $240''-9''$, or $231''$, as may easily be collected from Art. 37. And when *Venus* is a crescent ever so narrow, this longest diameter must be something less than $240''$. But the difference of $9''$, or $18''$, in $222''$, is less than $\frac{1}{24}$, or $\frac{1}{12}$ part. And an oval, whose transverse diameters are 11 and 12, can hardly be distinguished from a perfect circle, especially when very small.

Appearance of the fixed Stars.

61. A fixed Star appears to the eye under such an angle, as answers to the diameter of the *circle of dissipation*.

For by Art. 37, the diameter of the *faint false image* of a star is $2\rho-2r$.

But by the observations of the best astronomers, the apparent diameter of the brightest fixed stars is so exceedingly small, that even in looking thro' the longest Telescope no estimate can be made of its magnitude, the stars appearing only as lucid points. Therefore, neglecting the apparent diameter of the star, or the diameter of the *true image*, $2r$, the diameter of the *faint false image* which is equally bright in all parts, or $2\rho-2r$, Art. 37, will be very nearly 2ρ.

62. The *faint false image* of a star, or the appearance of a star to the eye, is attended with no sensible *penumbra*.

For by Art. 37, the diameter of the whole appearance of a circular object in the eye, taking in the *annular penumbra* surrounding it, is $2\rho+2r$, and the breadth of the *annular penumbra* is $2r$. But by reason of the exceeding smallness of the apparent diameter of a star, $2r$, the *annular penumbra* surrounding it will be utterly insensible, and the whole appearance in the eye will to our sense be the same as that of the *faint false image*, whose diameter we have shown to be the same with that of the *circle of dissipation*.

63. The distance between two stars will appear to the eye less than it really is, by twice the *radius of dissipation*.

Let the two circles *cfgh*, *mnop*, Fig. 23. whose radii *cf*, *mn* are severally equal to the *radius of dissipation*, Art. 61, represent the images of two stars upon the *Retina*; let the line *fn* represent the distance between those images, and the line *cm* the distance between their centers. Then it is plain, that the apparent distance *fn* will be less than *cm* the distance between the centers of the two stars, by *cf* and *mn* added together, that is by twice the *radius of dissipation*.

From this it evidently follows, that when the distance between two stars is not more than twice the *radius of dissipation*, the two stars must appear contiguous.

64. If the distance between two stars be less than twice the *radius of dissipation*, the two stars will appear to the eye as one star, brighter than either of the two taken alone.

For since each star appears under such an angle as answers to twice the *radius of dissipation*, by Art. 61, if two stars are less asunder than that angle, the two radii will meet in the middle, and part of the two *faint false images* will coincide, and where they coincide will be nearly twice as luminous as the rest of the image, so that the two stars will have the same appearance as if one brighter star appeared in the middle of the space, which is taken up by the two stars.

65. After the same manner two small circular objects seen very near, as two points in printing, may appear to run into one another, and compose a stronger image in the middle, when the *radius of dissipation* much exceeds the diameter of the objects. As the two points *a*, *b*, in Fig. 24, when seen very near, put on the appearance *A*, Fig. 25, or *B*, Fig. 26.

A Star seen within the limb of the Moon.

66. A star approaching very near to the edge of a planet may appear within the limb of the planet, just as if the planet were transparent and the star were seen through it.

Let the circle *ABD* Fig. 27, represent the Moon, and let *c* be a fixed star very near it, as within $8''$ or $10''$ of the edge of the Moon.

Then, I say, the star will appear within the limb of the Moon, as in Fig. 28, where *c* represents the star and *FGH* the limb of the Moon.

For in Fig. 29, let the circle *CABD* represent the *true image* of the Moon upon the *Retina*, and concentrick with this draw the circle *CFGH*, at the distance *AF* equal to the *radius of dissipation*. Then by Art. 20, the

the circumference *FGH* will appear as the limb of the Moon. And if the point *e* be taken very near to the circumference *ABD*, and from this center, with the *radius of dissipation ef*, be drawn the circle *efgh*, that circle, by Art. 61, will represent the image of the star upon the *Retina*. But that image will be almost wholly comprehended within the limb *GFH*. Therefore the star must appear within that limb.

As for the circular segment *fg*, which is without the limb, it will be insensible to the eye, both because of its smallness, and because the limb of the Moon itself is not even.

But as the *faint false image* of the star, when invironed with so strong a light as that of the Moon, will for reasons hereafter to be given, Art. 220, contract itself from the circle *efgh* to a lesser circle as *emno*, the star will appear not only within the limb of the Moon, but at a considerable distance within it.

67. This accounts for the Observations [a] SCHICKARD speaks of in the following passage. *Cynthia enim, quando stellis appropinquat, cernitur advenientes amplecti & aliquantulum intra peripheriam perspicuam admittere; ultrinsecus vero exeuntes visui reddere prius quam pervenerint ad oram: quod* Maestlinus *exemplo* Martis, *item Cordis* Scorpionis *animadvertit Anno* 1595. Disput. de pass. Planet. Thes. 158, *unde collegit, quodam diaphano velut aere ambiri: sed hæc experientiæ ulteriori relinquo.*

68. After the same manner we may account for *Mars* or *Venus*, &c. appearing within the limb of the Moon.

69. The same appearance of a star within the limb of the Moon may happen in viewing the Moon thro' a telescope, if the telescope be not good, or the eye of the observer have not the exact conformation necessary for *Perfect Vision*.

For though the object appear sufficiently distinct thro' the telescope, yet if there be not *Perfect Vision*, which by Art. 8 may easily happen, there must be a *circle of dissipation*, and the radius of the *faint false image* of the star must not be greater than the *radius of dissipation*, by Art. 61, and the ring between the *true image* of the Moon and its apparent limb must be equal to the same radius. Consequently the image of a star may appear within that ring, that is, within the apparent limb of the Moon, as well as if the ring and the *faint false image* had both been greater.

This may account for the Observation of [b] PERE FEUILLEE, and likewise for that of [c] MONS^r DELAHIRE, who both saw a star within the Moon's limb thro' a telescope, and the last named Gentleman has touched upon the true cause of that *phænomenon*, as Mr. HORROX, and more especially the incomparable GALILEO had done before him.

70. If a rectangular object be viewed at a distance proper for *Perfect Vision*; its picture upon the *Retina* will be rectangular, (I mean as far as the spherical figure of the *Retina* will admit) and will be proportional to the angle which the object subtends at the eye; its limb will be well defined; and all parts of the rectangular picture will be equally strong. Consequently, the idea thereby excited in the mind will be that of a rectangle equally strong in all parts, and well defined.

Phænomena of indistinct Vision in rectangular objects.

71. If the same rectangular object be viewed at a distance much too small, or much too great for *Perfect Vision*; its picture upon the *Retina* will still be rectangular: but the length and breadth will be greater than in proportion to the angles subtended at the eye by the length and breadth of the object. Nor will the picture be equally strong in all its parts, but the middle part will be the strongest, and will be surrounded with a *penumbra* growing gradually fainter towards the outside, whereby the limb will appear indistinct and ill-defined. Consequently, the idea thereby excited in the mind will be that of a rectangle too large, and too faint and indistinct towards the limb. That is, instead of the appearance *A*, Fig. 30, it will have the appearance *B*, or *C*, or *D*.

For let the rectangle *ABDC*, Fig. 31, represent that rectangular space upon the *Retina*, which the image of the object would take up if that image were distinct: Or, which comes to the same thing, let *ABDC* represent that rectangular space upon the *Retina*, which is occupied by the centers of all the pencils of rays belonging to the indistinct image of the rectangular object. This rectangle *ABDC* we shall call the *true image* of the object. Also, let the circle *fghe*, having its center *e* in the perimeter of the rectangle *ABCD*, represent that circular space upon the *Retina*,

[a] Respons. ad Gassendum. [b] Hist. de l'Academie Royale des Sciences. 1699. [c] Ibid. Item Art. 1088 of this Book.

Retina, which is taken up by one of the extreme pencils of rays issuing from the object, that is, let the circle *fghe* be the *circle of dissipation*, and its radius *ef* the *radius of dissipation*. Then draw the line *bd* parallel to *BD*, touching the *circle of dissipation fghe* in the point *f* within the rectangle, and compleat the rectangle *abdc* having its sides parallel to those of the rectangle *ABDC*, and every where distant from them by the *radius of dissipation ef*.

72. Then I say, 1. That part of the rectangular object, which is represented by the rectangle *abdc*, will be equally strong in all parts, and will be of the same strength, as if the image of the object had been perfectly distinct.

To prove this, let the rectangles *ABDC*, *abdc*, Fig. 32, represent the same things as before, and taking any point at pleasure, as *e*, within the rectangle *abdc*, from that point as a center, with the *radius of dissipation ef*, draw the *circle of dissipation efgh*.

Then it is plain, that the point *e* must receive light from every point of the circle *efgh*, into which it scatters its own light, and consequently must receive back the same quantity of light as is loses by dissipation. It must therefore be as strongly illuminated, as if there had been no dissipation of light, but the image had been perfectly distinct. And as this is true of every point in the rectangle *abdc*, that whole rectangle must be as strongly illuminated as if the image had been perfectly distinct, and must be equally strong in all parts. This rectangle, *abdc*, we call the *false image*.

73. I say, 2. The rectangular area, or border *ABDCcabd*, comprehended between the perimeters of these two rectangles, will not be so strongly illuminated as the *false image abdc*, and will grow gradually weaker towards its outer extremity.

For let the rectangles *ABDC*, *abdc*. Fig. 33, represent the same things as before, and and taking two points in the rectangular area *ABDCcabd*, one more inward as *e*, and the other more outward as *m*, from the centers *e* and *m*, with the radii *ef*, *mn*, each equal to the *radius of dissipation*, draw the two circles *efgh*, *mnop*, cutting the lines *AC*, *BD*, in the points *h* and *f*, *n* and *o* respectively.

Then it is manifest, that the points *e* and *m*, will receive light not from the whole circles of which they are the centers, but only from the segments *hgf*, *npo* respectively. Therefore each of these points will be less luminous than any point of the rectangle *abdc*. But also the segment *hgf*, which throws light upon the more inward point *e*, is larger than the segment *npo* which throws light upon the more outward point *m*. Therefore the more inward point *e* will be more strongly illuminated rhan the more outward point *m*, or the rectangular area *ABDCcabd* will decrease gradually in light towards the outer extremity.

74. I say, 3. Besides the rectangular area last described, which is darker than the *false image abdc*, there will appear another rectangular area, situated without the *true image*, or rectangle *ABDC*, which is darker than the former, and whose light does also diminish gradually towards the outside, till it wholly disappear and vanish.

For let the rectangle *ABDC*, Fig. 34, again represent the *true image*, the rectangle *abdc* the *false image*, and *efgh* the *circle of dissipation*, having its center *e* in *BD* one of the sides of the *true image*. Then parallel to *BD* draw the line *GF* touching the circle *efgh* in its outer point *f*, and compleat the rectangle *GFHI* distant every where from the rectangle *ABDC* by the *radius of dissipation ef*.

Then between the two perimeters *ABDC* and *GFHI*, will be comprehended a new rectangular area, which will receive light from the pencils whose centers are situated in the former rectangular area *ABDCcabd*; but it will be darker than the former, and its light will gradually diminish towards the outer edge till it entirely disappears and vanishes. This is manifest from the inspection of the figure, where the light received by the point *e* situated in the inner edge of this area is measured by the segment or semicircle *hg*, and the light received by the point *m* situated near the outer edge is measured by the smaller segment *no*.

75. From what has been said, Art. 71, 72, 73, 74. and from consideration of Fig. 31, 32, 33, 34, it is plain that the rectangle *abdc*, or the *false image*, is the only part that has its full quantity of light, and that the two rectangular area's *abdc CABD*, and *ABDCHIGF*, do both decrease in light just after the same manner

manner from $abdc$ to $IGFH$, for which reason, and because we shall have no occasion afterwards to consider these two rectangular area's separately from one another, we shall here look upon them as one area $abdcHIGF$, which we shall call by the name of the *rectangular penumbra*.

76. But it may not be amiss to observe, that this *penumbra*, which we call rectangular, is not strictly so, nor exactly as we have represented it, but cut off at the corners nearly in the following manner.

Let cab represent one angle of the *false image*, CAB one angle of the *true image*, and HIG one angle of the *rectangular penumbra*; and from the center A with the *radius of dissipation* Ag draw the *circle of dissipation* $Agfeh$, touching the lines IG, IH, in the points g and h respectively; and produce the line ca till it intersect GI in the point l, and produce the line ba till it intersect HI in the point m. Then it is manifest from the construction, that these two lines cl, bm must touch the circle $ehgf$ in the points f and e respectively.

Now it is manifest, that the point I being more remote from any even the nearest point of the *true image*, as A, than by the *radius of dissipation*, can receive no light from that image. And this is equally true of every point in the space gIh, comprehended between the two tangents gI, hI, and the arc gh. Consequently the *rectangular penumbra* does not extend to any part of that space.

Farther, the point f receives light from a semicircle of the *true image*, as is plain from this Figure compared with Fig. 34: But the point A receives light only from a quadrant of the *true image*. Therefore the light of the *penumbra* from f to A must be weaker than from f towards B, and must gradually decrease all the way from f to A.

It is easy to see likewise, that the outer edge of the *penumbra*, which is every where equally luminous in the line Gl, does begin to decline about the point l, and decreases gradually from l to g, where it vanishes.

And also, it is plain from Art. 73, that the light, which is equally strong along the edge of the *false image* from b to a, begins to decline at the point a, and grows gradually weaker from a to e.

It is therefore manifest, that in every parallel to the line AB, the *penumbra* grows weaker without the line lfa, or towards b, than within that line, or towards B.

In like manner it will be found, that in every parallel to the line AC, the *penumbra* is weaker without the line mea, or towards g, than within that line, or towards C.

Therefore, the lines lfa, mea, may be looked upon as the bounds of the *penumbra*, since that part of it which appears within the angle lam, is considerably weaker than what appears within the angles lab, and mac.

77. Hence, the *false image*, with the *penumbra* about it, must put on the appearance represented in Fig. 36, where the same letters denote the same things as before. And this in fact is nearly the case, when the corner of a distant high wall, or square tower, is looked upon by a short-sighted person, or by one long-sighted using a pair of spectacles, or a convex glass, against the sky-light.

78. For much the same reasons, a pyramid or spire of a church, will, under the same circumstances, appear, as in figure 37, after the manner of three different spires one stronger and lower as cab, the other two fainter and more elevated as Hmb, Glc, as in fact will nearly be found by those who make the tryal.

I say nearly, because this appearance, as well as that of the preceding article, will be a little varied from a cause hereafter to be assigned. See Art. 199, 200, &c.

79. If the *radius of dissipation* increase, the *false image* $abdc$, will decrease both in length and breadth; but the breadth will decrease more in proportion than the length; for as much as the equal radii ef, ef, are taken both out of the breadth and length, as in Fig. 38.

80. If the *radius of dissipation* be equal to half the breadth of the *true image*, the *false image* will degenerate into the right line ae, Fig. 39.

81. If the *radius of dissipation* exceed half the breadth of the *true image*, there will be no *faint false image* equally luminous in all parts, as in the case of the circular object, Art. 24, but the whole image of the rectangle will be one *penumbra*, strongest in the middle, and decreasing gradually in light on both sides towards the outer edge.

For in Fig. 40, the point e situated in the middle of the breadth of the rectangle, receives light from every point in its *circle of dissipation*, except the two segments fg, and hi.

 The

The measure of its light therefore is the space $fghi$.

The measure of the light thrown upon the point k, situated more towards the edge than the point e, is the space $lmon$, less than $fghi$, as is easy to perceive.

The measure of the light thrown upon the point p, still more outwardly situated than the point k, is the space, or segment rqs, less than $lmon$.

And the measure of the light thrown upon the point t, situated in the very edge of the *true image*, is the segment or semicircle vw, still less than rqs.

Therefore the light of the whole appearance will decrease from the middle both ways breadthwise.

Towards the ends of the *true image* AB and CD, the *penumbra* will decrease lengthwise also. For the point x, in the line AC, whose distance from A is xA equal to the *radius of dissipation*, will be illuminated by the semicircle Axy; but the points between x and A, will be illuminated by less than a semicircle, and the point A will be illuminated only by a quadrant, and any point of the *penumbra* beyond A in the line xA produced, will be illuminated by less than a quadrant. Therefore the ends of the *penumbra* about AB and CD will be weaker than the other parts.

82. If the *radius of dissipation* exceed the breadth of the *true image*, the *penumbra* will be fainter in proportion to that excess, but still the middle will be the strongest, and the light will decrease from the middle towards the outer extremity, as is manifest from the consideration of Fig. 41.

83. When the *radius of dissipation* is very large in proportion to the breadth of the *true image*, the *penumbra* will be very faint, but will decrease very slowly and almost insensibly, except very near the outer edge.

For in Fig. 41, let us imagine the two sides of the *true image*, AC and BD, to approach very near together, and then it is plain, that the several portions of equal circles, $fgih$, $lmon$, $qrst$, and yz, whereby the points e, k, p, and x are respectively illuminated, will decrease very slowly in respect to the distances of the points k, p and x from the middle of the image.

84. Very near the edge the *penumbra* will decrease very fast.

For in Fig. 42, the light thrown upon the point e very near the edge of the *penumbra* will be measured by the segment fg, and the light thrown upon the point h a little nearer that edge, will be measured by the segment ik. But the segment ik is much less than the segment fg.

85. That part of the *penumbra* near its edge, where the light decreases very fast, is nearly equal to the breadth of the *true image*. This easily appears from the consideration of Fig. 42. For if the point e were distant from the line GF by just the length of CD, its *circle of dissipation* would touch the line AC; and by removing the point e more outwards, its segment of light fg would decrease very fast.

86. When the *radius of dissipation* is vastly great in proportion to the breadth of the *true image*, that part near the edge where the *penumbra* decreases very fast, will be insensible, and the whole *penumbra* will decrease insensibly in light from the middle towards the outer edge.

87. A narrow line, as a stroke in this print, when seen very near, will appear very broad and faint, and almost equally faint throughout the whole breadth. This follows from Art. 83, 84, 85, 86.

88. Two narrow parallel lines drawn near to each other, when viewed very near, may put on the appearance of one line, with a *penumbra* on each side of it. The two lines A, Fig. 43, may, by viewing them very near, appear as B. Two parallel lines appearing as one line.

For, by Art. 87, each of these lines must appear as one broad faint line, and being near together their *penumbræ* will meet in the middle, and the part where they meet must appear nearly of a double strength.

89. Two narrow lines meeting in a small angle, when seen very near, may form the appearance of an inverted wedge surrounded with a *penumbra*.

The two lines C in Fig. 44, may appear as D, when viewed very near.

This is easily proved in the same manner with Art. 88.

90. A circle upon white paper, bounded by a narrow black line as its circumference, when viewed very near, will appear less than if it were seen distinctly: But the narrow line that encompasses it, will appear broad and faint, and will be stronger on the inner or Circular lines seen by indistinct Vision.

or concave side, than on the outer or convex side; and the whole breadth of this faint appearance, or *penumbra*, will be equal to twice the *radius of dissipation*, added to the breadth of the circular line. That is, instead of the appearance A, Fig. 45, we shall have the appearance B.

For, let AB in Fig. 46, represent a part of the circular narrow line bounding the circle described with the radius EA: And let $ebgi$ be a *semicircle of dissipation*, having its center e in the outer edge of the narrow circular line AB, and cutting the radius Ee produced in the point g. Also let $ebfi$ be another *semicircle of dissipation* having its center e on the inner edge of the same circular line AB, and cutting the radius Ee in the point f.

Then it is manifest, that the faint appearance, or *penumbra*, of the circular line AB must extend inwards to the point f, and outwards to the point g: Or, the whole breadth of the *penumbra* must be equal to the two *radii of dissipation* ef and eg added to the breadth of the circular line AB.

Consequently, the radius of the white circle, or Ee will in this appearance be diminished to Ef, the part fe being taken up by the *penumbra*.

91. I say further, this *penumbra* will be stronger on its inner edge than on the outside.

For in Fig. 47, let the narrow circular line be represented by the ring comprehended between the two circular lines AB and CD, which ring is here made very broad to render the parts we are going to speak of the more discernible: And from the center E draw the radius Eiq, cutting the inner and outer edge of this ring in the points i and q respectively. Then setting off in this radius the two equal distances ie and qm, each less than the *radius of dissipation*; from the centers e and m, with the *radius of dissipation* draw the arcs fbg, npo, cutting the circular lines CD, AB, in the points f and g, n and o, respectively.

Then will the circular segment $figb$ represent the quantity of shade thrown from the ring upon the inner point e.

And the circular segment $nqop$ will represent the quantity of shade thrown from the ring upon the outer point m.

And if thro' the point i be drawn the right line rs perpendicular to the radius Ei, and thro' the point q be drawn the right line vt perpendicular to the same radius: the space intercepted between these lines rs, vt may be considered as a rectangular object of the same breadth as the circular object, or ring, $ABDC$.

But by the inspection of the figure the circular segment $figb$ is greater than what would be cut off from the rectangular object by the *circle of dissipation*, whose center is e, and whose radius is eb.

And the circular segment $nqop$ is less than what would be cut off from the rectangular object, by the *circle of dissipation*, whose center is m, and whose radius is mp.

Therefore the shade thrown upon the point e from the circular segment $figb$, is greater than what is thrown upon the point m from the circular segment $nqop$: that is, the inner point e will appear darker than the outer point m, or the *penumbra* will be stronger on the inside than on the outside.

A dark spot in the center.

92. Since the breadth of the *penumbra* on the inside is equal to the *radius of dissipation*, it is manifest that the more this radius increases, the more will the radius Ef, Fig. 46, of the white circle be diminished: And when the *radius of dissipation* becomes equal to the radius Ee, the white circle will intirely disappear, and in the room of it a black spot will appear in the center E, surrounded with a *penumbra* whose radius is double to Ee, the radius of the circular line. And this is found true in fact, when such a circular line is either much too near, or much too remote to be seen distinctly: For then the object A appears as B, E as C, and I as D, Fig. 48: but to make the experiment rightly, the radius of the circular line must be proportioned to the distance.

To measure the radius of dissipation

93. Hence we have a good method of finding by experiment the *radius of dissipation* at any distance without the limits of *Perfect Vision*.

Upon white paper draw the circumference of a circle with a strong black line, and place the paper by guess about the farthest distance at which your eye can see an object distinctly. Then retiring gradually farther from the paper, observe at what distance the white circle in B, Fig. 48, appears equal in breadth to the *penumbra* on either side of it. At that distance the *radius of dissipation* is nearly equal to half the radius of the *true image* of the circle.

Again, retire still farther from the paper,

till the white circle vanishes and the central black spot just begins to appear. Then the *radius of dissipation* must be precisely equal to the radius of the *true image* of the circle.

But the radius of the *true image* of the circle at any distance is easily found by Art. 380, of this Book of Opticks, and consequently the magnitude of the *radius of dissipation* at the same distance may be determined.

94. When the *radius of dissipation* exceeds the radius of the *true image*, there will appear in the middle a circular spot equally strong in all parts, surrounded with a *penumbra* fainter than the spot, so that this spot will be a *faint false image* of the circular line, like the *faint false image* of the circular object in Art. 23. And the breadth of this circular spot, or *faint false image*, will be equal to the difference between the *radius of dissipation* and the radius of the *true image*.

This is easily shown after the same manner as we proved the 23d Article by Fig. 11, where the circumference ABD represents the *true image* of the circular line, whose radius is r; $cbfgh$ the *circle of dissipation*, whose radius is ϱ; the circle $Cabd$ represents the central spot, whose radius is $\varrho - r$; the circle $CHGF$ represents the whole appearance whose radius is $r+\varrho$, and the ring $abdFHG$ represents the *penumbra* round the central spot, whose breadth is $2r$.

Another way to measure the same radius.

95. When the *radius of dissipation* is equal to the diameter of the *true image* of the circular line, the central spot will then be equal in breadth to the *true image*, and equal in breadth to the *penumbra* encompassing the central point.

For $\varrho - r$ the radius of the spot will now be equal to $2r - r = r$, and its diameter will be $2r$ equal to the breadth of the *penumbra*.

Hence we have another way of finding the *radius of dissipation*, which must be equal to twice the radius of the *true image*, when the *faint false image*, or central spot appears equal in breadth with the *penumbra*.

A purple spot in the center.

96. The central spot we have been speaking of, does sometimes appear of a purple colour, instead of black.

This, I think, happens for the following reason. At the time of this appearance, the *radius of dissipation* exceeds the radius of the *true image* by a little more than the breadth of the circular line, as in Fig. 49, where ABD represents the circular line, and $Cfgh$ represents the *circle of dissipation*, which takes in a narrow ring of white paper beyond the circular line.

This ring of white will therefore throw a portion of its light upon the central spot, which portion of light being the farthest dissipated from the centers of the pencils in the white ring, must consist of the least refrangible rays, or the red rays. But this red, mixing with the bluish black of the central spot, must exhibit the appearance of purple.

97. When this central spot begins to appear, if the eye continue attentively fixed upon it, or if the sun break out on a sudden from a cloudy sky, or if at night a candle be snuffed so as to burn more brightly, in any of these cases the spot will disappear, and a small white circle will appear in the room of it.

For in all these cases the pupil contracts to a narrower apperture, and the *radius of dissipation*, which is always proportional to that aperture, is thereby lessened, so that this radius, which at first was equal to the radius of the *true image*, now falls short of it, and consequently there must appear no spot, but only a small white circle, by Art. 92.

98. The circumference of an oval will exhibit much the same *phaenomena*, as the circumference of a circle, and for the same reason. Only, the central spot will now be oval, and when there is no central spot, the *penumbra* will be stronger in the more concave part of the oval than in the less concave part.

Ovals indistinctly seen.

The oval A will appear as B, or C, Fig. 50.

99. From the preceding articles it is easy to see the reason, why a book of a small print held very near the eye appears quite confused. For, as the letters therein consist either of lines parallel to one another, as *m, n, il, it*, &c. or of lines inclined to each other, as v, w; or of circular, or oval, or partly oval lines, as o, *o*, *c*, *e*; or of a mixture of some of these, as *b*, *d*, *p*, *q*; it is plain from what has been above delivered, that when viewed very near, they will not only form large *penumbrae*, so as to render them ill defined, but will exhibit the appearance of foreign lines formed by the union of these *penumbrae* between the parallel or inclined lines of the letters, as also central spots in the circular, or oval, or mixed letters, so as to render

Letters of a book seen by indistinct Vision.

render the true shape of the letters utterly confused and indiscernible.

100. But a print considerably larger than that of the preceding article, when viewed at the same distance, will not be attended with the appearance of these foreign lines or spots, because the *penumbræ* will not now meet in the middle, the space being too great between them, and consequently the letters will appear only ill defined, and somewhat indistinct; but not confused.

101. If the small print be viewed at a distance somewhat greater than in Art. 99, it will now appear only ill defined and somewhat indistinct on account of the *penumbræ*: But it will not appear quite confused, since it is not now attended with any foreign lines or spots, the pencils not spreading far enough for the *penumbræ* to coincide from opposite sides and thereby to form those lines or spots.

102. If the larger print be viewed from the same distance with the smaller in the preceding Article, it will appear less indistinct than the smaller, for two reasons.

1. The lines being thicker will make a stronger appearance in the middle, by Art. 71, and this will in some measure efface the *penumbra*, especially towards the edge, where it is weakest, so that the whole *penumbra* will appear both fainter and narrower, and consequently will take up less of the void interval between the strokes of the letters than in the smaller print.

2. That interval is in itself larger than in the smaller print, and consequently appears more conspicuous, and thereby shews the strokes of the letters more separate and distinct from each other.

103. If the two prints be viewed at a distance still greater, the *penumbræ* will grow less, so that the smaller print will appear but little indistinct, and the greater not at all so, for much the same reasons as were given in the preceding Articles.

And in this case, as the eye perceives no indistinctness in the larger print, this sort of Vision, although, on account that the rays of a pencil are not accurately collected into a point upon the *Retina*, we call it imperfectly distinct, cannot by the sense be distinguished from *Perfect Vision*.

Points proposed to be examined.

104. We have hitherto considered *Perfect* and *Distinct Vision* only upon the foot of a given disposition of the eye, in which case it has been shown, that *Perfect Vision* depends upon the distance of the object solely, and that *Distinct Vision* depends upon the distance and magnitude of the object jointly.

It remains to consider how either of these sorts of Vision may be procured by a change in the disposition of the eye, and here some very curious points offer themselves to be discussed.

I. Whether *Perfect Vision* in a given eye is confined to one determinate invariable distance, or may be obtained at different distances by a change in the conformation of the given eye.

II. Within what limits these different distances are comprehended, or what is the greatest and least distance at which *Perfect Vision* can be obtained in a given eye, by changing its conformation.

III. What is that change, which is made in the conformation of the eye, in order to obtain *Perfect Vision* at different distances.

IV. By what means Vision, when it cannot be *Perfect*, is rendred *Distinct*, or at least not so indistinct as it would otherwise be.

V. What change is made in the eye, either by habit and custom, or by our growing into years.

VI. What is the least object, or angle, that the eye is capable of perceiving.

Of all these I intend to treat in their order.

I.

Whether the eye changes its conformation for Perfect Vision at different distances.

105. In the first of these points all Authors, that I know of, agree together, except only the famous Mons[r] De la Hire. He is of opinion,* that there is no accurate collection of the rays of a pencil into a point upon the *Retina*, that is, no *Perfect Vision*, except when the object is at some one determinate distance suited to the eye of the observer. The others maintain, that the distance of the object may vary, and yet the rays of a pencil may be accurately united upon the *Retina*. The experiment upon which Mons[r] De la Hire founded his opinion, has been briefly related in the *Remarks*, Art. 15, &c. and the insufficiency of his argument drawn from that experiment has been fully shown. The same thing has likewise been done by D[r] Porterfield in the *Medical Essays* of *Edinburgh*,

* Traitè des differens accidens de la Veüe.

Edinburgh, Vol. 4, where this learned and judicious Gentleman by a very well contrived experiment has plainly demonstrated the truth of the common opinion, that the eye has a power of altering its conformation so as to see distinctly at different distances.

I had myself, at leisure times for several years past, made a great number of tryals, in looking thro' two and sometimes more pin-holes, sometimes thro' two narrow slits made near to each other in a card, sometimes by a pin held directly before the eye, which answers the same purpose as the narrow interval between the pin-holes, or between the slits in a card. And the objects I looked at, were sometimes round, as a pin-hole in a card set against a candle; sometimes long, as a narrow slit in a card set against a wainscot strongly illuminated by a candle hid from the eye, or a strong black line upon white paper set against a window. And by those tryals I found, that the nearest distance at which with one eye I could see any of these objects single thro' the two pin-holes, or thro' the two slits, or on each side of the pin, was 40 inches; but that I could sometimes see them distinct at much greater distances, as 50, 60, *&c.* to 90 inches or more. But as my eyes are now considerably decayed with regard to their power of seeing distinctly at small distances, and as D^r Porterfield's experiments seem to be better contrived, and were more methodically made than mine, I fully acquiesce in the proof he has given, that the eye has a power of altering its conformation so as to see objects perfectly distinct at different distances.

II.

Limits of Perfect Vision. 106. We pass therefore to the second point, to enquire within what limits these different distances are comprehended, or what is the least and the greatest distance at which *Perfect Vision* can be obtained.

The first of these limits, or the least distance at which we can see with *Perfect Vision*, is easily settled. For besides D^r Porterfield's experiment, who determines it to be about 7 inches to his own eye, we have the concurrent experience of the generality of persons who are at their full growth, and whose eyes are no way impaired by age; for as much as in looking at minute objects, as the divisions upon a *Gunter's* line, or those of a diagonal scale for half an inch, or in examining the fineness of linnen and cambrick, or chusing lace, *&c.* they hold the objects at 5, 6, or 7 inches from the eye. Whence it may reasonably be presumed, that the nearest distance for *Perfect Vision* is commonly 5, 6 or 7 inches. And this I shall afterwards prove by another method.

107. But as for the other of these limits, or the greatest distance at which *Perfect Vision* can be obtained, it is a matter of somewhat more difficulty.

This greatest distance is by D^r Porterfield determined for his own eye to be 27 inches. But that to the generality of eyes this distance is much greater, may reasonably be presumed from the distinctness which we see a small misling rain, when walking in a piazza, or coming out of a church and within not less than six or eight feet from the door; or the small filaments of silk upon which spiders transport themselves thro' the air, at a greater distance; or the string of a boy's kite at a great height in the air. And to find what this distance is for a given eye, it may be proper to determine the *radius of dissipation* at some great distance by Art. 93, or 95, or rather by Art. 63, or 64, and from thence to compute at what lesser distance the *radius of dissipation* must vanish, that lesser distance being the utmost limit of *Perfect Vision*. But in order to do this it will be necessary to consider the measures of some of the parts of the eye.

108. In these measures I shall generally follow the famous Monf. * Petit, he having bestowed more pains upon the examination of them than any other Author I know of; but shall reduce them from the *French* lines in which he has given them, to the tenths of the *London* inch and decimals of those tenths, following herein the proportion established by my ingenious Friend, M^r. George Graham, between the *London* yard and the half toise of *Paris*, in some very accurate measures that have been mutually exchanged between the *Royal Society* and the *Royal Academy of Sciences* at *Paris*, namely that of 36 to 38, 355.

	tenths.
The radius of the convexity of the *cornea* is commonly	3, 3294

* Memoires de l' Acad. Royale 1728, 1730.

	tenths.
The radius of the anterior convexity of the crystalline at a medium from 26 eyes is	3, 3081.
The radius of the posterior convexity of the crystalline at a medium from all the same eyes is	2, 5056.
The axis or greatest thickness of the crystalline from all the same eyes at a medium is	1, 8525.
The joint axis of the *cornea* and the aqueous humour is commonly	1, 0358.

109. The refraction of the aqueous humour, or the proportion between the sines of incidence and refraction, we here suppose to be the same with that out of air into water, namely that of 4 to 3.

110. The refraction of the *cornea* we suppose to be the same with that of the aqueous humour, so that the incident rays suffer no new refraction in passing from the *cornea* into the aqueous humour.

111. The proportion between the sines of incidence and refraction, upon passing out of the aqueous humour into the crystalline, we take to be that of 13 to 12, upon passing out of the crystalline into the vitreous humour, that of 12 to 13.

Mr. *Hawksbee* * from an oxe's crystalline make this proportion a little more; Dr *Pemberton* † seems to think it a little less, but says his experiment was not accurately made.

112. By computing upon these measures and refractions it will be found by Art. 369 of this Book of Opticks, that such an eye will collect parallel rays into a point at the distance AM from the outer surface of the *cornea* of 8,9993 tenths, that fM is 23,9562, AL=5,3732, IL=2,0559, and the rectangle $IL \times fM$=49,2526.

113. Hence, if AP, the greatest distance at which the rays of an object can by the eye be collected into a point, be 27 inches, or 270 tenths, MS will by Art. 370 of the Book of Opticks be found equal to 0,1861; and by so much must the *Retina* be situated behind the point M in order to have *Perfect Vision* at the distance of 27 inches; and at all greater distances than 27 inches the pencils cannot be collected into single points, but must each of them occupy a circular space upon the *Retina*

114. Let us now examine what space the image of a lucid point supposed at an infinite distance, as a fixed star, will upon this supposition occupy upon the *Retina*, and what angle the star must appear to subtend.

By Art. 386 of the Book of Opticks, when PL is infinite, $Xx = \frac{AB \times MX}{Z}$ and by Art. 380. of the same Book, $\frac{Pp}{Pl} = \frac{Xx}{V} \times \frac{\pi M}{\pi X}$. Therefore, supposing this Xx equal to the other Xx, $\frac{Pp}{Pl} = \frac{AB \times MX}{ZV} \times \frac{\pi M}{\pi X}$. And by Art. 388, $Z \times V = LI \times fM$. Hence $\frac{Pp}{Pl} = \frac{AB \times MX}{LI \times fM} \times \frac{\pi M}{\pi X}$. And if we neglect the *ratio* of $\frac{\pi M}{\pi X}$, $\frac{Pp}{Pl}$ is nearly equal to $\frac{AB \times MX}{LI \times fM}$.

Now, if a star be considered as a lucid point, and MX be the distance of the *Retina* from the point M, the semidiameter of the image of the star upon the *Retina* will be Xx. And if Pp be the diameter of an object seen at a very great distance, Pl, whose image upon the *Retina* is equal to that of the star, it is plain that this object and the star must appear of the same magnitude, and to subtend the same angle. But the half angle which the object subtends at the eye, or $\frac{Pp}{Pl}$ is measured by $\frac{AB \times MX}{LI \times fM}$: therefore this quantity is likewise the measure of the half angle under which the star appears. Hence, if AB, or the half aperture of the pupil be one tenth of an inch, which I think by star-light must be pretty near the truth, and MX be equal to MS, or by the preceding article, to 0,1861, we shall find the angle $\frac{Pp}{Pl}$ to be nearly 13′. So that a star must appear to be about 26′ in diameter, or equal to between 8 and 9 digits of the full Moon, which is contrary to experience.

115. Likewise two stars, which are not more asunder than 26 minutes, will by Art. 63, appear contiguous, which is also contrary to experience. For ζ *Bayeri*, or the middle star in the tail of the great Bear is distant from *Alcor* but a little more than 12′, and yet the distance

* Physico-Mechanical Experiments. † Dissertatio Physico-Medica.

distance between them is easily perceived.

116. Also the distance between the two stars, which compose the double star *α* in the head of *Capricorn*, is little more than six minutes, yet that distance is easily seen by a common eye.

117. But farther, the interval between the two stars in the *Hyades*, *υ Bayeri*, is very conspicuous, and might, as I judge by my own eye, and am imformed by several other persons, young and old, be plainly distinguished, if it were less than it is, though it be only 5'. 40''.

118. From all which it follows, that the eye can distinguish a much less interval between two stars than that of 26', and consequently that the eye can have *Perfect Vision* at a much greater distance than 27 inches.

119. And if we suppose a star to appear under an angle of 6', and consequently that we can but just discern the interval between two stars that are a little more than 6' asunder, it will be found by computation, that the eye is suited to see an object by *Perfect Vision* at no less a distance than 9 feet 7 inches.

120. But if a star appears under an angle of 4', and we can see the interval between two stars little more than 4' asunder, which by the tryals I have made, seems generally to be the case in good eyes, it will follow from computation, that the eye in seeing such an interval is accomodated to see an object with *Perfect Vision* at the distance of 14 feet five inches.

From what has been delivered in the preceding articles may be drawn a decisive argument against the opinion of Mons^r DE LA HIRE mentioned in Art. 105.

If the eye cannot alter its conformation so as to obtain *Perfect Vision* at different distances, but is confined to some one determinate distance only, and at all other distances vision is more or less indistinct, without any other help than the contraction of the pupil, let us suppose that one determinate, invariable distance for *Perfect Vision* to be 27 inches.

Then by Art. 114, a star must appear under an angle of 26', and by Art. 115, the eye will not be able to perceive an interval between two stars that are 26' asunder, supposing the radius of the apperture of the pupil to be by star-light one tenth of an inch.

And if we were even to suppose the diameter of the pupil to be by star-light so small as one tenth of an inch, which I take to be as small a size, as it can be contracted to in most eyes by a moderate day-light, or a strong candle-light, yet still we shall not be able to perceive an interval between two stars that are not more than 13' asunder, which by Art. 115, 116, 117, is contrary to experience.

Also, by Art. 8 of the *Remarks*, by a moderate day-light, or strong candle-light, a point in this print will at the distance of 13 $\frac{1}{2}$ inches appear under an angle of 13', that is, as large as a black circle of half a tenth of an inch in diameter seen at that distance by *Perfect Vision:* And the *penumbræ* of two parallel lines in an *n*, or *m*, will meet in the middle of the space between them, unless the intervals of those lines be half a tenth of an inch asunder, that is, the print of this *Essay*, or of the *Book of Opticks* itself, will not be at all legible at the distance of 13 $\frac{1}{2}$ inches, which is also contrary to experience.

And, if instead of 27 inches a larger distance be pitched upon for the invariable distance of *Perfect Vision*, this will a little help the matter with regard to the intervals of the stars; but will increase the confusion at the distance we usually read at. If a smaller distance be pitched upon, we shall read more easily at our usual distance; but shall not see the interval between two stars, unless they are more than 13' asunder.

III.

121. We come now to the third point proposed in Art. 104, namely, What is that change, which is made in the conformation of the eye, in order to obtain *Perfect Vision* at different distances. How the eye alters its conformation.

Concerning this point Anatomists and Opticians are divided into many opinions, such of which as I can recollect either from reading or conversation, I shall here propose, and shall briefly examine.

122. One sentiment is, that the eye in a state of inaction, when both itself and the parts about it are in perfect rest, is accommodated to see with perfect distinctness at the greatest distance: And that in order to view nearer objects, the globe of the eye is compressed by its muscles into an oblong figure,

figure, ſo as to render the axis ſo much longer as is neceſſary to unite the pencils into points upon the *retina*.

But to this opinion it may reaſonably be objected, that in many animals the *ſclerotica* is ſo hard, as not to be capable of changing its figure by this preſſure. And that even in men and other animals, where the *ſclerotica* is leſs hard, yet this preſſure of the muſcles can never be ſo equal as to affect the fibres of the *retina* alike in all parts, but that its fibres muſt in ſome places be crowded cloſer together, or more preſſed inwards, than in others, which muſt needs diſturb the viſion. To which we may add, that no ſlight preſſure will be ſufficient for the effect propoſed. In order to render viſion perfectly diſtinct at all diſtances from ſix inches to 14 feet five inches, it muſt be ſuch a preſſure as will lengthen the axis of the eye one tenth part, and to do this the *retina* muſt be reduced from its ſpherical figure to ſuch an oval, as very much to diſorder the range of its fibres.

123. A ſecond opinion is, that the eye, when at reſt, is fitted for ſeeing the neareſt objects diſtinctly; and that in order to diſtinct viſion of remote objects, it is preſſed againſt the back of the orbit, ſo as to render it flatter and its axis ſhorter.

But againſt this *hypotheſis* all the ſame objections, as we have urged againſt the former, may juſtly be made.

124. A third opinion is, that the eye, when at reſt, is ſuited to the moſt diſtant objects, and that in order to ſee the nearer ones diſtinctly, the cryſtalline humour is by means of the *ligamentum ciliare* drawn forwards, ſo as to increaſe the diſtance between its back ſurface and the *Retina* ſufficiently to unite the pencils into points upon that membrane.

But to ſee objects with *Perfect Viſion* from 14 feet five inches to ſix inches, it would be neceſſary that the cryſtalline ſhould be drawn forwards by about 0,87, which the *uvea* will not permit, there being no more than the diſtance of 0,22 at the moſt between the *uvea* and the cryſtalline.

125. A fourth *hypotheſis* is, that the eye, in a ſtate of reſt, is accomodated to the neareſt objects, and that in order to ſee diſtinctly the more diſtant ones, the *ligamentum ciliare* contracts, and thereby draws out the cryſtalline into a leſs convexity.

But to ſee objects perfectly diſtinct from ſix inches to 14 feet five inches, the alteration in the convexity of the cryſtalline muſt be very great. For the radius of each of its ſurfaces muſt increaſe more than $\frac{3}{7}$ of what they are at preſent. But the cryſtalline is of too firm a texture, and the *ligamentum ciliare* ſeems much too weak, for ſo great an effect to be expected.

126. A fifth opinion has been advanced by the Learned and Ingenious D^r * *Pemberton*, that in order to ſuit the eye to the neareſt objects, one ſurface of the cryſtalline is to be rendred more convex, while the other grows flatter. And that, to ſuit the eye to the more diſtant objects, one ſurface of the cryſtalline is rendred flatter, while the other becomes more convex. And this alteration is ſuppoſed to be made by certain muſcular fibres within the ſubſtance of the cryſtalline. But this ſentiment has not been ſo fully explained by the Learned Author, as we could wiſh.

We ſhall therefore only take notice, that if the eye is to be accommodated to near objects by rendring the anterior ſurface of the cryſtalline more convex, while the hinder ſurface grows flatter, which for many reaſons ſeems to be the moſt advantageous and convenient method; then in order to ſee an object with *Perfect Viſion* at a diſtance of 5 or 6 inches, the radius of that anterior ſurface muſt be leſſened from 3,3081 to 2 nearly, if the radius of the hinder ſurface be increaſed only from 2,5056 to 3. And in order to ſee an object with *Perfect Viſion* at the diſtance of 14 feet 5 inches, the radius of the anterior ſurface muſt be increaſed to 5 nearly, if the radius of the hinder ſurface be diminiſhed no more than to 2. So that while the hinder radius changes from 3 to 2, the anterior radius, in order to ſee perfectly at all diſtances from between 5 and 6 inches to 14 feet 5 inches, muſt be more than doubled. But this ſurely is too great a change for a ſubſtance of ſuch a conſiſtence as the cryſtalline humour to admit of.

I might add, that the demonſtrations, upon which that Learned Gentleman founds his opinion, are all built upon this ſuppoſition, that in viewing objects which appear confuſed, the pupil is always contracted to the

* Diſſert. Phyſico-Medica.

least size it is capable of. For that this supposition is contrary to experience, will appear anon.

127. Meeting with no satisfaction in any of the *hypotheses* above related, I have applied myself to a diligent consideration of the parts of the eye, in order to find out, if possible, some power or powers seated within it, by which its conformation may be so altered, as adequately to answer the effects observed. And in order to enable the reader to judge how far I may have succeeded in this research, I shall, before I lay down my own opinion, examine a little into those parts of the eye, which I think subservient to the effect in question.

128. The *cornea* is a compressible and springy membrane, easily giving way to any force external or internal, and easily restoring itself to its former figure by its own spring assisted by the pressure of the aqueous humour within it.

129. The *uvea* is a muscular membrane, and as such is capable of contracting itself into less dimensions. It arises from a circular ridge or protuberance running all along the inside of the *cornea* at its juncture with the *sclerotica*, which ridge I do not remember to have seen hitherto taken notice of by any Anatomist.

That the *uvea* is furnished with a narrow ring of circular muscular fibres on the edge next the pupil, is now generally agreed by Anatomists, though, I think, not so much from their being able to demonstrate those fibres, as from reason; for as much as the contraction of the pupil upon a strong light, or upon attentively viewing a very near and small object, is plainly visible, and that contraction is justly presumed to be owing to such a muscular ring. Mr. *Ruysch* indeed has represented this ring of muscular fibres in one or two of his figures, but he tells us at the same time, *Sculptor hic justo distinctius repræsentavit, nam in objecto ipso non ita luculenter visuntur.* * And in another place he ingenuously declares, † *Fateor hasce fibras circulares non tam luculenter conspici posse, quin oculi mentis in subsidium sint vocandi.*

It is likewise an agreed point, that the *uvea* is furnished with streight fibres inserted into this ring, and having their origin from that part of the *uvea* which is connected to the inner edge of the *cornea*, and that these streight fibres, which are put upon the stretch and drawn out into a greater length when the ring contracts, do again by their spring, or by their muscular force, restore themselves to their former dimensions, and thereby serve to dilate the pupil, when the abovementioned muscular ring ceases to contract, and is in a state of inaction.

But it is here to be considered, that, when these streight fibres are thus put upon the stretch by the contraction of the muscular ring, they must necessarily draw the edge of the *uvea*, which is connected to the *cornea*, and likewise the edge of the *cornea* itself, a little inwards at the same time. And this edge of the *uvea* cannot be drawn inwards, without contracting into a less circumference than it had before. Must not therefore this edge of the *uvea*, which is next the *cornea*, be furnished with a ring of circular fibres, whereby it may contract itself into a less circumference, as well as that edge of the *uvea* which is next the pupil? To me this part of the *uvea* appears of such a strength, and to adhere so strongly to the *cornea*, by the resistance it makes in tearing them asunder, that I make no doubt of its being muscular, there seeming in this place to be no occasion for a membrane of that strength, unless it were to exert a muscular force, and such a one as might overcome a considerable resistance. I shall therefore make no scruple of qualifying this limb of the *uvea* next the *cornea*, by the name of the greater muscular ring of the *uvea*, to distinguish it from the other ring next the pupil, which I shall hereafter call the lesser muscular ring.

It will perhaps be objected to me, that the existence of this supposed greater muscular ring has not yet been proved by ocular demonstration. I answer, neither has the existence of the lesser muscular ring been yet proved in the same manner.

But it may be said, although the existence of the lesser muscular ring has not been demonstrated by ocular inspection, yet it is justly inferred from its effect, the contraction of the pupil, which is visible, and is no other way to be accounted for, but by supposing the existence of such a muscular ring.

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* Thes. Anatom. II. p. 87. † Ibid. p. 14.

I anſwer, the change of conformation, in adapting the eye to very near objects, is no leſs certain than the contraction of the pupil: And this change of conformation has not yet been adequately accounted for, but may be fairly made out by ſuppoſing the exiſtence of the greater muſcular ring, as I propoſe to ſhew by-and-by.

130. The cryſtalline humour is contained in a very fine membranous *capſula*, with a water between them, after the manner of the heart in the *pericardium*.

This I take from the obſervations of the late Anatomiſts, particularly the famous Monſ. * PETIT, from whom I muſt likewiſe obſerve, that the † back part of this *capſula*, or that part which inveſts the hinder ſurface of the cryſtalline humour, adheres to the membrane encloſing the vitreous humour, yet ſo as to be ſeparated from it without cutting: but that all along the limb or edge of the cryſtalline theſe two membranes adhere ſo firmly together, as not to be parted without the knife.

I muſt alſo take notice that, from the meaſures taken by this diligent and accurate Anatomiſt, as well as from autopſy, it appears that the figure of this compound body conſiſting of the cryſtalline humour, the water ſurrounding it, and the *capſula* containing them both, is ſuch as would ariſe from two ſegments of equal breadth, but of unequal ſpheres, clapt together on their plane ſides, and having the ſharp edge rounded off, ſo as to leave an obtuſe limb or edge of ſome conſiderable thickneſs, by which means the attachment of the edge of the *capſula* to the membrane of the vitreous humour all along that edge is rendred much ſtronger than it could otherwiſe be.

And to render this attachment ſtill ſtronger, I have obſerved the limb of the *capſula* to be indented all round with ſhallow tranverſe *ſulci*, or furrows, ſeemingly perpendicular to the limb, into which furrows I ſuppoſe the membrane of the vitreous humour is all along inſerted.

Thus much from obſervation, and till it ſhall be otherwiſe determined by experiment, I take leave for facility of calculation to ſuppoſe, that the *capſula*, the water within it, and the cryſtalline humour itſelf have all of them one and the ſame refractive power.

131. The *ligamentum ciliare* is a muſcle compoſed of longitudinal fibres, and is much weaker than the *uvea*. It ariſes cloſe behind the *uvea*, from the abovementioned circular ridge at the juncture of the *cornea* and *ſclerotica*, and running over the outer edge of the vitreous humour is inſerted all round the anterior ſurface of the *capſula*, upon which, ſays Monſ. ‡ PETIT, this ligament prolongs its fibres and the veſſels which it furniſhes to the *capſula*.

Now as that part of the *capſula*, into which theſe muſcular fibres and veſſels are inſerted, muſt thereby be rendred ſomething leſs diaphanous than the reſt, it is probable that this inſertion does not extend far enough towards the middle of the *capſula*, to be in the way of the rays that paſs the pupil in its greateſt dilatation.

132. If what is contained in the four preceding articles be allowed me, I think the change of conformation in the eye, to ſee objects diſtinctly at different diſtances, may be explained in the following manner.

When the eye is perfectly at reſt, no force, ſtrain, or effort of any kind being uſed by any of its parts, it is then ſuited to ſee with *Perfect Viſion* at ſome one determinate moderate diſtance.

This diſtance, I ſuppoſe, is for moſt eyes about 15 or 16 inches, the uſual diſtance for reading a print of a middle ſize. For it is likely, we uſually read at that diſtance, where viſion is perfect without any ſtraining of the eye, and at which conſequently we read with moſt eaſe, and can continue it longeſt.

Eye how altered for near objects.

133. When we view objects nearer than the diſtance of 15 or 16 inches, I ſuppoſe the greater muſcular ring of the *uvea* contracts, and thereby reduces the *cornea* to a greater convexity. And when we ceaſe to view theſe near objects, this muſcular ring ceaſes to act, and the *cornea* by its ſpring returns to its uſual convexity ſuited to 15 or 16 inches. In which condition the elaſticity of the *cornea* on the one ſide, and the tone of the muſcular ring on the other, may be conſidered as two antagoniſts in a perfect *equilibrium*.

How altered for remote objects.

134. When the eye is to be ſuited to greater diſtances

* Memoires de l'Acad. Royale 1730. † Ibid. p. 436. ‡ Ibid. p. 438.

distances than 15 or 16 inches, I suppose the *ligamentum ciliare* to contract its longitudinal fibres, and by that means to draw the part of the anterior surface of the *capsula*, into which these fibres are inserted, a little forwards and outwards. And at the time this is done, the water within the *capsula* must necessarily flow from under the middle towards the elevated part of the *capsula*, and the aqueous humour must flow from above the elevated part of the *capsula* to the middle. Consequently, the middle part of the anterior surface of the *capsula* must a little sink, while the other is elevated, or the whole anterior surface within the insertion of the *ligamentum ciliare* must be reduced to a less convexity. And when the contraction of the *ligamentum ciliare* ceases, the anterior part of the *capsula*, which has been put a little upon the strain by that contraction, will by its elasticity recover itself and return to its former figure. In which condition the elasticity of the *capsula* and the tone of the ligament may also be looked upon as two antagonists perfectly *in æquilibrio* with one another.

This *capsula* as it is a very tender membrane, and contains a water between its inner surface and the crystalline, can readily obey the effort of so weak a muscle as the *ligamentum ciliare*, which would not be sufficient to flatten the crystalline itself, considering the firmness of its contexture. And hence appears the true use of the *capsula* and the water within it.

Here possibly it may be thought, that the *ligamentum ciliare* might as well have been made stronger, and have been inserted into the crystalline itself, in which case it had been sufficient to have drawn outwards and flattened the crystalline without all this *apparatus* of a *capsula* and water within it.

But this would not so well have answered the end proposed. For the *ligamentum ciliare* arises from the edge of the *cornea* at its union with the *sclerotis*, close to the *uvea*; and consequently when the longitudinal fibres of such a stronger *ligamentum ciliare* had shortened themselves, they must not only have drawn the crystalline outwards, but must have drawn the *cornea* inwards, that is, they must not only have lessened the convexity of the crystalline, but must have increased the convexity of the *cornea*: And these two effects would have been contrary to one another, the flattening of the crystalline tending to suit the eye to remoter objects, and the increasing the convexity of the *cornea* tending to suit the eye to nearer objects. Whereas the *ligamentum ciliare*, being made so weak, cannot sensibly affect the *cornea*, and yet by means of this admirable contrivance of a *capsula* and water within it, is sufficient for the intended effect.

I need not take notice, that such a stronger *ligamentum ciliare* might by its contraction have endangered the disuniting the crystalline humour from the vitreous.

I had once thought, that both surfaces of the *capsula* were rendred less convex by its edge being drawn a little outwards, and had formed my computations from that notion. But upon considering the close attachment of the hinder surface of the *capsula* to the membrane of the vitreous humour, particularly the firm adhesion of these two membranes at the edge of the *capsula*, as likewise the situation of the anterior and outer part of of the vitreous humour, such as that it must necessarily obstruct the drawing the edge of the *capsula* outwards, and especially upon calling to mind the situation and insertion of the *ligamentum ciliare*, I found the suiting the eye to distant objects could not be performed but by the flattening of the anterior surface only.

135. This may suffice to give a general notion of the manner, by which the eye alters its conformation according to the different distances of objects; but in order to shew that the means we have proposed, are adequate to the intended purpose, it will be necessary to enter into particulars.

In Fig. 51, let *BAB* represent the *cornea*; *BSB* the *sclerotica* joining to the *cornea* in *B* and *B*; *Bu*, *Bu* the *uvea*; *uu* the aperture of the pupil; *DCD* the anterior surface of the *capsula* of the crystalline humour; *DED* the hinder surface of the same *capsula*; *Bd*, *Bd* the *ligamentum ciliare*; *AS* the axis of the eye from the outer surface of the *cornea* to the *retina*; *AC* the axis of the *cornea* and aqueous humour; *CE* the axis of the *capsula*, the water within it, and the crystalline humour; *ES* the axis of the vitreous humour; *M* that point of the axis of the eye at which parallel rays will be collected into a point, or the principal focus of the eye; *f*, *L*, *I* the other foci of Art. 370. of this Book; *AP* the distance of an object from the eye, when

when the pencils are collected into points on the *retina* at *S*.

Now let us suppose, as before in Art. 108, the radius of $BAB=3,3294$; that of $DCD=3,3081$; that of $DED=2,5056$; $AC=1,0358$; $CE=1,8525$; the refraction at *A* as 4 to 3; that at *C* as 13 to 12; and that at *E* as 12 to 13.

Then will AM be 8,9993; the rectangle $LI\times fM=49,2526$; and it remains to find *MS*, by deducting AM from *AS*.

136. Monf. PETIT * from the measure of a single eye makes $AS=10,0578$: but this seems to differ greatly from the common length of an eye, and if there is no error of the press, could belong only to an eye very shortsighted.

In six eyes of adult persons, which we have measured, the axis from outside to outside was as follows, 9,3; 9,8; 9,6; 9,3; 9,4; 9,0; the medium of which is 9,4.

From this length the thickness of the *sclerotica* is to be deducted; and upon cutting thro' this coat at the termination of the axis, we placed two pins near its divided edge, and looking at them thro' a magnifying glass, the thickness of the coat seemed equal to one of the pins, whose diameter we before knew to be $=0,25$. Hence the axis of the eye, from the outer surface of the *cornea* to the *retina*, may be supposed usually to be 9,15, and $MS=0,1507$.

137. Hence, by Art. 370 of the Book of Opticks, $\frac{LI\times fM}{MS}$, or $PL=326,7$, and $AP=332$ tenths, or 33 inches nearly.

This therefore is the distance, at which an eye of these dimensions, and with the refractions here supposed, must see an object with perfect distinctness, and without any straining or effort of any of its parts, that is, with perfect ease; and may therefore justly be called the natural distance of such an eye.

138. But we apprehend this distance is too large for common eyes, and that for two reasons.

1. It seems reasonable to suppose, that the distance, at which we usually read a fair large print, is the distance we find by experience to be the most easy to us; and this is about 15 or 16 inches.

2. It is to be presumed, that the natural distance at which we see distinctly with perfect ease, is but about double the least distance we see distinctly at. For by Art. 8. of the *Remarks*, it requires nearly as great a change of conformation to lessen the natural distance to one half, as to increase the natural distance to infinity. Whence it is reasonable to conclude, that the natural distance is such, as that no greater change of conformation is required to reduce it to the least distance, than to increase it to the greatest distance at which we can see distinctly. But if the natural distance be 33 inches, it will follow from the same Art. of the *Remarks*, that it requires above four times as great a change of conformation to reduce it to the least distance of 5 or 6 inches, as to increase the natural distance to infinity. Whereas, if we suppose the natural distance to be about 15 or 16 inches, there will be required not much more change of conformation to see distinctly at about 6 inches, than to inlarge that distance to the greatest distance *Perfect Vision* can extend to.

139. This seems to make it necessary to increase some one at least of the refractions we have before supposed. And indeed, as the aqueous humour is not a bare water, but an animal liquour in some degree charged with salts and sulphurs, it seems reasonable to allow it a refraction somewhat greater than that of 4 to 3, or more accurately than that of $80\frac{3}{4}$ to 60, which is a mean between the most refrangible and least refrangible rays on passing out of air into water. See S[r] Is. NEWTON's Opticks. pag. 114.

Let therefore the refraction of this humour and of the *cornea* be supposed that of 81 to 60, or of 27 to 20, instead of that before supposed of 80 to 60, and AM will now be 8,8202, MS 0,3298, and $LI\times fM$ will be 46,7621. Whence $PL=141,8$, $AL=5,2$, and AP, or the natural distance of the eye, will be 147 tenths, or 15 inches nearly, much the same as we find by experience the most easy distance to read at. Natural distance of objects from the eye.

But here it may be proper to observe, that though this distance of 15 inches is the most easy to read at, when the print is large, or at least of a middling size, yet it is not the easiest distance to read at, when the print is small.

For let a young person, but of full growth, read

* Memoires de l' Acad. Royale 1728.

read a middling print, and you will find him hold the book at about 15 inches more or leſs from his eye. Then give him a book of a ſmaller print, and you will find he holds it nearer, perhaps about 12 or 13 inches.

The reaſon of this is plain. When the print is ſmall, the book is held nearer to inlarge the image upon the *retina*, that being reciprocally as *PL*, by Art. 376. of this book; ſo that by bringing the book nearer, the image is as large as that of the larger print was before. And to remedy the confuſion of the image, if it be conſiderable, either the *cornea* is rendred a little more convex by a gentle and ſmall contraction of its muſcular ring, or the pupil is a little contracted.

140. Having now ſettled the natural diſtance of objects from the eye, or the diſtance at which objects are moſt diſtinctly ſeen, when the eye is perfectly at reſt and at eaſe, without any the leaſt ſtrain or effort of any of its parts, we come next to examine whether the means we have propoſed for changing the conformation of the eye for viewing near objects, be ſufficient to accomodate that organ to *Perfect Viſion* at the leaſt diſtance, namely that of five or ſix inches. In order to which, beſides the refractions and meaſures we have already laid down, it will be neceſſary to give one or two meaſures of other parts.

141. The chord of the *cornea*, or *BB*, Fig. 51, is uſually, according to Monſ. PETIT, 5 lines *Paris* meaſure, that is in our meaſure, 4,4392. Hence the verſed ſine belonging to this chord, or that part of *AC* which is intercepted between *A* and *BB*, is 0,8481.

Let now the greater muſcular ring of the *uvea* contract itſelf, ſo as to leſſen its circumference about $\frac{1}{47}$ part, or in the proportion of 4,4392 to 4,5462: Then muſt the chord of the *cornea*, which is the diameter to that circumference, be leſſened in the ſame proportion, or from 4,4392 to 4,3462, by the edge of the *cornea* being drawn ſo much inwards. And as *BAB* the arc of the *cornea* continues of the ſame length as before, but is rendred only more convex, the radius to this arc and this chord will be 3, inſtead of 3,3294, being leſſened about one tenth part; and the verſed ſine of the *cornea* will be increaſed by 0,0835, that is, the line *AC* will be increaſed from 1,0358 to 1,1193, as will eaſily be found by computation.

142. Computing therefore by the new radius of the *cornea*=3, and this new line *AC*=1,1193, and uſing the other meaſures and refractions as before in Art. 139, we ſhall find *AM*=8,3594; *MS*=0,8741; *AL*=4,8970; *LI*×*fM*=41,8714. Whence *PL*=47,9; and *AP*=52,8, or a little more than 5 inches.

Therefore the means we have propoſed, for reducing the natural diſtance of the eye, to the ſmalleſt diſtance we ſee diſtinctly at, are ſufficient for that purpoſe, few grown perſons having *Perfect Viſion* nearer than between five and ſix inches. Tho' by a very little farther contraction of the muſcular ring the diſtance may ſtill be ſomewhat leſſened, where the *cornea* is ſufficiently flexible.

Nor can any juſt objection be drawn againſt the change of conformation we have here ſuppoſed in the eye, as being greater than can reaſonably be admitted. For the radius of the *cornea* alters only a tenth part, and this ariſes from the contracting the greater muſcular ring of the *uvea* only $\frac{1}{47}$ part, which is vaſtly leſs than the contraction of the leſſer muſcular ring, that being able to contract into half its dimenſion, when the eye is expoſed to a ſtrong light.

But poſſibly ſome doubt may ariſe about that circumference of the *cornea* into which the *uvea* is inſerted, whether by reaſon of its union with the *ſclerotica*, it can comply with the contraction of its muſcular ring, ſo as to be drawn inwards towards the pupil, and likewiſe to contract itſelf into a leſs circumference.

To this therefore we reply, that the ſpace, by which it approaches the pupil, is by our ſuppoſition very ſmall, being leſs than $\frac{1}{[illegible]}$ part of an inch: and that this ſmall motion is favoured by the obliquity of its junction with the *ſclerotica* obſerved by * Monſ. PETIT: And that the ſpace, by which that circumference ſhortens its length, is leſs than $\frac{1}{100}$ of an inch, which in a compreſſible and dilatable membrane is not hard to conceive.

But the quantity both of this approach to the pupil, and of this contraction in circumference, will be very conſiderably leſſened, if the arch *BAB* be ſuppoſed a little to ſtretch

* Memoires de l'Acad. Roy. des ſciences. 1728.

and

and dilate, when the points *B*, *B* are drawn inward. And this we take to be the truth of the caſe, though, not to tire our reader with too many *minutiæ*, we have above omitted this conſideration.

And laſtly, as we have above taken notice, Art. 134, that the elaſticity of the *capſula* and the tone of the *ligamentum ciliare* antagonize each other, it follows that, when the edge of the *cornea* is drawn inwards by the contraction of the *uvea*, and conſequently the *ligamentum ciliare* is relaxed, the *capſula* will then grow more convex. On which account a ſomewhat leſs convexity of the *cornea*, and a leſs contraction of the *uvea* will be neceſſary, than is above ſuppoſed.

143. We proceed now to examine into the ſufficiency of the means we have propoſed, to accommodate the eye to greater diſtances.

The chord of the cryſtalline humour, or *DD*, is according to Monſ. Petit, † at a medium from 26 different eyes, in our meaſure, 3,7321. And the radius of the anterior ſurface being 3,3081, the verſed ſine to that chord is 0,5765. And the radius of the hinder ſurface being 2,5056, the verſed ſine to that ſame chord is 0,8355.

Now theſe two verſed ſines, 0,5765 and 0,8355 added together, make only 1,4120, falling ſhort of 1,8525 the axis of the cryſtalline by 0,4405, which plainly proves what we had before aſſerted, Art. 130, that the cryſtalline is compoſed of two ſegments of ſpheres joined together on the flat ſides, with the edge rounded off.

For a ſolid of the ſame dimenſions with the cryſtalline, I mean the compound cryſtalline conſiſting of the true cryſtalline, water and *capſula*, may be compoſed, either by prolonging the common chord of the two ſegments, till their verſed ſines added together make up the axis 1,8525, and then rounding off the edge till the common chord is again reduced to 3,7321:

Or by ſuppoſing two ſegments of the dimenſions aſſigned above to be clapped one on the upper ſurface, the other on the under ſurface of a cylinder having the ſame breadth with themſelves, and the altitude 0,4405, which added to the verſed ſines of the two ſegments makes up the axis 1,8525.

We have above taken notice, Art. 131, that the anterior ſurface of the *capſula* muſt probably have its middle part free from the inſertion of the *ligamentum ciliare*, to ſuch a breadth as to receive all the rays that paſs the pupil when moſt dilated. Now it is obvious, that none of the rays that paſs the pupil, can fall upon this middle part, although its breadth were conſiderably leſs than the diameter of the pupil, which hardly ever exceeds 2,22, or half the breadth of the *uvea*.

However, we ſhall here ſuppoſe the breadth of the anterior ſurface of the *capſula*, or *dd*, Fig. 51, to be 2,5, ſo that the right ſine of half the arc *dd* will be 1,25; the half arc will be 22°. 12′; and its verſed ſine will be 0,2452.

Let now the *ligamentum ciliare* contract itſelf ſo, as that the point *d* may be drawn forwards and outwards, and let its motion forwards, parallel to the axis of the eye, be about $\frac{1}{400}$ part of an inch, or 0,0255; and in conſequence of this motion let the vertex of the *capſula*, by Art. 134, be moved as much backwards. Then will the radius of this arch *dd* be increaſed from 3,3081 to 4,2000, and the verſed ſine will be leſſened from 0,2452 to 0,1941, the difference being 0,0511.

But hereby the axis of the aqueous humour, or *AC*, Fig. 51, will be increaſed by 0,0255, that is, from 1,0358 it will become 1,0613, and the axis of the whole cryſtalline, or *CE*, will be as much leſſened, that is from 1,8525 it will become 1,8270.

Then if we compute from this new radius 4,2000, and theſe new axes *AC* and *CE*, the other meaſures and refractions being the ſame as in Art. 139, we ſhall find $AM = 9,1209$; whence $MS = 0,0291$; and $LI \times fM$ will be 50,4316; and $AL = 5,4321$. Whence $PL = 1733,0000$, and $AP = 1738$, or 14 feet, 5,8 inches.

Whence it appears, that the means we have here ſuppoſed, are fully ſufficient to enable the eye to extend its natural diſtance of 15 inches to that of 14 feet 5 inches, and that without any the leaſt motion of the cryſtalline humour itſelf, and with a very ſmall one of the anterior ſurface of the *capſula*, the point *d* and the vertex being moved only $\frac{1}{400}$ part of an inch, and the intermediate parts moving leſs.

† Mem. 1730.

When

When the *ligamentum ciliare* ceaſes to contract, the *capſula*, whoſe parts, eſpecially between *d* and *d*, have been put a little upon the ſtretch, will by its elaſticity reſtore itſelf to its former dimenſions.

144. Having ſhown how the natural diſtance of an object from the eye to be ſeen with *Perfect Viſion*, may by rendring the *cornea* more convex be reduced to five or ſix inches, and on the other hand may by rendring the anterior ſurface of the *capſula* leſs convex be extended to 14 feet 5 inches, we are to obſerve, that though the eye is poſſeſſed of this power to extend *Perfect Viſion* from one of theſe limits to the other, yet it does not always exert that power to the utmoſt; but contents itſelf with attaining to *Diſtinct Viſion*, which as we have taken notice in Art. 7 and 103, cannot by the ſenſe be diſtinguiſhed from *Perfect Viſion*. And this muſt happen chiefly near the limits of *Perfect Viſion*, where the ſtraining either of the greater muſcular ring of the *uvea*, or the *ligamentum ciliare*, to the utmoſt they are capable of, muſt be ſomewhat laborious and uneaſy.

IV.

Indiſtinct viſion how rendred diſtinct.

145. We proceed now to the fourth point we propoſed to inquire into, namely, By what means Viſion, when it cannot be *Perfect*, is rendred *Diſtinct*, or at leaſt not ſo indiſtinct as it would otherwiſe be.

This we apprehend to be performed by two ſeveral ways.

1. The firſt is for the eye to apply the ſame power, as it uſes to obtain *Perfect Viſion*.

For inſtance, let us ſuppoſe the eye to be in a ſtate of inaction, and conſequently ſuited to have *Perfect Viſion* at the natural diſtance of 15 inches, and let any object, as a book of a ſmall print, be placed within 4 inches from the eye, and the eye be attentive to read it. Then immediately the *cornea* will be rendred ſo convex, as to ſuit the eye for ſeeing with *Perfect Viſion*, at ſome diſtance not leſs than five or ſix inches, and by this means the viſion at four inches, though it cannot be rendred perfectly diſtinct, yet will be leſs indiſtinct than it would otherwiſe be, and perhaps diſtinct enough to read the book with eaſe.

Again, let another object, as a playhouſe bill paſted againſt a wall, be preſented to the eye at the diſtance of ſixteen feet. Then as ſoon as we attempt to read this bill, the anterior ſurface of the *capſula* will be rendred flatter, ſo as to accomodate the eye to ſome diſtance not exceeding its utmoſt limit of 14 feet 5 inches; and though by this means viſion at the diſtance of 16 feet cannot be rendred perfectly diſtinct, yet will it be rendred leſs indiſtinct than before, and perhaps diſtinct enough to read the Bill with eaſe.

Here it may be proper to remark, that as, by Art. 133, the elaſticity of the *cornea* is an antagoniſt to the contraction of the greater muſcular ring of the *uvea*, and by Art. 134, the elaſticity of the *capſula* antagoniſes the contraction of the *ligamentum ciliare*, it follows that a greater contraction of either of theſe muſcles muſt be more laborious than a leſs contraction. Therefore in many caſes neither of them will be contracted to ſo great a degree as would be neceſſary to procure either *Perfect Viſion*, or viſion as little indiſtinct as poſſible, but to ſome leſſer degree only, ſuch as may procure *Viſion* ſufficiently *Diſtinct*.

For inſtance, if a young adult perſon in reading holds his book at 10 inches diſtance, it will not be neceſſary to contract the greater muſcular ring of the *uvea* ſo much, as to procure *Perfect Viſion* at the diſtance of 10 inches: for a middling print it may be enough to contract the *uvea* ſo much only, as would procure *Perfect Viſion* in caſe the book were at the diſtance of 13 or 14 inches; or at the diſtance of 11 or 12 inches, if the print be ſmall; and in this conformation of the eye he may ſee with ſufficient diſtinctneſs at 10 inches for the book to be eaſily read. And this leſſer contraction, being leſs laborious, will be uſed inſtead of the greater contraction, which is more fatiguing, eſpecially if he reads a great while.

And if he were to read at ſix inches diſtance, it will not be neceſſary to have *Perfect Viſion* at that diſtance; a leſs contraction of the *uvea* may ſuffice, ſuch as accommodates his eye to have *Perfect Viſion* at eight or nine inches diſtance, or poſſibly at ſeven inches diſtance, if the print be very ſmall.

For if the print be given, the leſſer diſtance he reads at, the more the *uvea* will contract: And if the diſtance be given, then the leſs the print, the more the *uvea* wlll contract. But it will never contract ſo much as to procure *Perfect Viſion*, at the diſtance of the book, but will ſtop at a degree of contraction

leſs

leſs laborious, and yet enough to render Viſion ſufficiently diſtinct; or the eye will not always receive one and the ſame conformation at the ſame diſtance; but the conformation will vary as the object is varied, though the diſtance be the ſame.

Likewiſe, in looking at an object at the diſtance of 14 feet, it will ſeldom be neceſſary for the *ligamentum ciliare* to contract ſo much as to procure *Perfect Viſion* at that diſtance. If the object be ſingle and uncompounded and ſufficiently large, or ſufficiently luminous, it may be ſufficient to contract the ligament ſo much only as would ſuffice to procure *Perfect Viſion*, at three or four foot diſtance. And if the object be compounded, but of few parts and thoſe large and eaſily diſtinguiſhable, it may not be neceſſary to contract the ligament more than would be ſufficient for *Perfect Viſion* at the diſtance of ſix or eight feet.

And in viewing objects much more remote than the diſtance *Perfect Viſion* can be extended to, it will not always be neceſſary to contract the *ligamentum ciliare* to the utmoſt, or to ſuch a degree as to procure *Perfect Viſion* at 14 feet five inches: but according to the largeneſs of the object, or of the parts it conſiſts of and to which the eye is attentive, ſometimes one, ſometimes another, leſſer and leſs laborious contraction of the ligament will ſuffice to ſhew the object with ſufficient diſtinctneſs, eſpecially when the pupil is contracted at the ſame time, as will often happen.

Conformation of the eye not always the ſame for the ſame diſtance.

So that in looking at very diſtant objects, the eye will not always have one and the ſame conformation, namely the flatteſt poſſible conformation of the *capſula*; but that conformation will be different for different objects at the ſame diſtance, as well as for the ſame object at different diſtances. And from this conſideration we are led to ſuſpect, that the Ingenious Dr [a] PORTERFIELD was miſtaken, when he fixed the utmoſt limit of his eye for *Perfect Viſion* at 27 inches.

2. The other way is the contraction of the pupil by the leſſer muſcular ring of the *uvea*. For by art. 382 of the Book of Opticks, the *radius of diſſipation* is, *cæteris paribus*, always proportional to the radius of the pupil. Conſequently, when the pupil is narrower, the *radius of diſſipation* and the *penumbra* ariſing from the diſſipation will be ſmaller, that is, viſion will be rendred either diſtinct, or at leaſt leſs indiſtinct than it would otherwiſe be.

146. Here it may be proper to take notice, that in a weak light the firſt of theſe means, namely the altering the convexity of the *cornea*, or of the *capſula*, muſt be uſed. For in ſuch a caſe the pupil is ſo far from contracting, that there is rather a neceſſity of dilating it to take in more light.

147. But in a ſtrong light the contraction of the pupil is chiefly made uſe of. For then that contraction anſwers two purpoſes; one to exclude an over great quantity of light, which would be offenſive to the eye; the other, to leſſen the indiſtinctneſs. And when the light is very intenſe, the pupil may contract ſo much as of itſelf to cauſe *Diſtinct Viſion*, and to render the other means altogether unneceſſary. So that theſe two ſeveral means of procuring diſtinct, or leſs indiſtinct viſion, may ſometimes be uſed jointly, that is, each in a moderate degree, and ſometimes ſingly.

Contraction of the pupil not always dependent on the will, or appearance of confuſion.

148. The degree, to which the pupil contracts, does not abſolutely depend either upon our will, or upon the ſenſation of confuſion in the object; but partly upon the degree of light.

This is eaſily proved in the following manner. By day-light take any book, and ſtanding about the middle of a room, with your back to the window, hold the book ſo near, as that the letters may appear indiſtinct, and yet not ſo much, but that you can read, though with ſome difficulty: Then turn your face to the light, and the book will be read with more eaſe. Again, holding the book at the ſame diſtance from your eye, go into the darkeſt part of the room, and ſtanding with your back to the light, you will find the book not at all legible: But upon coming to the window, with your face to the light, you will be able to read, eſpecially if the ſun ſhines, with great eaſe and diſtinctneſs.

Alſo, a perſon, who has been obliged for ſome years to uſe ſpectacles in reading, will in the ſunſhine be able to read very eaſily without them.

Whence it appears, that in a ſtrong light

[a] Medical Eſſays of Edinburg. Vol. IV. p. 169.

the pupil contracts to a greater degree, than it can be made to do by the act of our will, or by the ſenſation of confuſion only; conſequently it is a miſtake to think, that, upon ſeeing an object confuſedly, the pupil does always contract itſelf to the leaſt ſize it is capable of.

V.

Changes in the eye by cuſtom, or by age.

149. The fifth point we propoſed to examine into, was, What change is made in the eye either by habit and cuſtom, or by our growing into years.

In the eyes, as well as in all other parts of the body, the muſcles by conſtant exerciſe are enabled to contract themſelves with more ſtrength, and with more eaſe, and to a greater degree: And by diſuſing exerciſe they are reduced to leſs ſtrength, and perform their office with more difficulty, and in a leſs degree.

150. Alſo, the elaſtick parts of the eye, like thoſe of the reſt of the body, if they are often and long kept in tenſion, do more eaſily obey that tenſion, and do by degrees loſe ſomething of their elaſticity, ſo as not to be able to reſtore themſelves, or at leaſt not to do it with the ſame readineſs or to the ſame degree, when the tenſion ceaſes or abates: And if they are too ſeldom put in tenſion, they grow ſtiffer and are not ſo eaſily diſtended to the ſame degree.

151. From theſe cauſes ſeem to ariſe thoſe changes of the eye, that habit and cuſtom introduce.

Perſons, who are much and long accuſtomed to view remote objects, and but ſeldom to conſider near ones, ſee better at great diſtances, and not ſo well at near diſtances as other people.

This is often the caſe of Travellers, Sailors, Sportſmen, *&c.* In them the *ligamentum ciliare* being much uſed, comes by degrees, by art. 149, to acquire more ſtrength, ſo as to draw out the *capſula* of the cryſtalline farther, and thereby to reduce it to a leſs convexity than in other perſons. Whence they are enabled to ſee diſtinctly at greater diſtances than other perſons.

But the *capſula* being thus often put upon the ſtretch, and long detained ſo, will thereby in time loſe ſomewhat of its elaſticity, ſo as not to return to its natural convexity, when the contraction of the *ligamentum ciliare* ceaſes, by art. 150. And this is one reaſon, why theſe perſons do not ſee ſo well as the reſt of Mankind, at ſmall diſtances.

Alſo, the greater muſcular ring of the *uvea*, by diſuſe, will grow weaker, by art. 149: And the *cornea* by being ſeldom bent into a greater convexity, will in time grow ſtiffer and leſs capable of obeying the contraction of the muſcular ring, by art. 150, which is another cauſe why theſe perſons do not ſee ſo well at ſmall diſtances.

152. On the other hand, Perſons, who are much and long accuſtomed to view objects at ſmall diſtances, as Students in general, Watchmakers, Gravers, Painters in miniature, *&c.* ſee better at ſmall diſtances, and not ſo well at great diſtances, as the reſt of Mankind.

For in them the greater muſcular ring of the *uvea* contracts more eaſily and ſtrongly, by art. 149: And the *cornea*, by art. 150, more readily obeys the contraction of that ring. Whence they ſee better at ſmall diſtances.

But the *cornea*, by being thus often and long bent into a greater convexity, does by degrees loſe ſomething of its elaſticity, ſo as not to return to its natural convexity, when the muſcular ring ceaſes to act upon it, by art. 150. This is one cauſe of their not ſeeing ſo well at great diſtances.

Alſo, the *ligamentum ciliare*, being ſeldom employed to leſſen the convexity of the *capſula*, does by degrees become leſs capable of performing that office, by art. 149: And the *capſula*, being ſeldom drawn out and put into tenſion, muſt, by art. 150, loſe ſomething of its diſtenſile quality, ſo as leſs eaſily to comply with the action of the ligament. And this is another cauſe of their not ſeeing ſo well at great diſtances.

153. Perſons, who begin to uſe ſpectacles early, and keep conſtantly to the uſe of them, do in a little time find themſelves under a neceſſity of continuing to uſe them. For, by uſing ſpectacles, they have not occaſion to contract the greater muſcular ring, to bend the *cornea*, ſo much as before, and when they have forborn this contraction for ſome time, they are not ſo well able to perform it, as before they uſed ſpectacles.

154. In children the pupil is uſually more dilated than in grown perſons. This is eaſily ſeen; for in grown perſons the diameter of the

the pupil ſeldom appears equal to the breadth of the ring of the *uvea* on either ſide of it, that is, is ſeldom equal to one third of the breadth of the *cornea*, and is often much leſs, eſpecially in a good light. But in children the diameter of the pupil ſcarce ever appears ſo little as one third of the breadth of the *cornea*, and often exceeds half that breadth.

The reaſon of this we apprehend to be, that in children the *cornea* is extremely flexible, ſo as to be very eaſily bent by its muſcular ring, into any curvature that is neceſſary for ſeeing diſtinctly in reading, and conſequently their pupil has leſs occaſion to contract for *Diſtinct Viſion*.

But in grown perſons the *cornea* is ſomewhat ſtiffer and does not ſo eaſily bend, whence they have occaſion more frequently to contract the pupil.

And in elderly perſons the *cornea* grows ſtill more rigid, ſo that they can hardly read without ſpectacles, unleſs the print be very large, or the light be very ſtrong, ſo as to cauſe a great contraction of the pupil. It is for this reaſon they are obliged to hold the candle between the eye and the paper they read; and their doing ſo is a certain ſign that they begin to want ſpectacles.

155. Children read much nearer than grown perſons. This happens for two reaſons.

1. Their eyes are ſmaller, and the leaſt diſtance any eye can ſee diſtinctly at, is proportional to the length of the eye.

2. Their *cornea*, being very flexible, is eaſily accomodated to a leſs diſtance. And at a leſs diſtance the print appears larger, and is eaſier to read, than at a greater.

156. Elderly perſons ſee better at a great diſtance than younger people. The matter of fact I think is pretty generally agreed upon, and if queſtioned, may thus be examined. Let a perſon, whoſe ſight for near objects is impaired by age, obſerve the Moon at three or four days old, and take notice whether the illuminated part appears of a larger diameter than the dark part, and how much is the exceſs. And let him recollect, if he can, what was the appearance of thoſe two parts of the Moon, when he was much younger. Then, I am perſuaded, he will think the exceſs of the light part above the dark is either none at all, or much leſs than formerly. At leaſt this is my caſe. I well remember, that formerly the diameter of the light part of the Moon much exceeded that of the dark part: Whereas now the limbs of thoſe two parts appear nearly to unite in the ſame circle.

The cauſe why perſons advanced in years ſee better at a diſtance, has been reckoned to be the ſhrinking of the coats and humours of the eye. But this I think cannot be ſatisfactory. If the eye were to ſhrink in all parts proportionably, this would have a contrary effect, the eye would then become ſhortſighted, as has been obſerved of children, whoſe eyes are ſmaller than thoſe of men.

But the true reaſon I take to be this. The *cornea*, as it is of a rarer texture, and is more expoſed to the air than the *ſclerotica*, will in length of time ſhrink a little more than the *ſclerotica*, and will by that means grow a little flatter than it was before. For in Fig. 51, if the line $BuuB$ continue of the ſame length, and the arc BAB be a little leſſened, the vertex of this arc at A muſt be a little depreſſed towards C, that is, the arc BAB muſt become flatter or leſs convex. And if the length of this arc of the *cornea* be leſſened by ſhrinking, about the $\frac{1}{1500}$ part of an inch, this will be ſufficient to encreaſe the radius of the *cornea* from 3,3294 to 3,3500, and thereby to increaſe AM, in art. 143, from 9,1209 to 9,1497. And as by this ſhrinking of the *cornea*, its verſed ſine will be leſſened by 0,0070, and conſequently the axis of the eye will now be leſſened from 9,1500 to 9,1430, AM may ſomewhat exceed this axis, if the *capſula* of the cryſtalline be flattened as much as we have ſuppoſed in art. 143, and if it be ſomething leſs flattened, AM may be exactly equal to the axis, that is, parallel rays may be collected into a point upon the *retina*, or the fartheſt limit of *Perfect Viſion* may be extended to an infinite diſtance.

157. Elder perſons do not ſee ſo well at ſmall diſtances as thoſe of leſs age. This happens partly from the ſhrinking, and partly from the rigidity of the *cornea*, which increaſes with our age, and may carry out the neareſt limit of *Perfect Viſion* from 3 or 4 inches, as in children, and from about 5 or 6 inches in young adult perſons, to 20, 30, 40 inches, or a greater diſtance. And in this caſe the eye has no aſſiſtance in viewing near objects, but only from the contraction of the pupil, and this is not ſufficient for *Diſtinct Viſion*, unleſs in a ſtrong light.

If the arc of the *cornea* ſhrink $\frac{1}{200}$ part of an inch, this will remove the natural diſtance from 15 to 77 inches. And the *cornea* being now grown more rigid, the *uvea* will be leſs able to contract it into a greater convexity. While the *cornea* was more flexible, the *uvea* was able to render it ſo convex as to reduce the natural diſtance from 15 inches to 5, that is to a third part: but now probably the new natural diſtance of 77 inches can hardly be reduced to leſs than one half, that is to 38 or 39 inches.

Now as this is probably the caſe of many perſons above 50 years of age, and particularly my own, not to have *Perfect Viſion* at a diſtance leſs than 38 or 39 inches, it might be expected that ſuch perſons might have *Perfect Viſion* at an infinite diſtance, inaſmuch as we have juſt now ſhown, that upon the ſhrinking of the *cornea* only the $\frac{1}{1500}$ part of an inch, *Perfect Viſion* might be extended to an infinite diſtance, provided the *capſula* of the cryſtalline humour could be flattened to the ſame degree as was ſuppoſed in art. 143.

But perſons, whoſe *cornea* is thus flattened, are thereby enabled to ſee diſtinctly at much greater diſtances than before, without flattening the *capſula*; and therefore have leſs occaſion to contract the *ligamentum ciliare*; and that muſcle by diſuſe becomes leſs able to contract itſelf to the ſame degree as before, and conſequently this inability will hinder the extent of *Perfect Viſion* to the utmoſt diſtance it would otherwiſe reach to.

158. As the juices of grown animals are ſtronger and more loaded with animal ſalts and oils, than thoſe of younger ones, it is probable, that the refraction of the coats and humours of the eye becomes ſomething greater as we advance in age, than when we are children. For all ſulphureous ſubſtances refract more ſtrongly, *cæteris paribus*, than others. And Monſ. Petit * obſerves, that the cryſtalline humour, which is at firſt perfectly limpid, has in adult perſons a yellowiſh tinge, which grows ſtronger as they advance in years.

The gradual increaſe of this yellowiſh tinge, and the increaſing refraction conſequent thereupon, may, I ſuppoſe, in ſome meaſure countervail the increaſing ſtiffneſs and flatneſs of the *cornea*. Otherwiſe we ſhould grow longſighted and want ſpectacles ſooner than we do.

VI.

159. The ſixth and laſt head of our enquiry is, What is the leaſt object, or angle, that the eye is capable of perceiving? The leaſt viſible angle.

The learned Dr. † Hook aſſerts, that when an object ſubtends a leſs angle than a minute, it is to the generality of eyes wholly inviſible. And by the experiment related in art. 97 of this book, and one that I have made myſelf, within the limits of *Perfect Viſion*, I am inclined to think, that when the object is round, as a black circular ſpot upon a white ground, or a white ſpot upon a black ground, an eye muſt be exceeding good to perceive it under an angle much leſs than a minute.

160. But there are other caſes, in which a much leſs angle can be diſcerned by the eye, ſome of which we ſhall here conſider, after premiſing one obſervation, which ſeems neceſſary for explaining the reaſons of them.

In order to our perceiving the impreſſion made by an object upon any of our ſenſes, the impreſſion muſt be either of a certain degree of force, or of a certain degree of magnitude.

For inſtance, a very ſmall drop of dew, or rain, may fall upon the hand without our feeling it wet; and a very ſmall particle of ſugar may be laid upon the tongue without our taſting it ſweet; but a ſpark of fire of the ſame magnitude with the drop of rain, by falling upon the hand will ſenſibly affect it, becauſe the impreſſion of the ſpark of fire is of a greater force than that of the drop of water, or particle of ſugar.

And for the ſame reaſon, a ſtar, which appears only as a lucid point thro' a teleſcope, not ſubtending ſo much as an angle of one ſecond, is viſible to the eye, though a white or black ſpot of 25 or 30 ſeconds is not to be perceived.

161. Alſo, though one very ſmall drop of water will not ſenſibly affect the hand, yet a number of ſuch drops falling together, or one larger drop falling alone, will affect the hand with a ſenſe of wet, becauſe the quantity of the impreſſion is greater.

* Memoires de l'Acad. Roy. des Sciences. 1726, 1730. † Animadvers upon *Hevelius's Machina Coeleſtis* pag. 7, 8, and Poſthumous Works, pag. 97.

And

A line visible farther off than a spot of the same breadth.

And for the ſame reaſon, a line, of the ſame breadth with a circular ſpot, will be viſible at ſuch a diſtance, as the ſpot is not to be perceived at, becauſe the quantity of impreſſion from the line is greater than the quantity of impreſſion from the ſpot. And a longer line is viſible at a greater diſtance, than a ſhorter line of the ſame breadth.

162. It has been by ſome perſons ſuppoſed, that the impreſſions of viſible objects are received upon certain *villi* of the optick nerve, imagined to ſtand erect upon the *retina*, like the pile on velvet. And from the experiment that a ſpot leſs than one minute in diameter cannot be perceived by the eye, it would follow that the thickneſs of one of theſe *villi* is about $\frac{1}{5000}$ or $\frac{1}{6000}$ part of an inch.

163. But admitting the ſuppoſition, the diameter of one of theſe *villi* will be found to be much ſmaller.

For, I find, that a bit of ſilver wire, of the thickneſs of $\frac{1}{485}$ of an inch, laid upon a white paper, is viſible at the diſtance of 10 feet. Hence the angle ſubtended by the diameter of the wire is about $3''\frac{1}{2}$. Conſequently, the thickneſs of a *villus* muſt be 17 times ſmaller, than in art. 162.

164. I took a ſingle filament of ſilk, and laid it cloſe to this bit of wire, then viewing them with a deep magnifying glaſs, I judged the diameter of the wire to be equal to 4 diameters of the ſilk. The diameter of the ſilk muſt therefore be about $\frac{1}{1940}$ part of an inch.

This ſilk and wire, being laid upon a white paper, were both viſible at the diſtance of 40 inches from the eye, and the ſilk appeared plainly leſs than the wire.

Here the ſilk ſubtended an angle of $2''\frac{1}{2}$ only. Conſequently, the thickneſs of a *villus* is 24 times ſmaller than that determined in art. 162.

165. It will be objected perhaps, after DES CARTES's way of reaſoning, that there is no neceſſity of ſuppoſing the diameter of the *villi* to be ſo ſmall as the diameter of the ſilk. For if the diameter of a *villus* were 24 times as great as that image, ſo that the image took up only $\frac{1}{24}$ part of each *villus*, yet the whole of every *villus* the image fell upon, would be affected by the impreſſion upon that $\frac{1}{24}$ part. Conſequently, the ſilk muſt be equally perceivable, as if its image had taken up the whole of each *villus*.

We anſwer, If this be the caſe, then the ſilk ought to appear equal in breadth with the ſilver wire, whoſe image takes up only a ſixth part of each *villus*. But, in fact, the wire appears broader than the ſilk.

166. This greater viſibility of a line, than of a ſpot of the ſame breadth, within the limits of *Perfect Viſion*, ſeems to ariſe only from the cauſe we have here laid down, *viz.* the greater magnitude or quantity of the impreſſion upon the *retina* by the line, than by the ſpot. But without the limits of *Perfect Viſion* another cauſe concurs, whereby the difference of viſibility between the line and the ſpot is rendred much more conſiderable. For the impreſſion npon the *retina* made by the line, is then not only much greater, but likewiſe much ſtronger than that of the ſpot.

167. Let *ABC*, Fig. 52, repreſent a white circular ſpot upon a black ground, and let it be placed ſo much beyond the utmoſt limit of *Perfect Viſion*, as that the *radius of diſſipation*, art. 17, may conſiderably exceed the radius of the ſpot. Then as every point within this circle *ABC* diſſipates its rays over a circle conſiderably greater, and can receive no ſcattered light but only from the other points ſituated within the ſame circle *ABC*, it is plain that the whole circle *ABC* muſt appear conſiderably fainter than if it were ſeen by *Perfect Viſion*.

168. But, in Fig. 53, let the ſame circle *ABC* be contiguous to two other equal circles *DEF*, *GHI*, their centers being all ſituated in one right line, and let theſe three circles be viewed from the ſame diſtance as before. Then the circle *ABC* will not appear ſo faint and weak as before, becauſe it will receive part of the rays diſſipated from the adjacent circles *DEF*, *GHI*. And if the lines *GD*, *IF* be drawn touching theſe three circles, and the whole ſpace between thoſe tangents be made white, the circle *ABC* will be ſtill farther illuminated by the light which is diſſipated from the additional ſpaces compriſed between the circles and the tangents. But this will compoſe a line of the ſame breadth with the circle *ABC*, and by the addition of more circles this line may be extended to what length we pleaſe, and the whole line will be more illuminated than the ſingle circle *ABC*.

Therefore a line placed beyond the limits of *Perfect Viſion* will appear more ſtrongly illuminated, and will make an impreſſion of

more

more force upon the *retina*, than a circular ſpot of the ſame breadth. Conſequently, upon this account, as well as upon that of the quantity of impreſſion, the line will be viſible at a greater diſtance than the ſpot.

169. As we have ſhown above, art. 163, 164, that a dark line upon a white ground is viſible, within the limits of *Perfect Viſion*, when it ſubtends an angle of no more than 2 or 3 ſeconds; and as there can be no reaſon to doubt but that a white line upon a dark ground muſt be viſible under as ſmall an angle; it might naturally be expected, that a white ſpace between two black parallel lines, the ſpace being of the ſame breadth as each of the lines, ſhould alſo be viſible within thoſe limits, when the ſpace ſubtends an angle of only 2 or 3 ſeconds, or that this ſpace ſhould be viſible at the ſame diſtance as one of the lines ſingly taken is perceivable. There might even be reaſon for thinking, that this ſpace, as it is white, ſhould be perceivable at a greater diſtance than a black line of the ſame breadth and length. But the caſe is far otherwiſe.

Different viſibility of ſingle and compound objects.

170. For in Fig. 54, let *AB* be two black lines drawn upon white paper with a ſpace between them equal in breadth to each of the black lines; and let *CD* be another black line drawn at ſome conſiderable diſtance from the two former, but equal in breadth and length with either of them. Then, if the paper be ſet againſt a wall, and you retire backwards from it, you will find that at ſome certain diſtance ſuited to your eye, even within the limits of *Perfect Viſion*, the white ſpace between the two lines *AB* will not be diſtinguiſhable, the two lines appearing as one broad line only; but at the ſame diſtance the ſingle line *CD* will be manifeſtly perceived, and will continue to be ſo, though you retire to a conſiderable diſtance farther backwards.

171. This will ſeem the more ſurprizing, when we conſider the following expetiment.

Upon the ſame paper with the two black lines *AB*, and the ſingle black line *CD*, let *IK*, in Fig. 55, be a white line, of the ſame length and breadth with *CD*, but lying between the two black ſpaces *IKEF*, *IKGH*, whoſe ſides next the white line are parallel to each other. Then, if the paper be fixed againſt a wall, and you retire backwards from it, the white line *IK* will be viſible at as great a diſtance as the black line *CD*, that is, at a much greater diſtance than the white line *AB* can be diſtinguiſhed.

172. Here it may reaſonably be aſked, Why the white line *AB* is not viſible at as great a diſtance as the black line *CD*, ſince thoſe two lines are equal both in length and breadth, and are both ſeen within the limits of *Perfect Viſion*? Alſo, Why is the white line *IK* ſeen at a greater diſtance than the white line *AB*, ſince theſe two white lines are equal in length and breadth, are both of them terminated by parallel black lines, and are both ſeen within the limits of *Perfect Viſion*?

173. In order to give a ſatisfactory anſwer to theſe queſtions, it will be neceſſary to premiſe another obſervation.

The more compounded any object is, or the more parts it conſiſts of, it will, *cæteris paribus*, be more difficult for the eye to perceive and diſtinguiſh its ſeveral parts.

For inſtance, it is ſomewhat difficult for the eye to judge how many figures are contained in the following numbers, 1111111111; 1000000000. But if we divide the figures in this manner, 11111,11111; 10000,00000; ſo as to conſtitute ſeveral objects leſs compounded, we can more eaſily eſtimate the number of figures contained in each of thoſe numbers; and more eaſily ſtill, if we thus divide them, 1,111,111,111; 1,000,000,000.

174. The cauſe of this, we apprehend, is the difficulty of keeping the eye perfectly ſteddy.

For in counting theſe figures, 111111111, if the eye be arrived, for inſtance, at the fifth figure from the right hand, and by any imperceptible motion of the body, or involuntary fluctuation of the eye itſelf, its axis happen to be directed to the fourth or ſixth figure, or to the interval between the fifth figure, and either of theſe, we find ourſelves immediately at a loſs, not knowing from which figure to proceed in our counting, and are therefore obliged to begin again at the right hand.

That this is the true reaſon, may appear from its being eaſier to count the figures in the number 121212121212, than in the number 111111111111; and from its being ſtill eaſier to count them in the number 123123123123, or 123412341234, where from the diſſimilitude of the figures the eye more

more eaſily recovers its place when once loſt.

175. From the ſame cauſe of the inſtability of the eye it muſt be, *cæteris paribus*, more difficult to perceive and diſtinguiſh the parts of any compound object, when each of thoſe parts ſubtends a very ſmall angle, than to ſee a ſingle object of the ſame magnitude as one of thoſe parts.

For inſtance, the hour I. upon a dial-plate may be ſeen at ſuch a diſtance, as the hours II, III, IIII. are not to be diſtinguiſhed at, eſpecially if the obſerver be in motion, as in a coach, or on horſeback, or even in a boat upon the water. This may eaſily be experienced in looking at a dial where the intervals between the black or gilt ſtrokes are equal to the breadth of thoſe ſtrokes; and much more eaſily where the intervals are of a leſs breadth, which is a defect in large dials that are to be ſeen at a great diſtance. For in theſe, the intervals ought to be conſiderably broader than the ſtrokes.

Likewiſe, *AB*, in Fig. 54, is a compound object conſiſting of three parts, *viz.* the two black lines and the white line lying between them: But *CD* is a ſingle object conſiſting of one black line only upon a white ground: And *IK*, Fig. 55, is to be conſidered as a ſingle object conſiſting of one white line only upon a black ground.

Now in viewing either of theſe ſingle objects, if the eye be imperceptibly moved, all the effect from that motion will be only, that the object will be painted upon a different part of the *retina*; but wherever it be painted, there will be but one picture, ſingle and unconfounded with any other.

But in viewing the compound object *AB*, if the eye be ſuppoſed to fluctuate ever ſo little, the image of one or other of the black lines will be ſhifted on to that part of the *retina*, which was before poſſeſſed by the white line; and this muſt occaſion ſuch a dazzle in the eye, that the white line cannot be diſtinctly perceived and diſtinguiſhed from the black lines, which by a continual fluctuation will alternately occupy the ſpace of the white line, whence muſt ariſe an appearance of one broad dark line without any manifeſt ſeparation.

176. By trying this experiment with two pins of known diameters, ſet in a window againſt the ſky-light, with a ſpace between them equal in breadth to one of the pins, I find the diſtance between the pins can hardly be diſtinguiſhed, when it ſubtends an angle leſs than 40″, though one of the pins alone can be diſtinguiſhed under an angle vaſtly leſs.

177. But though a ſpace between two pins cannot be diſtinguiſhed by the eye, when it ſubtends a leſs angle than 40″, it would be a miſtake to think, that the eye muſt neceſſarily commit an error of 40″ in eſtimating the diſtance between two pins, when they are much farther off from one another.

For if the ſpace between the two pins be ſuppoſed to ſubtend an angle of 1°, and each of the pins to ſubtend an angle of 4″, which, by art. 163, 164, is greater than the leaſt angle the eye can diſtinguiſh, it is manifeſt, that the eye may judge of the place of each pin within 2″ at the moſt, and conſequently the error committed in taking the angle between them cannot at the moſt exceed 4″, provided the inſtrument be ſufficiently exact.

178. And yet upon the like miſtake was founded the principal objection of Dr. Hook, againſt the accuracy of the cœleſtial obſervations made by the famous Hevelius.

179. By art. 149, a black ſpot upon a white ground, or a white ſpot upon a black ground, can hardly be perceived by the generality of eyes, when it ſubtends a leſs angle than one minute.

And if two black ſpots be made upon white paper with a ſpace between them equal in breadth to one of their diameters, that ſpace is not to be diſtinguiſhed, even within the limits of *Perfect Viſion*, under ſo ſmall an angle, as a ſingle ſpot of the ſame ſize can be diſtinguiſhed. To ſee the two ſpots diſtinct, therefore, the breadth of the ſpace between them muſt ſubtend an angle of more than a minute. It would be very difficult to make this experiment accurately within the limits of *Perfect Viſion*, becauſe the objects muſt be extremely ſmall; but by a rude tryal with ſquare bits of white paper placed upon a black ground, I judge that the leaſt angle, under which the interval between the two objects can be perceived, is at leaſt a quarter part greater than the leaſt angle under which a ſingle object can be perceived. So that an eye, which cannot perceive a ſingle object under

under a leſs angle than one minute, will not perceive the interval between two ſuch objects under a leſs angle than 75″.

180. Without the limits of *Perfect Viſion*, the diſtance at which a ſingle object ceaſes to be perceivable, will be much greater in proportion, than the diſtance at which a ſpace of equal breadth between two ſuch objects ceaſes to be perceivable.

For without theſe limits the image of each of the objects will be attended with a *penumbra*; and the *penumbræ* of the two near objects will take up part of the ſpace between them, and thereby render that ſpace leſs perceivable; but the *penumbra* will add to the breadth of the ſingle object, and will thereby make it more perceivable, unleſs its image be very faint.

Hevelius defended againſt Dr Hooke.

181. Dr. Hook aſſerts, that the ſharpeſt eye cannot diſtinguiſh the interval between two ſtars, that are leſs than half a minute aſunder, and that not one eye in an hundred can diſtinguiſh an interval of leſs than a minute.

If we ſuppoſe therefore that Hevelius, whoſe eye undoubtedly was very good, as appears from his obſervations, and particularly from Dr. * Halley's teſtimony, could diſtinguiſh two ſtars that were one minute aſunder, it follows, from art. 64, that the *falſe image* of a ſtar to Hevelius's eye did not exceed half a minute.

But if 40″ be ſuppoſed the leaſt angle in a compound object, that Hevelius's eye could perceive in caſe of *Perfect Viſion*; then in order for him to perceive an interval between two ſtars that were 1′, or 60″ aſunder, 40″ at leaſt of that interval muſt have been clear of the two *falſe images* of the ſtars, ſo that the *falſe image* of each ſtar could at the moſt take up no more than 10″ of the interval between them. Conſequently, a ſingle ſtar to Hevelius's eye could appear only under an angle of 20″.

182. And from other conſiderations it is not improbable, that his eye muſt have been fitted to ſee a ſtar under ſo ſmall an angle as 20″, at the time of his controverſy with Dr. Hook. For this noble Aſtronomer was then far advanced in years, and his eye muſt thereby have been ſomewhat flattened; and by a continued courſe of obſervations for near fifty years, it muſt have been as much adapted for viewing celeſtial objects diſtinctly, as practice and habit could make it.

183. Let us therefore conſider, what error in obſerving either the altitude of a ſtar, or the interval of two diſtant ſtars, muſt neceſſarily be committed by ſuch an eye; ſuppoſing the inſtrument he obſerved with to be perfectly exact.

And here we are firſt to take notice, that the ſights he made uſe of next his eye, were narrow rectangular ſlits ſo placed upon his inſtrument, that the length of the ſlit was always perpendicular to the plane of the angle he was to obſerve, and the breadth of the ſlit was in the ſame plane with that angle. So that by this means the eye received a ſufficient quantity of light along the ſlit to render the ſtar viſible, while at the ſame time that apparent diameter of the ſtar, which lay in the plane of the angle intended to be meaſured, was conſiderably leſſened. On which account, if not by reaſon of his age alone, we may reaſonably ſuppoſe a ſtar appeared to him under no greater angle than 20″, but rather leſs.

Let AOB, Fig. 56, be the altitude of the ſtar to be obſerved, whoſe center is C, and whoſe apparent vertical diameter is Aa, ſubtending an angle of 20″.

Here it is manifeſt, that if, inſtead of aiming at C, the center of the ſtar, or the middle of its light, he directed his ſight to the lower edge of the light at a, or to the upper edge of the light at A, he could err but 10″ either way, and that the greateſt difference between two obſervations of the ſame ſtar, could not exceed 20″.

But as he would undoubtedly aim at the middle of the light as near as he could, and we cannot eaſily ſuppoſe him to deviate from the middle more than half the radius of the ſtar, he could hardly commit an error of more than 5″ either way.

184. Let Cc, Fig. 57, be the interval between the centers of two ſtars C, c; and let COc be the angle which that interval ſubtends at the eye of the obſerver, and which Hevelius was to meaſure by his inſtrument; alſo, let Aa, Bb repreſent the apparent diameters of the two ſtars in the plane of the angle to be meaſured, each of them ſubtending an angle of 20″.

* Hevelii Annus Climactericus.

Then

Then if the axis of one observer's eye were directed to *c* the center of the star *Bcb*, along the line *Oc*; and the axis of the other observer's eye were directed to *C* the center of the star *ACa*, along the line *OC*; it is plain that the angle found by the instrument, must be the very angle *cOC* subtended by *Cc* the interval between the centers of the two stars, without any error.

Also, if the eye of one observer were directed to *b*, the right hand extremity of the star *Bcb*; and the eye of the other observer were directed to *a*, the right hand extremity of the star *ACa*; the angle *bOa*, found by the instrument, must be equal to the angle *cOC*, without any error.

So likewise, if the eye of each observer were directed to the left hand extremities of the two stars, *B* and *A*; the angle *BOA*, found by the instrument, must be precisely equal to the angle *cOC*, without any error.

185. And if the two observers were so careless, and withal so unhappy, as to direct their eyes to the two remotest extremities of the two stars, *b* and *A*, the angle thereby found *bOA*, would exceed the true angle *cOC*, by an error of 20″, and no more.

And just the same error would arise if they took their aim at the two nearest extremities of the stars, *B* and *a*, the angle being then only 20″ two little.

186. But this could harldly happen, especially to so experienced an observer as HEVELIUS, who would undoubtedly direct his line of collismation *Oc* to the middle of the light of the star, as near as he could: and it can scarce be supposed that he could deviate from *c* the center of the star *Bcb*, more than half the distance *cb*, or *cB*, which could alter the place of the star no more than 5″. And if the same mistake were committed the contrary way in the place of the other star *ACa*, this could increase or diminish the interval between them by no more than 10″, supposing that interval to have been taken no more than once.

187. But by repeated observations, and taking a *medium* between them, as was his practice, he must settle the interval very near the truth; so that scarce any single observation could differ from that *medium* more than 10″, and the rest must generally approach the *medium* within 5″ or 6″, as appears to have been the case, both by comparing his observations one with another, and by the testimony given him by Dr HALLEY, who affirms that his observations made by plain sights did not differ from the truth, *nisi contemnendâ minuti parte*. Therefore the observations of this illustrious Astronomer could not differ from the truth, 1, 2 or 3 minutes, nor was the largeness of his instruments, and the accuracy of their divisions, a needless charge, or vain curiosity, much exceeding what nicety the eye could observe to, as was injuriously suggested by the learned Dr HOOK.

188. To what we have thought fit to say in vindication of this noble and renowned Astronomer, whose memory will always be honoured by ingenuous minds, it may perhaps be objected, that the eyes of those assistants, who observed with him, might possibly not be so good as his own; and consequently the error committed in taking the distances of two stars, might be considerably greater than what we here suppose.

We answer, that one of those assistant observers was his Lady, probably not much younger than himself, and much accustomed to observation, as were likewise his other assistants: and if some of these were younger persons, and consequently might see a star under a greater angle than 20″, by the bare eye, yet in looking through his sights the angle might little exceed that magnitude: at least, as they were trained up by him and used to observation, they could hardly deviate from the middle of the light so much as one half of the radius, as we have above supposed. And our liberality in this concession may very well compensate for the somewhat greater angle under which they might see the star. To which we may add, that any error an assistant observer could commit, must affect the angle on one side only, and could not be doubled, as was above supposed. And upon the whole, the near agreement of his observations one with another, is a sufficient proof, that his assistants were capable of observing with sufficient accuracy.

189. Mr. *Huygens* in his *Systema Saturnium*, among many curious points, which with his usual sagacity and penetration he has happily discussed, to the satisfaction of the Learned World, Outer edge of Saturn's ring why invisible.

World, proposes a very difficult [a] question, Why the outer edge of *Saturn*'s ring is not visible in the form of *brachia*, when the earth is in the plane of the ring, *Saturn* then appearing round.

This he finds no other way of solving, than by supposing that this outer edge is of such a nature, or is covered with such a matter, as does not at all, or very little reflect the light that falls upon it from the Sun: and in this supposition he has been followed by some later Philosophers.

190. Now, it has indeed been observed by Mr. [b] *Huygens* himself, as well as by signor *Cassini* and some of our *English* Astronomers, that the outer part of the planes of the ring is less luminous than the inner part; and therefore it is not unlikely, that the outer edge of the ring may also be less luminous than the inner and brighter part of those planes, or even than the body of *Saturn*, which is something darker, at least towards the outside, than the inner and more lucid part of those planes.

But that this outer edge should be of so different a nature from those parts of the upper and under plane, between which it lyes, and to which it is immediately contiguous, as that those should reflect the light in the manner they are found to do, and this should reflect it very little, or not at all, is difficult to suppose.

191. It may therefore be worth while to try, whether this question can be answered any other way, without admitting so hard a supposition.

Mr. *Huygens* supposes the thickness of the ring to be about 600 *German* miles, and as he makes the diameter of *Saturn* equal to 15 diameters of our Earth, which is about 2000 *German* miles over, this thickness of the ring must by his estimate be abont $\frac{1}{50}$ part of the diameter of *Saturn*.

Now the apparent diameter of *Saturn*, by the observations of the late accurate Astronomer, Mr. *James Pound*, is at his mean distance from the earth, 18''; therefore at his least distance it must be 20'', and consequently the greatest apparent thickness of the ring must be estimated at $\frac{1}{50}$ part of 20'', when the earth is in the plane of the ring.

And if the telescope used by Mr. *Huygens* in these observations did, as he supposes, magnify an hundred times, the apparent thickness of the ring through that telescope must have been at the most $\frac{20}{50}$'' × 100, that is 40''.

Now we conceive, that in such a compound body as *Saturn* appearing under an angle of 20'' × 100, or of 33', that is, about the size of the full Moon, with two *brachia* proceeding from him of 40'' in apparent breadth, those *brachia* cannot be very distinguishable, even though they were to reflect the light as strongly as the body of the planet; much less, if they are not so luminous as the planet. And less still can those *brachia* be visible, if when the Sun is in the plane of the ring, the Earth be in quadrature with him and *Saturn*, in which case the edge of the ring, being obliquely seen, will subtend an angle at the eye less than 40'' thro' the telescope.

But if the breadth of this edge be much less than $\frac{1}{50}$ part of *Saturn*'s diameter, as will appear probable from what follows; then it must have subtended a much less angle in Mr. *Huygens*'s telescope than 40'', so that the *brachia* must undoubtedly be invisible, and *Saturn* ought to appear round, as he is found to do by observation.

192. It may be here objected, that in the year 1656, Mr. [c] *Huygens* saw the projection of this edge as a dark list upon the body of *Saturn*, at a time when the Earth and Sun were both nearly in the plane of the ring; and consequently that the thickness of the ring is not so small as to render it invisible.

We answer, there is reason to suspect this illustrious Astronomer was mistaken, not in the observation itself, but in the judgment he made upon that observation.

That he saw a dark list traversing the middle of the planet, is not to be doubted: but as a like dark list, or belt, has since been discovered about the middle of *Saturn*, at such time as the Earth is out of the plane of the ring, so that the ring is plainly visible, and the middle of *Saturn* appears within the ellipse; we suspect that this appearance, or such part of it as was most distinguishable thro' his telescope of 23 feet, either alone, or jointly with the darkish line arising from the projection of the edge of the ring upon the body of *Saturn*, might be mistaken for the

[a] System. Saturn. p. 61. [b] Cosmotheoros. p. 110. [c] System. Saturn. p. 16, 17, 61, 62.

projection

projection of that edge alone; which must cause him to conclude the thickness of the ring much greater than it really is.

193. And when the Earth and Sun were not exactly in the plane of the ring, but one was a little elevated above it, and the other a little depressed below it, or both were either a little elevated above it, or a little depressed below it, and yet so little that the plane of the ring was very faintly illuminated by the Sun; in all these cases that plane of the ring, which was exposed to the eye of the observer, would be projected upon the body of *Saturn*, as a narrow dark list, contiguous to the narrow dark list formed by the projection of the outer edge of the ring. And these two narrow dark lists must appear as one, and might be taken by Mr. *Huygens* for the projection of the edge alone, which consequently must have been reckoned much thicker than it really is. To which I must add, that when the Earth and Sun were on opposite sides of the plane of the ring, or when both being on the same side, the Earth was a little more, or a little less out of that plane than the Sun, in both these cases the eye might perceive a small part of the shade cast by the ring upon the body of the planet, which being contiguous to the other two narrow dark lists would appear as one with them, and must increase the error.

194. But it will be farther objected to me, that when *Saturn* appears with *brachia*, or even with *ansæ*, in which case the same side of the ring is turned to the Earth, as is illuminated by the Sun, yet such a dark list has been observed traversing the body of *Saturn* sometimes on the [a] northern part, and sometimes on the [b] southern, but always in such a position as answers to the projection of the ring upon the body of the planet, and consequently arising only from that projection.

I answer, that the outer part of the plane of the ring, which by Art. 190, in all probability is equally dark with the edge of the ring, will be projected upon the body of *Saturn*, contiguous to the projection of that edge, and together with it will form the appearance of one dark list.

To which we may add, that when the eye is any thing less elevated than the Sun above the plane of the ring, some part of the shadow mnst appear contiguous to the two projections abovementioned, and must increase the breadth of the list.

Consequently, what was observed in this case by Mr. *Huygens*, was not the projection of the edge alone, but the united projection of that edge and of the outer and darker part of the plane of the ring, to which sometimes a part of the shadow might be added: So that the ring may be much less in thickness than $\frac{1}{50}$ part of *Saturn*'s diameter, and might subtend a much less angle than 40″, when magnified an hundred times through the telescope.

195. *Galileo*, in his *Nuncius Sidereus*, proposes an objection made by a great number of learned Persons, against his new discovery, that the surface of the Moon was not even, but full of high mountains in all the brighter parts of it.

Moon's edge why even, and not ragged.

If, say they, the limb of the Moon, which is generally very bright, be full of mountains, why does it appear even and smooth thro' the telescope, and not ragged like the teeth of a saw, or a wheel in clock-work?

To which that most accute and ingenious Philosopher replies, that one range of hills only upon the very edge of the Moon might create such an appearance as they supposed; but that several ranges of hills being placed one behind another, the tops of the nearer hills must be projected by the eye into the apertures or vallies between the hills of the outer ranges, and consequently must cause the limb of the Moon to appear even, as it is found by observation.

196. To this it may be added, that if there were only one range of hills on the edge of the Moon, and the height of those hills were about $\frac{1}{1000}$ part of the Moon's diameter, as he supposes in another place; then, the Moon's apparent diameter being about 30′, or 32′, the perpendicular height of those hills, which alone is here to be considered, would be presented to the naked eye under an angle of $\frac{3}{100}$ or $\frac{32}{1000}$ of a minute, and thro' his telescope magnifying about 30 times, would be presented under an angle of $\frac{9}{10}$ or $\frac{96}{100}$ of a minute, which in such a compound object could by art. 173, hardly be discerned. Much less could the smaller inequalities,

[a] Syst. Saturn. p. 10, 11. [b] Ibid. p. 18, 21, 24.

 which

which must be formed by several ranges of hills, be visible thro' his or a much longer telescope.

Circle of dissipation not uniformly luminous.

197. In accounting for the *phænomena* of indistinct Vision above recited, we have made use of only this single principle, that the rays of a pencil are not accurately collected into a point upon the *retina*, but occupy a circular space thereon, which we have called the *circle of dissipation*. This circle we have hitherto treated of, as if it were uniformly luminous, or as if the rays of the pencil were equally and uniformly spread over the whole circle. But in reality the fact is otherwise: the rays are not evenly disposed all over this circle, but are denser in some parts of it than in others. And though in many cases, and indeed in most of the *phænomena* above related, this inequality of the density of the rays is not very considerable, nor occasions any great change in the appearance, yet there are some few of them that will be remarkably affected by it; and there are some other very uncommon and surprizing appearances, that are not to be accounted for from the common laws of Opticks, and depend wholly upon this inequality of the rays in different parts of the *circle of dissipation*.

A line indistinctly seen appears double.

198. Monf. *De la Hire* * takes notice, that to shortsighted persons, narrow objects at a distance, as the black lines for the hours upon a white ground in a large sun-dial, appear double. This *phænomenon*, he observes, is one of the most difficult to account for; and finding no way of solving it by the common and known make of the eye, he is reduced to suppose, that in persons, who see this appearance, the hinder surface of the crystalline humour is made after a peculiar manner, so that a perpendicular section thro' it resembles the conchoide of *Nicomedes*.

But this is certainly a very hard supposition, and does not appear to have any foundation in nature, no crystalline of such a make having ever yet been observed.

And if we should even allow of this licence, which is so often taken by ingenious men, of framing an *hypothesis* intirely out of their own brain, and without the least appearance of foundation in the nature of things, in order to account for such particular *phænomena* as they want to solve; yet this supposition of Monf. *De la Hire* will be found utterly insufficient to answer the different cases of this *phænomenon*.

199. For a long narrow object, as one of the abovementioned lines upon a sun-dial, or a sign-iron, or a pole on the top of a house or scaffolding, or a narrow distant spire, when seen against the sky-light, or the top of a broader spire, or a narrow interval between two chimnies, or one horn of the new Moon, or the whole illuminated part of the new Moon when very narrow, will appear double not only to shortsighted persons, but to such also as are longsighted, provided they apply a convex glass to the eye, so as to put themselves into the case of a person naturally shortsighted. Consequently this appearance is not occasioned by any particular and unusual form of the crystalline in shortsighted persons.

Appears triple, quadruple, &c.

200. But these objects will not only appear double, which is the case Monf. *De la Hire* wanted to account for. They will sometimes appear triple, quadruple, quintuple, *&c.*

201. And the same appearances will occur, not only when the object is at too great a distance for *Distinct Vision*, but also when it is too near for *Distinct Vision*. Let any person whatsoever, whether shortsighted or longsighted it matters not, look upon a narrow black line drawn upon white paper, and held too near his eye to see the line distinctly, and he will sometimes see the appearance of two, sometimes of three or more dark lines, separated by whitish lines.

202. This tryal will succeed still better, if instead of a black line upon paper, a fine needle be held very near the eye, and viewed against the sky-light, against a window, or against white paper strongly illuminated. For the needle will appear as two, three, four, five, or more different needles, especially towards the point.

203. But the best way I have met with of trying this experiment, is as follows. Take a parallel rule, and opening it to a small aperture, hold it directly before your eye, so as to look at the sky-light thro' the aperture. And let the distance you hold it at, at first, be the nearest distance at which you can see it distinctly. At that distance the aperture will appear as one luminous line. But if you

* Traitè des differens accidens de la vuë. pag. 243.

bring

bring the parallel rule nearer to your eye, the aperture will appear double, or as two luminous lines, with a dark line between them. And accordingly as you vary the aperture, or the distance from your eye, you will see the appearance not only of two, three, four, five, *&c.* luminous and dark parallel lines, alternately, but of a greater number than you can count, especially when you look thro' the aperture at the flame of a candle.

204. The parallel rule opened to a small aperture affords the best means of trying the experiment beyond the bounds of *Distinct Vision* likewise, by setting it in a window against the sky-light, and viewing it thro' a convex glass, if you are longsighted, or with the bare eye, if shortsighted. For then you will have all the same appearances as in the preceding article.

Other phænomena of a like nature.

205. Likewise when a broad luminous or white body is seen against a dark ground, as a piece of white paper, the tallow or wax in a candle next the flame, or the lower part of the flame itself, or when the sky-light is looked at contiguous to the dark edge of a building, and the object is either too near or too far off for *Distinct Vision*, the apparent edge of the luminous body will be bounded by a dark line, without which will be a luminous line parallel to it; and sometimes there will appear two or more such dark and luminous lines alternately, like so many fringes on the outside of the luminous body.

206. And when a dark body is seen against a light ground, as the edge of a building against the sky-light, or a flat rule against white paper, or against the sky-light, and the object is too near or too remote to be distinctly seen, the dark body will appear fringed with luminous and dark edges alternately. And all those appearances may be made to happen to any eye, by using a concave or convex glass as circumstances shall require. But the experiment will succeed best, when the light of the luminous body, or that against which the dark body is seen, is neither very strong, nor very weak.

207. As, I suppose, it will readily be allowed me, that the appearances mentioned in the eight preceding articles, are only different cases, or varieties, of one and the same *phænomenon* diversified by different circumstances, and must arise from one and the same cause; and that the *hypothesis* of Mons. *De la Hire* is utterly insufficient for their solution, since there is no eye whatsoever to which they may not all be made to happen, at least by means of a convex glass for one, or concave glass for another, I shall proceed to lay down what I apprehend to be the cause of them, and that not an imaginary or fictitious cause, but such an one whose existence has been demonstrably proved, long before those appearances were taken notice of.

Caused by the Fits of easy refraction and reflexion of light.

208. Sir * ISAAC NEWTON has demonstrated, that the rays of light are not, in all parts of their progress, in the same disposition to be transmitted from one transparent medium into another, but that sometimes a ray, which is transmitted thro' the surface of the second medium, would be reflected back from that same surface, if the ray had a very little farther to go before it fell upon that surface. This change of disposition in the rays of light to be either transmitted by refraction, or to be reflected by the surface of a transparent medium, he calls their *Fits of easy Refraction* and *Fits of easy Reflexion:* and he proves, that these Fits succeed each other alternately at very small intervals in the progress of the rays.

209. Thus if *A*, Fig. 58, be a luminous point, or center of a pencil of rays, which issuing from the point *A* fall upon *BaD* a refracting surface interceding two pellucid mediums of different density; and the ray *Aa*, perpendicular to that surface, be in a *Fit of easy refraction*, when it arrives at the point *a*; it will pass on and be transmitted thro' the medium *BDF*. And if the ray *Ab*, whose passage from the point *A* to *b* is a little longer than that of the ray *Aa* from *A* to *a*, be in a *Fit of easy Reflexion*, when it arrives at the point *b*; it will not be transmitted thro' the medium *BDF*, but will be reflected back again into the medium *BDA*. And if we suppose *Ab* to be the nearest ray to *Aa*, that is in a *Fit of easy reflexion*; and *Ac* the nearest ray to *Ab*, which is in a *Fit of easy refraction*; and *Ad* the next ray which is in a *Fit of easy reflexion*; and *Ae* the next ray which is in a *Fit of easy refraction*; and *Af* the next ray which is in a *Fit of easy re-*

* Opt. Book II.

flexion: Then all the rays between *a* and *b* will be transmitted thro' the medium *BDF*; all the rays between *b* and *c* will be reflected back; all the rays between *c* and *d* will be transmitted; all the rays between *d* and *e* will be reflected back; and all the rays between *e* and *f* will be transmitted. And thus we may imagine a great number of parcels of rays, which will be transmitted or reflected alternately by the surface *B a D*.

210. Now if all the parcels of rays transmitted thro' the surface *B a D* be accurately collected into a point, or focus, there will no other consequence arise from the other parcels of rays having been reflected backwards, than that this focus will be less luminous than it would have been, had every parcel of rays been transmitted to it.

211. But if the parcels of transmitted rays be received upon a plane *MN*, before they arrive at their focus, *F*, this plane will be distinguished alternately into luminous and dark spaces, luminous where the parcels of transmitted rays fall upon it, and dark where the other parcels of rays ought to have fallen, had they not been reflected backwards at the surface *BaD*, as is manifest from Fig. 58 and 59, in which last the middle circle is surrounded by several rings luminous and dark alternately. And it is to be remarked, that those parcels of rays which fall upon the right hand side of the surface *BaD*, do likewise fall upon the right hand side of the plane *MN*, and constitute the right hand side of the rings.

212. If the transmitted rays meet with the plane *mn*, Fig. 60, after they have passed the focus, the effect will be just the same as in the preceding article, except only that the parcels of rays, which fall upon the right hand side of the surface *BaD*, will now, after decussating in the focus, fall upon the left side of the plane *mn*, and will constitute the left side of the rings in Fig. 61, as is manifest from inspection of the figures.

213. Here it is to be remarked, that the middle part of the image, in the four last figures, will not always be luminous as is there represented; but will sometimes be dark, as in Fig. 62, that is, when the midddlemost rays about *Aa* are in a *Fit of easy reflexion* upon their arrival at the surface *BaD*.

214. As these effects of the *Fits of easy refraction* and *easy reflexion* of the rays of light must happen at all refracting surfaces whatsoever, it is manifest that, when a pencil of light falls upon the *cornea*, some parcels of its rays will be transmitted into the eye, while other parcels are reflected back into the air; and consequently that the picture of a luminous point, if it were received upon a plane placed before the crystalline, would consist of a middle circle, surrounded with rings dark and luminous alternately, as in Fig. 59, 61, 62.

215. But the transmitted rays, which constitute the luminous rings, will, upon their arrival at the anterior surface of the crystalline, be some of them in *Fits of easy refraction*, and some of them in *Fits of easy reflexion*, that is, some of them will be transmitted, and others will be reflected back. The consequence of this will be, that the luminous rings arising from the refraction at the the *cornea*, will now be subdivided into narrower rings dark and luminous alternately; and some of these narrower lucid rings will fall upon the spaces possessed by the dark rings caused by the reflexion of the *cornea*, and will subdivide them likewise into dark and lucid rings alternately; and such would be the picture of a lucid point, when received upon a plane within the crystalline.

216. But when the transmitted rays arrive at the hinder surface of the crystalline, those lucid and dark rings will again be subdivided, as at the anterior surface, by which means the dark or luminous rings will sometimes be rendred smaller and more in number than before, and sometimes by the junction of new dark rings to the former dark rings, or of new lucid rings to the former lucid rings, those rings will be rendred larger and reduced to a lesser number.

Such therefore must be the image of a lucid point upon the *retina*, when that point is either too near, or too remote to be seen by *Distinct Vision*.

217. As the interval between a *Fit of easy transmission* and the next *Fit of easy reflexion* is exceeding small, being in air but about $\frac{1}{180000}$ part of an inch, and in water but about the $\frac{1}{240000}$ part of an inch, it is obvious that the least motion of the body, or of the eye, or of any of the parts within it, must make a change in these rings, making dark parts lucid, and lucid parts dark, whence must arise an infinite variety in the number, magnitude and order of the rings.

218. From

Appearance of a lucid point by indistinct Vision.

218. From what has been ſaid, it plainly appears, that when a lucid point is too far off, or too near, for *Perfect Viſion*, its image upon the *retina*, will be diverſified with light and dark rings alternately, after the manner repreſented in Fig. 59, 61, 62, or, the *circle of diſſipation*, which we have hitherto conſidered as uniformly luminous, will not be ſo, but will be alternately divided into light and dark rings, ſometimes more, ſometimes fewer, ſometimes broader, ſometimes narrower.

219. When any parcel of theſe rings alternately light and dark, are ſo very narrow and cloſe together, as that they cannot ſingly be perceived, they will all appear as one ring, which our ſenſe will judge of as light or dark, according as the lucid rings in that parcel for number and breadth either exceed or fall ſhort of the rings adjacent.

220. It may be proved, that when rays iſſue from ſo diſtant a point as to unite in the eye before the *retina*, their denſity in the ſeveral lucid rings decreaſes all the way from the center to the circumference, and conſequently that the outer lucid rings are leſs luminous than the inner ones.

Therefore the middle part of the image of a ſtar will be the ſtrongeſt, and conſequently may be viſible when ſurrounded with ſuch a light as effaces the weaker appearance of the outermoſt lucid rings.

Stars appear ſmalleſt by day-light.

For this reaſon ſtars muſt appear ſmaller while day-light continues, than they do at night. For the day-light effacing their outer lucid rings, deprives them of their *crines ſeu fulgores adſcititii*, as *Galileo* [a] calls them, or the *radii adventitii* as they are termed by Mr. *Horrox*. [b]

So likewiſe a ſtar appearing within the Moon's edge, muſt have the outer part of its image effaced by the ſtronger light of the Moon, and conſequently the ſtar thus leſſened may appear at ſome diſtance within the limb, as was obſerved by Monſ. *De la Hire*, Art. 69.

Why ſtars twinkle.

221. If by Art. 213, the middle part of the image of a ſtar be changed from light to dark, and the adjacent ring be at the ſame time changed from dark to light, as muſt happen from the leaſt motion of the eye towards or from the ſtar, this will occaſion ſuch an appearance as we call the ſcintillation or twinkling of the ſtars.

Why radiate.

222. And if the axis of the eye do not continue directly and ſteddily pointed at the ſtar, but have the leaſt nutation to the right or left, or upwards or downwards, this will diſturb the uniform appearance, by cauſing the light to project out from the lucid rings into the darſt rings, whereby the continuity of the rings will be broken. And if theſe nutations to different parts do ſwiftly ſucceed one another, this will make the light ſeem to project out different ways at the ſame time, that is, it will occaſion what we call the radiation of a ſtar.

223. In our former articles about the fixed ſtars from Art. 61 to 69, a correction is to be made by leſſening the *faint falſe image*, probably about one half, or at leaſt one third for moſt eyes; excepting only the caſe where two ſtars are near enough for part of their *faint falſe images* to coincide: for there, an outer lucid ring of one ſtar, tho' too weak to affect the eye ſingly, yet by coinciding with the oppoſite lucid ring of the other ſtar, may become ſtrong enough to be perceived.

Proportion of refracted and reflected rays near the perpendicular.

224. When light paſſes out of one tranſparent medium into another of a different refraction, and the rays are partly tranſmitted and partly reflected back at the ſurface of the ſecond medium; at incidences near the perpendicular the quantity of the tranſmitted rays will much exceed the quantity of reflected ones.

Upon a table I placed two candles of equal height and burning with equal brightneſs, at equal diſtances from a ſheet of white paper: and ſet a book upright between one of the candles and the paper, in ſuch manner as to cut off the light of that candle from one half of the paper, while the other half was illuminated by both candles. By this means one half of the paper was nearly twice as luminous as the other half. Then I took a flat, thin piece of clear glaſs, and held it in ſuch manner, as to look perpendicularly thro' it at the more luminous half of the paper with one eye, the other being ſhut, and to ſee the darker half of the paper with the ſame eye beſide the glaſs. In doing thus, I obſerved that the more luminous half of the paper, ſeen perpendicularly thro' the glaſs, appeared much brighter than the other half of the paper ſeen with the bare eye on the ſide of the glaſs. Conſequently leſs than half the

[a] Nuntius Sidereus, pag. 23. [b] Venus in ſole viſa.

light

light was reflected back from the two ſurfaces of the glaſs, and more than half was tranſmitted thro' thoſe two ſurfaces to the eye.

Taking then ſuch another piece of glaſs, I held it parallel to the firſt piece, ſo as to look thro' them both at the more luminous half of the paper, which ſtill appeared conſiderably whiter than the other half of the paper ſeen with the bare eye beſide the glaſſes; ſo that leſs than half the light was reflected back from the two glaſſes.

In looking thro' three ſuch glaſſes in the ſame manner, the more luminous half of the paper had now ſomething of a greeniſh caſt, the glaſſes not being of a perfect clearneſs; but ſtill that half appeared full as light, if not lighter, than the other half of the paper ſeen with the bare eye beſide the glaſſes. Conſequently not more than half the light was reflected back from the three glaſſes put together.

Now if we ſuppoſe a certain part of the incident light to be reflected back from the firſt ſurface of the firſt glaſs, and the remainder of the light to be tranſmitted thro' that firſt ſurface and to fall upon the ſecond ſurface; and a proportionable part of that remaining light to be reflected back at the ſecond ſurface, and the reſt to be tranſmitted to the firſt ſurface of the ſecond glaſs; and a proportionable part of the remaining light to be reflected back at its incidence upon that ſurface, and upon every one of the three following ſurfaces of the glaſſes; and one half of the whole light to be reflected back from the ſix ſurfaces put together, and the remaining half to be tranſmitted thro' the laſt ſurface to the eye: Then calling the whole light 1, and the part reflected back at the firſt ſurface, x, we ſhall have $x = \frac{\sqrt[6]{2}-1}{\sqrt[6]{2}} = 0,1091$, that is, about $\frac{11}{100}$ of the incident light is reflected back at the firſt ſurface, and about $\frac{89}{100}$ is tranſmitted thro' that ſurface.

At oblique inciden-ces.

225. As the incident rays deviate more and more from the perpendicular, the proportion between the quantity of reflected rays and the quantity of rays tranſmitted will increaſe more and more: but leſs than half the quantity of the incident rays will be reflected, and more than half will be tranſmitted thro' ſubſtances tolerably clear, unleſs the angle of incidence be very great.

In the experiment of the laſt article, if the ſingle glaſs, inſtead of being perpendicular to the axis of the eye, be gradually more and more inclined to that axis, the brighter half of the paper ſeen thro' the glaſs will gradually appear leſs and leſs luminous, and will more and more approach to the ſame degree of whiteneſs with the other half of the paper, which is illuminated with one candle only and viewed with the bare eye. And at ſome certain degree of obliquity the one half of the paper ſeen thro' the glaſs, and the other half of the paper ſeen with the bare eye will appear equally luminous: But even then only about $\frac{29}{100}$ of the incident light is reflected back from the firſt ſurface, x being equal to $\frac{\sqrt{2}-1}{\sqrt{2}}$; and about $\frac{21}{100}$ is reflected back from the ſecond ſurface. And this, I think, does not happen, till the angle of incidence is about 70 or 80 degrees.

By this article the lucid parts near the center in Fig. 59, 61, 62, will bear a greater proportion to their contiguous dark parts, than the lucid parts near the circumference to the dark parts contiguous to them.

Obſervations on the Fits of eaſy refraction and reflexion.

226. Here it may be of uſe, not only for the better underſtanding of what is to follow, but for the fuller comprehenſion of this curious doctrine of the *Fits of eaſy refraction* and *eaſy reflexion*, to lay down two or three obſervations, which though not explicitely delivered by Sir ISAAC NEWTON, yet may eaſily be deduced from his Theory, eſpecially when compared with the two preceding articles.

1. When a ray of light is ſaid to be in the *Fit of eaſy refraction*, it is not meant, that the ray muſt neceſſarily be tranſmitted thro' every pellucid medium whatſoever, and at any whatſoever obliquity of incidence upon the ſurface of that medium; but only that the ray is more eaſily tranſmitted, and more difficulty reflected, when in that Fit, than when it is in the *Fit of eaſy reflexion*. And when a ray is ſaid to be in the *Fit of eaſy reflexion*, it is not meant, that the ray muſt neceſſarily be reflected back from every pellucid medium whatſoever, and at any whatſoever obliquity of incidence upon the ſurface of that medium; but only that the ray is more eaſily reflected, and more difficulty tranſmitted, when in that Fit, than in the *Fit*

Fit of eaſy refraction. For this reaſon theſe Fits are not called, abſolutely, Fits of refraction, and Fits of reflexion, but Fits of *eaſy* refraction, and Fits of *eaſy* reflexion.

2. During the time, or during the part of the progreſs of a ray of light, that the ray is in the *Fit of eaſy refraction*, or in the *Fit of eaſy reflexion*, it is not conſtantly in juſt the ſame diſpoſition to be refracted, or to be reflected: But there is one inſtant of time, or one point in the progreſs of the ray, at which it is in the middle of the *Fit of eaſy refraction*, and is then the moſt diſpoſed to be refracted, and the leaſt diſpoſed to be reflected; and from that inſtant, or that point, it becomes gradually leſs diſpoſed to be refracted, and more diſpoſed to be reflected, till another inſtant, or point, at which the ray is in the middle of the *Fit of eaſy reflexion*, and is then the leaſt diſpoſed to be refracted, and the moſt diſpoſed to be reflected; and from this ſecond inſtant, or point, it becomes gradually more diſpoſed to be refracted, and leſs diſpoſed to be reflected, till it arrives at a third inſtant, or point, like the firſt, when the ray is again in the middle of the *Fit of eaſy refraction*, or is the moſt diſpoſed to be refracted, and the leaſt diſpoſed to be reflected.

To explain this, let the right line *AAAA*, Fig. 63, repreſent part of the progreſs of a ray of light, in which line ſeveral points as *A*, *A*, *A*, *A*, are taken at equal diſtances from each other, and other points *E*, *E*, *E*, are taken at equal diſtances from each other, and from the points *A*,*A*,*A*,*A*; and let us ſuppoſe that in every one of the points *A*,*A*,*A*,*A*, the ray is moſt ſtrongly diſpoſed to be refracted, and in every one of the points *E*,*E*,*E*, it is moſt ſtrongly diſpoſed to be reflected; that is, in each point *A* let the ray be in the middle of the *Fit of eaſy refraction*, and in each point *E* let it be in the middle of the *Fit of eaſy reflexion*.

Then will the diſtance *AA* repreſent the interval between the middle points of two ſubſequent *Fits of eaſy refraction*, *EE* the interval between the middle points of two ſubſequent *Fits of eaſy reflexion*, and *AE* the interval between the middle point of a *Fit of eaſy refraction* and the middle point of the ſubſequent *Fit of eaſy reflexion*.

And the diſpoſition of the ray to be refracted, being ſtrongeſt in the middle of the *Fit of eaſy refraction*, or in the point *A*, and weakeſt in the point *E*, will by degrees diminiſh all the way from *A* to *E*; and will again by the ſame degrees increaſe all the way from *E* to *A*. And the diſpoſition of the ray to be reflected, being ſtrongeſt in the middle of the *Fit of eaſy reflexion*, or in the point *E*, and weakeſt in the point *A*, will gradually diminiſh all the way from *E* to *A*, and will again gradually increaſe all the way from *A* to *E*.

3. That part of the progreſs, in which a ray is in the *Fit of eaſy refraction*, is longer or ſhorter, as the medium it falls upon has a leſs or greater refractive power, and its ſurface is leſs or more obliquely ſituated with regard to the progreſs of the ray: And that part of the progreſs, in which the ray is in the *Fit of eaſy reflexion*, is longer or ſhorter, as the medium the ray falls upon, has a greater or leſs refractive power, and its ſurface is more or leſs obliquely ſituated with regard to the progreſs of the ray.

In every interval *AE* let a point be taken as *a*, ſo that the ray falling upon a medium of a given refracting power, and with a given obliquity, in the point *a*, ſhall be tranſmitted thro' that medium; but would have been reflected back, had it fallen upon the medium in any point between *a* and *E*. Then will the ray be in the *Fit of eaſy refraction* with regard to that medium and that obliquity of incidence, throughout the ſpace *a A a*; and will be in the *Fit of eaſy reflexion* throughout the ſpace *a E a*; ſo that the beginning of the *Fit of eaſy refraction* will happen in *a*, the middle in *A*, and the end in the ſubſequent *a*; and the beginning of the *Fit of eaſy reflexion* will happen in this laſt point *a*, the middle in *E*, and the end in the following point *a*: or the interval *AE* will be ſo divided in the point *a*, that throughout the ſpace *Aa* the ray will be in the *Fit of eaſy refraction*, and throughout the ſpace *aE* it will be in the *Fit of eaſy reflexion*.

But if the refracting power of the medium be leſſened, or if its obliquity to the progreſs of the rays be leſſened, the ray will be refracted, tho' it meet the ſurface of the medium below the point *a*, nearer to *E*; or the point *a*, at which the *Fit of eaſy refraction* ends and the *Fit of eaſy reflexion* begins, muſt be removed nearer to *E*, that is, *Aa* will be longer, and *aE* will be ſhorter, or

aAa the part of the progreſs, in which the ray is in the *Fit of eaſy refraction*, will be longer, and *aEa* the part of the progreſs, in which the ray is in the *Fit of eaſy reflexion*, will be ſhorter.

And juſt the contrary will happen, if the refracting power of the medium, or its obliquity to the progreſs of the rays be increaſed, the point *a* then being removed nearer to *A*, whereby *Aa* or *aAa* is made ſhorter, and *aE* or *aEa* is made longer.

Intervals of thoſe fits how divided.

227. If parallel rays, moving with an uniform velocity, fall upon a plane ſurface of a tranſparent medium with ſuch obliquity, that one half of the rays are tranſmitted thro' that ſurface, and the other half are reflected back; every ray muſt be in the *Fit of eaſy refraction* thro' one half of its progreſs, and in the *Fit of eaſy reflexion* thro' the other half of its progreſs.

For, ſince by the ſuppoſition the rays move with an uniform velocity, there muſt be as many rays in any one point of their reſpective intervals *AE* between the *Fits of eaſy refraction* and *reflexion*, as in any other point of the ſame intervals. Conſequently, the number of rays in the ſpace *Aa* muſt be to the number of rays in the ſpace *aE*, as *Aa* to *aE*. But ſince half the rays are refracted and half reflected, the number of rays in the ſpace *Aa* is equal to the number of rays in the ſpace *aE*: therefore *Aa* is equal to *aE*, that is, the rays are in the *Fit of eaſy refraction* through one half of their progreſs, and in the *Fit of eaſy reflexion* through the other half of their progreſs.

228. But if the obliquity of the plane ſurface be ſo much leſſened, as that three fourths of the incident rays are now tranſmitted thro' it, and only one quarter of them reflected, then every ray muſt be in the *Fit of eaſy refraction*, with regard to this obliquity of the plane ſurface, through three fourths of its progreſs, and in the *Fit of eaſy reflexion* through one quarter of its progreſs. That is, the interval *AE*, between the two points of eaſieſt refraction and eaſieſt reflexion, muſt be ſo divided in *a*, that *Aa* may be three fourths of that interval, and *aE* one fourth.

229. And in general, if the interval *AE* repreſent the whole quantity of the incident light, *Aa* the quantity of the refracted light, and *aE* the quantity of reflected light; *Aa* will be that part of the interval, in which every ray is in the *Fit of eaſy refraction*; and *aE* that part of the interval, in which every ray is in the *Fit of eaſy reflexion*.

In theſe three articles we have ſuppoſed the rays of light to move with an uniform velocity: but if they move more ſwiftly when in the *Fit of eaſy refraction*, and more ſlowly when in the *Fit of eaſy reflexion*, as Sir ISAAC NEWTON ſeems to have thought; we muſt then imagine the line *AAAA*, inſtead of the progreſs of a ray of light, to repreſent the time of that progreſs, *A* the inſtant of time when the ray is moſt ſtrongly diſpoſed to be refracted, *E* the inſtant when it is moſt ſtrongly diſpoſed to be reflected, *AE* the interval of time between thoſe two inſtants, and *a* the inſtant when the ray paſſes out of the *Fit of eaſy refraction* into the *Fit of eaſy reflexion*, and the conſequences will be juſt the ſame.

230. It follows from the preceding article, jointly with art. 224, that, in the caſe of perpendicular, or nearly perpendicular incidence of light from air upon glaſs, or from glaſs upon air, a ray of light is in the *Fit of eaſy refraction* thro' $\frac{89}{100}$ of the interval *AE*, and in the *Fit of eaſy reflexion* thro' no more than $\frac{11}{100}$ of that interval.

231. If two mediums differ leſs in their refractive power than air and glaſs, a ray of light, in paſſing out of one of thoſe mediums into the other almoſt perpendicularly, will be in the *Fit of eaſy refraction* more than $\frac{89}{100}$ of the interval *AE*, and in the *Fit of eaſy reflexion* leſs than $\frac{11}{100}$ of that interval.

232. Sir ISAAC NEWTON has [a] obſerved, that thoſe ſurfaces of tranſparent bodies reflect the greateſt quantity of light, which have the greateſt refracting power: And till this matter ſhall be more accurately ſettled by experiment, it may probably be ſuppoſed, that the quantity of light reflected back at the ſurface of any tranſparent body is proportional to the refracting power of that body, or to the ſquare of [b] *BR*.

233. Hence we may infer, that the ſquare of *BR*, or the refracting power of glaſs being to that of water, as [c] 1,4450 to 0,7845, and the quantity of light reflected back at a per-

[a] Opticks Book II. Part III. Prop. I. [b] Ibid. Prop. X. [c] Ibid.

pendicular incidence upon glaſs being [a] 0,1091, the quantity of light reflected back at a perpendicular incidence upon water will be 0,0592, or nearly $\frac{6}{100}$ of the incident light. Conſequently, a ray of light falling perpendicularly from air upon water, or from water upon air, will be in the *Fit of eaſy reflexion* thro' only $\frac{6}{100}$ of the interval AE, Fig. 63, and in the *Fit of eaſy refraction* thro' $\frac{94}{100}$ of that interval.

234. Likewiſe, the refraction of the aqueous humour of the eye, or of the *cornea*, being that of 27 to 20, by art. 110, and conſequently its refracting power, or the ſquare of BR being 0,8225, the quantity of light reflected back at a perpendicular incidence upon the *cornea* will be 0,064 of the incident light.

235. Alſo, the refraction out of the aqueous humour into the cryſtalline being that of 13 to 12, by art. 111, and conſequently the refracting power of the cryſtalline humour in this caſe being 0,1736, the quantity of light reflected at a perpendicular incidence from the aqueous humour upon the cryſtalline will be 0,0135 of the incident light. And the ſame proportion of the incident light will be reflected at the hinder ſurface of the cryſtalline, the refraction out of that humour into the vitreous being the ſame as that out of the aqueous humour into the cryſtalline.

236. Hence it follows, that if the ſurface of the *cornea* and the two ſurfaces of the cryſtalline were not ſpherical, but plane, like the piece of glaſs uſed in the preceding [b] experiment, then in looking directly at any object, $\frac{64}{1000}$ parts of the incident light would be reflected back at the *cornea*, and the remainder would be tranſmitted to the anterior ſurface of the cryſtalline; and of this remainder $\frac{135}{10000}$ parts would be reflected back at that firſt ſurface, and the remaining light would be tranſmitted to the ſecond ſurface; where again $\frac{135}{10000}$ parts of the remaining light would be reflected back, and the *reſiduum* would be tranſmitted to the *retina*. So that if the whole light incident upon the *cornea* be called 1, the quantity reflected back at the three ſurfaces put together would be 0,0891, or $\frac{1}{11}$ part nearly, and the quantity tranſmitted to the *retina* would be 0,9109, or about $\frac{10}{11}$.

But as theſe three ſurfaces are not plane, but ſpherical, on which account the greateſt part of the incident rays will fall upon them with ſomewhat greater obliquity, and conſequently a part ſomewhat greater will be reflected, we ſhall not be far from the truth, if we ſuppoſe $\frac{1}{10}$ part to be reflected back at all the three ſurfaces put together, and $\frac{9}{10}$ to be tranſmitted to the *retina*.

How much light loſt in viſion by thoſe fits.

237. From the preceding article jointly with art. 210, we may probably collect, that in looking at an object between the limits of *Perfect Viſion*, it appears leſs luminous by about $\frac{1}{10}$ part, than it would do were all the light incident upon the *cornea* tranſmitted to the *retina*.

238. And much the ſame thing happens in looking at a broad object, although too near or too far off for *Perfect Viſion*. For then every pencil will likewiſe loſe $\frac{1}{10}$ part of its light by the three reflexions abovementioned, and the remaining $\frac{9}{10}$ of the light will be ſcattered into the *circle of diſſipation:* But every pencil within the *falſe image*, art. 18, 72, will receive as much light from the adjoining pencils, as it beſtows upon them by this diſſipation. Therefore the center of every pencil, or, every point of the *falſe image* of the object upon the *retina*, will have juſt as much light thrown upon it, as if the object had been ſeen by *Perfect Viſion*, that is, $\frac{9}{10}$ of the whole incident light.

Appearance of a lucid phyſical line by indiſtinct Viſion.

239. A lucid phyſical line upon a dark ground, when much too near, or much too far off for *Diſtinct Viſion*, will appear ſometimes as three, ſometimes as five, ſeven, or more lucid parallel lines, with two, four, ſix, or more dark parallel lines between them.

For a lucid point, when indiſtinctly ſeen, will by art. 211, 212, 213, exhibit the appearance of a circle ſurrounded with dark and lucid rings alternately, and this muſt happen to every phyſical point in the lucid line.

Let therefore AaA, Fig 64, be a lucid phyſical line, a ſome point in that line from which a pencil of rays flows to the *cornea*. And let a portion of rays in the middle of this pencil be in the *Fit of eaſy refraction* at their incidence upon the *cornea*, and be thereby tranſmitted to the anterior ſurface of the cryſtalline; where let them again be in the *Fit of eaſy refraction*, and be thereby tranſmit-

[a] Art. 224. [b] Ibid.

ted to the hinder surface of the crystalline. And at that hinder surface let the same middle portion of rays be again in the *Fit of easy refraction*, and let the middlemost ray of this portion, *Aa*, Fig. 58, be precisely in the middle of the Fit, or in one of the points *A*, Fig. 63; and let the outermost ray, as *Ab*, or *Aβ*, Fig. 58, be at the end of the Fit, or in one of the points *a*, Fig. 63; and let this portion of rays be thereby transmitted to the *retina*, and form thereon the lucid circle *abβ*, Fig. 64. And let this lucid circle be surrounded, by art. 211, 212, with a dark ring, *bcγβ*, and this again with the lucid ring *cdδγ*, as represented in figure 64; and through the points *b*, *c*, *d*, *β*, *γ*, *δ*, draw the lines *BbB*, *CcC*, &c. parallel to the line *AaA*.

Then I say, instead of the one lucid line *AaA*, will appear three lucid lines *BBBB*, *CDDC*, and ΓΔΔΓ, separated from one another by the darkish lines *BCCB*, and *B*ΓΓ*B*.

For if a like middle portion of rays, in every pencil flowing from the lucid line *AaA*, be supposed to be in the *Fit of easy refraction* at their incidence upon the *cornea*, and upon the anterior and hinder surface of the crystalline, and at this last surface the middlemost ray be precisely in the middle of the Fit, and the outermost rays in the end of the Fit, so as that every point in the line *AaA* may form an image upon the *retina* exactly like that of the point *a*; then the lines *BbB*, *BβB*, must be tangents to all the lucid circles formed by those points: Also, the lines *BbB*, *CcC*, on the one hand, and *BβB*, ΓγΓ, on the other, must be tangents to the first dark ring formed by each of those points; and the lines *CcC*, *DdD* on the one hand, and ΓγΓ, ΔδΔ on the other, must be tangents to the first lucid ring formed by every one of those points.

From which it follows, that, as the space 1*b*11*β*1, which is cut off from the image of the point *a*, by the tangents *BbB*, *BβB*, has more light in proportion to its magnitude, than the space 1*b*12*c*2, which is cut off from the same image by the tangents *BbB*, *CcC*; the whole space comprised between the tangents *BbB*, *BβB*, will be more luminous than the whole space comprised between the tangents *BbB*, *CcC*, that is, the whole space *BbBBβB* will appear as a lucid line, and the whole space *BbBCcC* will appear as a dark line.

In like manner it will be found that the spaces *CcCDdD*, ΓγΓΔδΔ, ought to appear as lucid lines; and the space *BβB*ΓγΓ must appear as a dark line.

In this case therefore, instead of one physical lucid line *AaA*, we shall have the appearance of three lucid lines separated by dark lines parallel to them; and if the lgiht of the point *a* be strong enough to form more lucid rings than are here represented, and the breadth of those lucid rings be not too little to be perceived, art. 174, we may have the appearance of 5, 7, or more lucid lines separated by parallel dark lines.

240. But when the lucid line *AaA* is of any considerable length, it is not possible that in every pencil issuing from that line, things can happen exactly as we have supposed. Although a middle portion of rays in every pencil were to be transmitted through every one of the three surfaces abovementioned to the *retina*, and were to form thereon a lucid circle like *abβ*; yet the middlemost ray cannot in every pencil be precisely in the middle of the *Fit of easy refraction*, at its incidence upon the hinder surface of the crystalline. In some pencils the middlemost ray will not be come to the middle of the Fit, in others it will be past the middle of the Fit, at its incidence upon that surface.

Also, this middle portion of rays, which we have supposed to be wholly transmitted thro' each of the three refracting surfaces, and thereby to form on the *retina* the lucid circle *abβ*, cannot in every pencil be so transmitted. In some pencils a part of the middle portion of rays must necessarily be in the *Fit of easy reflexion*, at its incidence upon one or other of those surfaces, and consequently cannot be transmitted to the *retina*.

We are therefore to consider what alterations in the *phænomenon* will arise from these changes in our suppositions.

241. In those pencils where the middle portion of rays is wholly transmitted to the *retina*, but the middlemost ray is not yet arrived to the middle of the *Fit of easy refraction*, at its incidence upon the hinder surface of the crystalline, it is manifest that the luminous circle *abβ*, will be inlarged and will swell farther outwards, and the dark and luminous rings,

rings, together with the dark and luminous lines arising from them, will by this means be also carried farther outwards.

And in those pencils where, at the time of this incidence, the middlemost ray is past the middle of the *Fit of easy refraction*, the luminous circle $ab\beta$ will be lessened, and by this means the dark and luminous rings surrounding it, and the dark and luminous lines arising from those rings, will be drawn farther inwards.

But as the number of pencils in the one case must be equal to the number of pencils in the other, these two contrary effects will very nearly balance each other, and consequently the *phænomenon* will thereby undergo very little alteration, except only that the edges of the luminous and dark lines, where they entrench upon each other, will thereby be rendred somewhat less distinct than they would otherwise appear.

242. We come next to consider those pencils, where the middle portion of the rays, which we have supposed to form the luminous circle $ab\beta$, are not wholly transmitted to the *retina*; but some part of them, being in the *Fit of easy reflexion* at their incidence upon one or other of the three surfaces abovementioned, are reflected back and never arrive at the *retina*.

And here we are to remember that, by art. 236, hardly more than $\frac{1}{10}$ part of this middle portion of rays, taking all the pencils one with another, can be reflected back by the three surfaces put together.

Now in such pencils, where this tenth part of the rays of the middle portion, were it not reflected back, must have fallen upon the middle part of the luminous circle, there must be a small dark spot in the middle of the image, as *a*, Fig. 65; and the luminous circle will by this means be changed into a luminous ring, $b\beta$, whose breadth will be nearly equal to the radius of the luminous circle in Fig. 64. Consequently, this luminous ring, and the luminous line *BBBB*, will extend farther outwards than in Fig. 64; and the dark and lucid rings, *bc*, *cd*, and the dark and lucid lines arising from those rings will also be carried farther outwards, and will be rendred somewhat narrower.

And in such pencils, where this tenth part of the rays of the middle portion, were it not reflected back, must have fallen upon the outer part of the luminous circle, that circle and the luminous line *BBBB* will thereby be lessened, and the dark and lucid ring surrounding it, and the dark and lucid lines arising from those rings, will be removed farther inwards, and will be rendred something broader, as in Fig. 66.

But as the number of pencils in one of these cases must be nearly equal to the number of pencils in the other; and the effect of the one is contrary to that of the other; the change in the *phænomenon* will be little more, than to cause some farther indistinctness in the edges of the luminous and dark lines.

243. In such pencils where this tenth part of the middle portion of the rays is not all contiguous, but is divided into three or more disjoined parcels, which, had they not been reflected back at the three surfaces, must have fallen upon separate spaces in the luminous circle, it is manifest, that the change in the *phænomenon* will be still less than in the two preceding articles, only the luminous circle, and the lucid line arising from it, will be rendred something less luminous. And the dark ring, and the dark lines arising from it, will be rendred something less dark, in case the rays that tend thereto, be not all reflected, but some parts of them be transmitted.

244. This would be the case of a lucid physical line, such as one might imagine to be composed by a great number of stars disposed in a right line and contiguous to each other. The middle line would be always luminous, so that the number of lucid lines would always be odd, and the number of dark lines between them would always be even.

245. A lucid line upon a dark ground, of a small sensible breadth, but whose *true image* upon the *retina* is not so broad as the *circle of dissipation*, when too near, or too far off for *Distinct Vision*, will sometimes appear as two, sometimes as three, four, five or more lucid parallel lines, with dark parallel lines between them. Appearance of a narrow lucid line by indistinct Vision.

Let *a*, Fig. 67, be a point in the middle of the breadth of the lucid line, and let *e* and *i* be the two extreme points in the breadth of the same line, and let the axis of the eye be directed to the point *a*.

Then will the axis of the pencil of rays flowing from the point *a*, be perpendicular to the *cornea* and to both surfaces of the crystalline humour, and the image of that

point

point upon the *retina* will be agreeable to what is represented in Fig. 59, 61, or 62: And the physical line, which runs thro' the middle of the lucid line, and passes thro' the point *a*, will form an image upon the *retina* like that represented in Fig. 64.

But the axis of the pencil of rays flowing from the point *e*, will fall obliquely upon the *cornea*, and upon both surfaces of the crystalline humour. Consequently, not only the center of the image of the point *e* will fall upon a different part of the *retina* from the center of the image of the point *a*, and within the *circle of dissipation* of the point *a*; but the image of the point *e* will not be similar to the image of the point *a*, so that some lucid parts of one will fall upon some dark parts of the other, and on the contrary. And if a physical line be conceived to pass thro' *e*, parallel to the physical line which passes thro' *a*, some lucid parts of the image of one physical line will fall upon some dark parts of the image of the other. And what we have said of the image of the physical line passing thro' *e*, will be likewise true of the image of the physical line passing thro' the point *i*.

And accordingly as the breadth of the lucid line is greater or less, or these two points *e* and *i* are more or less distant from the point *a*, the dark lines of the images of the physical lines passing thro' those points, may either coincide in the middle of the *retina*, in which case there will appear a dark line in the middle of the image: Or, they may be contiguous only, or else leave a sensible space between them, in either of which cases a lucid line will appear in the middle.

And as the same reasoning will extend to the other points on the inside of *e* and *i*, it is manifest, that the appearance of the whole lucid line may consist of more or fewer, narrower or broader, lucid and dark lines, as in art. 203, 204, with infinite variety.

Other appearances by indistinct Vision.

246. From the same cause, and by a like way of reasoning it will follow, that when a broad lucid surface, as *BBbb*, Fig. 68, is contiguous to a broad dark surface as *BBdd*, and is seen by indistinct vision, the lucid surface will appear bordered with a dark line as *BB*ΓΓ, and that again with a lucid line beyond it; and sometimes there will be two or more of these dark and lucid lines alternately, as was related in art. 205.

247. Or the broad dark surface *BBdd*, contiguous to a broad lucid surface *BBbb*, when seen by indistinct vision, will appear narrower than when seen by *Distinct Vision*, that is, from *BBdd* it will be reduced to ΔΔ*dd*, and the line ΔΔ will be the apparent edge of the dark surface, which will be bordered with the lucid line ΔΔΓΓ, and that again with the dark line ΓΓ*BB*, and sometimes there will appear two or more of these lucid and dark lines alternately, as in art. 206.

248. From which it may easily be conceived, that when the dark surface is very narrow, as *A*, Fig. 69, it may appear divided into two dark lines with a lucid line between them, as *B*; or as three dark lines with two lucid lines between them, as *C*; or there may appear four, five, or more dark lines separated by lucid lines, as was mentioned in art. 199, 200, 201, 202.

But in all these cases it is to be observed, that vision must not be a little only, but greatly indistinct, and that the light must neither be exceeding strong, nor very weak. When the light is too strong, the eye is too much dazzled to perceive the appearance plainly; when too weak, the lucid lines between the dark ones are too faint to be discerned.

Experiments to confirm the Theory.

249. If the account we have given of these appearances, from a cause that was before known really to exist in nature, and which we have shown to be sufficient to produce the *phænomena* above recited, should seem to stand in need of any farther confirmation, it may receive a very strong one from the following experiments.

Let *Aa*, Fig. 70, be a lucid line formed by a narrow aperture of a parallel rule held too near the eye for *Distinct Vision*, and seen against the skylight or against white paper strongly illuminated; and let ΔΔ*DD* be the appearance of that lucid line consisting of three luminous lines and two dark ones; and let ΔΔ*EE* and *DDEE* be the appearance of the two parts of the parallel rule itself. Also, let *FFGG* be a flat rule held near the eye, on the right hand side, and parallel to the edge of the parallel rule.

Then if the flat rule *FFGG* be gradually moved transversly, keeping always its parallel position, as its edge *FF* comes a-cross the edge of the pupil, the apparent edge of the

parallel

parallel rule *DD* will advance to meet it, and will obliterate firſt the lucid line *DDCC*; and as the edge of the flat rule advances by degrees farther and farther a-croſs the pupil, the ſame edge of the parallel rule *DD* will advance farther and farther to meet it, obliterating the dark line *CCBB*, then the lucid line *BBBB*, , then the other dark line *BB*ΓΓ, and laſtly, part of the other lucid line ΓΓΔΔ, of which it will at laſt only leave ſo much as is equal in breadth to the lucid line *Aa*.

250. And if the flat rule be thus moved reciprocally forwards and backwards a-croſs the pupil, the apparent edge of the parallel rule *DD* will likewiſe appear to move reciprocally forwards and backwards, either advancing to meet the edge *FF*, or receding from it, ſometimes obliterating the lucid and dark lines of the image, ſometimes ſuffering them to re-appear; and that half of the parallel rule *DDEE* ſometimes appearing broader, ſometimes narrower. And juſt the reverſe will happen, if the flat rule be placed on the left ſide of the parallel rule, and be drawn the contrary way a-croſs the eye.

251. Now this is preciſely what ought to happen from the theory above laid down.

For, from art. 211, Fig. 58, 59, it is plain, that when an object is ſeen too near for the rays of each pencil to be united upon the *retina*, thoſe rays of every pencil, which fall on the right hand part of the *cornea*, muſt likewiſe fall on the right hand part of the *retina*, and conſequently muſt, from the known laws of viſion, conſtitute the left hand part of the appearance; and on the contrary.

So that, when the edge of the rule, *FF*, is drawn a-croſs the *cornea*, it firſt intercepts thoſe rays of every pencil, which fall the moſt to the right hand upon the *retina*, and which conſtitute the part of the appearance the moſt to the left hand, that is, the lucid line *DDCC*, which being obliterated, the apparent edge of the parallel rule muſt now be *CC*, inſtead of *DD*.

And the edge *FF*, by moving farther along the *cornea* will intercept the parcel of rays from every pencil, which fall next on the right hand part of the *cornea* and *retina*, and which conſtitute the dark line *CCBB*, dark in compariſon of the adjoining lucid lines, but more luminous by means of theſe rays, than the ſides of the parallel rule. And this dark line being obliterated, the apparent edge of the parallel rule will now be advanced from *CC* to *BB*.

And after the ſame manner it is eaſy to conceive, that by the continued motion of the flat rule a-croſs the *cornea*, the luminous line *BBBB*, the dark line *BB*ΓΓ, and the greater part of the luminous line ΓΓΔΔ muſt likewiſe be obliterated.

And at laſt, when the remaining aperture, or uncovered part of the *cornea*, is left very narrow, the edges of the parallel rule and the aperture between them muſt appear diſtinct, juſt as if they were ſeen thro' a narrow ſlit, or a pinhole in a card, and conſequently the lucid line muſt appear of its true and proper magnitude.

252. If the parallel rule be placed at too great a diſtance for *Diſtinct Viſion*, then upon drawing the edge of the flat rule *FF* a-croſs the *cornea*, the right hand ſide of the parallel rule will grow gradually broader, the edge ΔΔ ſeeming to advance the ſame way with *FF*, and thereby obliterating firſt the lucid line ΔΔΓΓ, then the dark line ΓΓ*BB*, the lucid line *BBBB*, the dark line *BBCC*, and laſtly reducing the lucid line *CCDD* to the magnitude of the lucid line *Aa*, when the edges of the two ſides of the parallel rule will appear diſtinct. This is eaſy to conceive from art. 212, and Fig. 60, 61. And juſt the reverſe of this will happen, if the flat rule be placed at firſt on the left ſide, and be drawn the contrary way a-croſs the eye.

253. As in both theſe caſes, when the edge *FF* is reciprocally and quickly moved backwards and forwards, the tremulous motion, art. 250, of the edge *DD*, or ΔΔ, is very conſpicuous, this affords one means of diſcovering when viſion is conſiderably indiſtinct, and thereby of fixing the limits of *Perfect Viſion* within ſome latitude.

254. After the ſame manner, and for the ſame reaſons, when the edge ΔΔ of a dark ſurface, Fig. 68, appears bordered with a lucid line, as ΔΔΓΓ, and a dark line without that, as ΓΓ*BB*, if the eye be too near, then upon placing the edge of a flat rule on the right hand, and drawing it a-croſs the *cornea*, the apparent edge ΔΔ will advance to meet the flat rule, and will thereby obliterate firſt the lucid line, and then the dark one, after which that edge will appear diſtinct in the line *BB*, contiguous to the lucid ſurface *BBbb*.

And

And if the eye be too far off, the flat rule must be placed on the left hand, and then upon drawing it the contrary way a-cross the *cornea*, the edge ΔΔ will advance along with it, obliterating first the lucid and then the dark line as before, till it arrives at the situation *BB*, when the confines of the dark and lucid surface will appear distinct.

255. Also, when a narrow dark object, as *A*, Fig. 69, appears double, as *B*, or treble as *C*, if the edge of the flat rule be moved a-cross the *cornea*, parallel to the image, then if the eye be too near, the opposite dark line will seem to advance towards the rule, obliterating the lucid or dark lines in its way, till it comes into the same situation with the dark line nearest the rule, when it will appear as one only distinct line.

But if the eye be too far off, the dark line nearest to the rule will advance along with it, obliterating the lucid or dark lines in its way, till it comes into the situation of the farthest dark line, when it will appear distinct and as one line only.

256. By the same means of covering part of the pupil with the edge of a flat rule, or with the finger only held parallel to the side of a book, a person who needs spectacles, or another who is short-sighted, may read at such a distance, that the book would otherwise be wholly illegible by the spreading of the *penumbræ* between the letters. For the aperture of the pupil being now lessened, the *circle of dissipation*, and the *penumbræ* arising therefrom between the letters, will be lessened in the same proportion. And if the light be strong enough to bear covering the greatest part of the pupil, then the *penumbræ* above and below the letters will likewise be considerably lessened, which will render the book still more easily legible.

257. A farther proof of the truth of the theory above delivered may be taken from the following experiment.

In looking at a lucid or dark line, which on account of its being too near, or too remote from the eye to be seen distinctly, exhibits the *phænomenon* of Fig. 69, or 70, move your head gradually sideways to the right hand or the left, that the rays from the object, which fall upon the *cornea*, may vary both the length of their passage and likewise the obliquity of their incidence, and consequently may vary their *Fits of easy refraction* and *easy reflexion*.

Then you will see the lucid and dark lines of the image perpetually changing their places, their magnitude, and their strength, and seeming to roll over one another, as they ought to do by this theory.

An extraordinary appearance.

258. This subject has unexpectedly carried me out to so great a length, that I should now think it high time to conclude, were it not proper to advertise the curious reader of an appearance, that in making these experiments may possibly mislead him, as at first it did me.

In looking at the confines of a dark and a lucid surface seen by indistinct vision, there will sometimes appear on the border of the lucid surface, without the utmost dark line of the image, or to the right hand of *BB*, Fig. 68, a line of some breadth much more luminous than the rest of the lucid surface. And on the border of the dark surface, within the utmost lucid line of the image, or to the left hand of ΔΔ, will appear a line of some breadth much darker than the rest of that surface.

This appearance I at first imagined to proceed from the same cause, as the other *phænomena* recited in the preceding articles, namely the vicissitude of the *Fits of easy refraction* and *easy reflexion* in the rays of light; but finding no way of accounting satisfactorily for it from that cause, I suspected some mistake, and therefore applied myself to consider that appearance more attentively.

Not owing to the Fits of easy refraction and reflexion.

259. Upon white paper I drew a rectangle as *ABCD*, Fig. 71, which with ink I made very black; then viewing it at too small a distance for *Distinct Vision*, I saw it bordered on all sides by two luminous and two dark lines alternately. But after looking attentively upon it for a little time, I perceived another luminous border without the utmost dark perimeter *abcd*, which new border was much brighter than the rest of the white paper.

Now, upon turning my eye a little towards that part of the new luminous border, which lay without the dark line *cd*, I found that border to increase in breadth, and to grow brighter than before. And in like manner, when I directed my eye to the new luminous border without the utmost dark line *ab*, or *ad*, or *cb*, I found the respective luminous borders to increase in breadth and in the strength of their light.

This

This put me upon turning my eye from the black rectangle to a distant part of the white paper, when immediately a luminous rectangle much brighter than the rest of the paper, and of the same dimensions nearly with the black rectangle, was presented to my eye, and continued for some time, after which it gradually vanished.

But to a different cause.

260. This occasioned me to reflect upon some like appearances I had met with before, either from my own observation, or the relation of some of my acquaintance, and to judge that both the one and the other must proceed from a different cause than what I had hitherto supposed. But for farther confirmation, I put on a pair of spectacles, by means of which the black rectangle appeared very distinct, and free from the alternate lucid and dark edges that had appeared before.

But after viewing the rectangle attentively for some time, such a luminous border as is above described, brighter than the rest of the paper, began to appear round it; and to which side soever of the rectangle I directed my eyes, the luminous border on that side appeared brighter and broader than before. Then turning my eyes quite away from the black rectangle to a distant part of the white paper, a bright rectangle, of the same size with the dark one, immediately appeared and continued for some time, after which it gradually vanished away.

261. I then drew two other black rectangles upon white paper, of the same height and breadth, and parallel to each other, with a narrow white space between them, as represented in Fig. 72.

Then viewing them attentively for some time, till the bright luminous border began to appear, I suddenly transferred my eyes to a distant part of the white paper, where immediately I perceived the appearance of a dark space between two bright rectangles, of equal magnitude with the white space and the two dark rectangles, respectively, in Fig. 72.

And as this equally happened, whether the object was seen by indistinct vision with the bare eyes, or by *Distinct Vision* through the spectacles, it was now manifest, both from this and the experiment of the preceding article, that this new appearance was no way owing to the indistinctness of the sight, or to the *Fits of easy refraction* and *reflexion*, but to another cause, which the intelligent reader may by this time guess at, and which will more plainly appear when I have related the other observations of myself and friends, which I hinted at in art. 260.

Other appearances of a like nature.

262. A person sitting to be shaved against a light sash window, fixed his eyes intently upon the window for some time, and afterwards shutting them, he had now the appearance of a window similar to that he saw before: only the glass panes were dark, and the wood between them was luminous.

263. Another fixed his eyes for some time upon the end of his pen black with ink, and held against white paper; and upon suddenly moving his pen, without stirring his eyes, he saw the appearance of a bright luminous end of a pen upon the same part of the paper. And since I was told of this appearance, I have often made the same observation.

264. In looking at the setting sun, in a coach upon the road, when the light is not so strong as much to offend the sight, if the eyes be suddenly directed to a distant part of the sky, we have the appearance of one, two, three or more dark circles of about the same size with the sun, upon different parts of the sky.

265. A person considering the reason of the vast length of the image of the setting sun seen by reflexion from the *Thames*, looked so long upon it, that when he took off his eye, every thing appeared to him with a long dark beam upon it for above a quarter of an hour.

266. Another, after looking too long at the Sun eclipsed, saw a dark spot upon every thing for several months after.

The general reason of these appearances.

267. These, and many more *phænomena* of a like kind, seem to depend upon this principle, that when we have been for some time affected with one sensation, as soon as we cease to be so affected, a contrary sensation is apt to arise in us, sometimes of itself, and sometimes from such causes, as at another time would not produce that sensation at all, or at least not to the same degree.

Every body knows, that the sudden cessation of intense pain after some continuance, is immediately followed with sensible pleasure.

In coming out of a strong light, into a room with the window-shutters almost closed, we have immediately a sensation of darkness; and this continues much longer than the pupil requires to dilate and accomodate itself to

that weak degree of light, which is almost instantly done.

But after staying some time in a much darker place, the same room, which appeared dark before, will be sufficiently light.

In coming out of a cold bath, the intense cold is immediately succeeded by a glowing heat. But this, as well as the effects recited in art. 265, 266, is partly owing to other causes than the bare ceasing of the contrary sensation.

If we drink a dish of coffee, or thea, without sugar, and afterwards taste of another dish with but little sugar in it, this last will appear very sweet.

But if we first eat any thing very sweet, a dish of coffee or thea moderately sweetened, will taste very bitter; and it were easy to bring a great number of like examples.

268. But such a change of sensation is not always general, so as that the whole organ of one of our senses is affected by it: it is oftentimes partial, so that from one and the same cause, one part of the organ undergoes a change of sensation, while the other part perceives no change.

For instance, the skin of our bodies, to speak loosely, may be considered as the whole organ of feeling, with respect to the heat and cold of the air we are environed with, or of the water we may happen to be immersed in: And the whole *retina* may be considered as the organ of sight.

But when we are in a moderate degree of warmth, a gentle, mild breeze of air may not affect our faces, which yet would strike us with a sense of chillness, if our naked bodies were exposed to it.

And a tepid bath, as that of *Buxton*, may feel neither warm nor cold to our hands, and yet upon immersion may feel cold to our bodies.

269. And this sensation may not only be partial, affecting one part of the organ, and not at all affecting the other: but one part of the organ may be affected one way, while another part is the contrary way affected by the same cause.

Thus the hand may be in such a temper, that upon dipping it into the bath of *Buxton*, the water shall feel warm: but upon immersion of the whole body, the same water shall feel cold.

And the air will sometimes feel warm to our faces, that would feel cold to our naked bodies.

270. We are therefore not to wonder, that either the whole *retina*, or that any part of it, after having been intensely affected with one sensation for some space of time, should upon the cessation of the cause that so affected it, immediately be affected with a contrary sensation. It is natural to expect, that those parts of the *retina*, which have for some time been impressed with a bright image, as those abovementioned of the setting Sun, the panes of glass, or the white space between two black rectangles, should upon the cessation of what so affected them, be affected with a sense of darkness, that is, that a dark image, of the same magnitude, should appear in the same place as the bright one had done before. And if a very bright image had, by the jolting of a coach, been transferred to several parts of the *retina* in a short space of time, several dark images ought afterwards to appear, one for every one of those parts of the *retina*.

271. And those parts of the *retina*, that have been for some time taken up with a dark image, as in the case of the black rectangles, the wood in the window-frame, or the black end of a pen, while the neighbouring parts have been occupied by bright images, that is, those parts which, one may say, have been in the dark, while the neighbouring parts have been in the light, will now, as soon as that inequality ceases, be affected with the contrary sensation; and this will cause the appearance of the bright images, similar and equal to the dark ones those parts were before possessed with.

272. Hence it is a necessary consequence, that after viewing intensely the black rectangle, art. 259, 260, for some time, upon the least unavoidable nutation of the head, some part of the *retina*, which had been occupied by the dark and faint lucid lines without the image of that black rectangle, would now be impressed by the light of the white paper, and this sudden transition from darkness to light would cause the light upon that part of the *retina*, to appear stronger than upon the other patrs, which had all along received the light of the white paper, that is, a luminous border would appear, brighter than the rest of the paper.

273. And when the eye was a little diverted to one side, in order to consider more attentively this luminous border, by this means

means a part of the *retina*, which was before poſſeſſed with part of the image of the black rectangle itſelf, would now be impreſſed with the light of the white paper, and by having undergone a greater degree of darkneſs than the former part, it would be affected with a ſtill ſtronger ſenſation of light, that is, the luminous border would not only appear broader, but ſtronger than before.

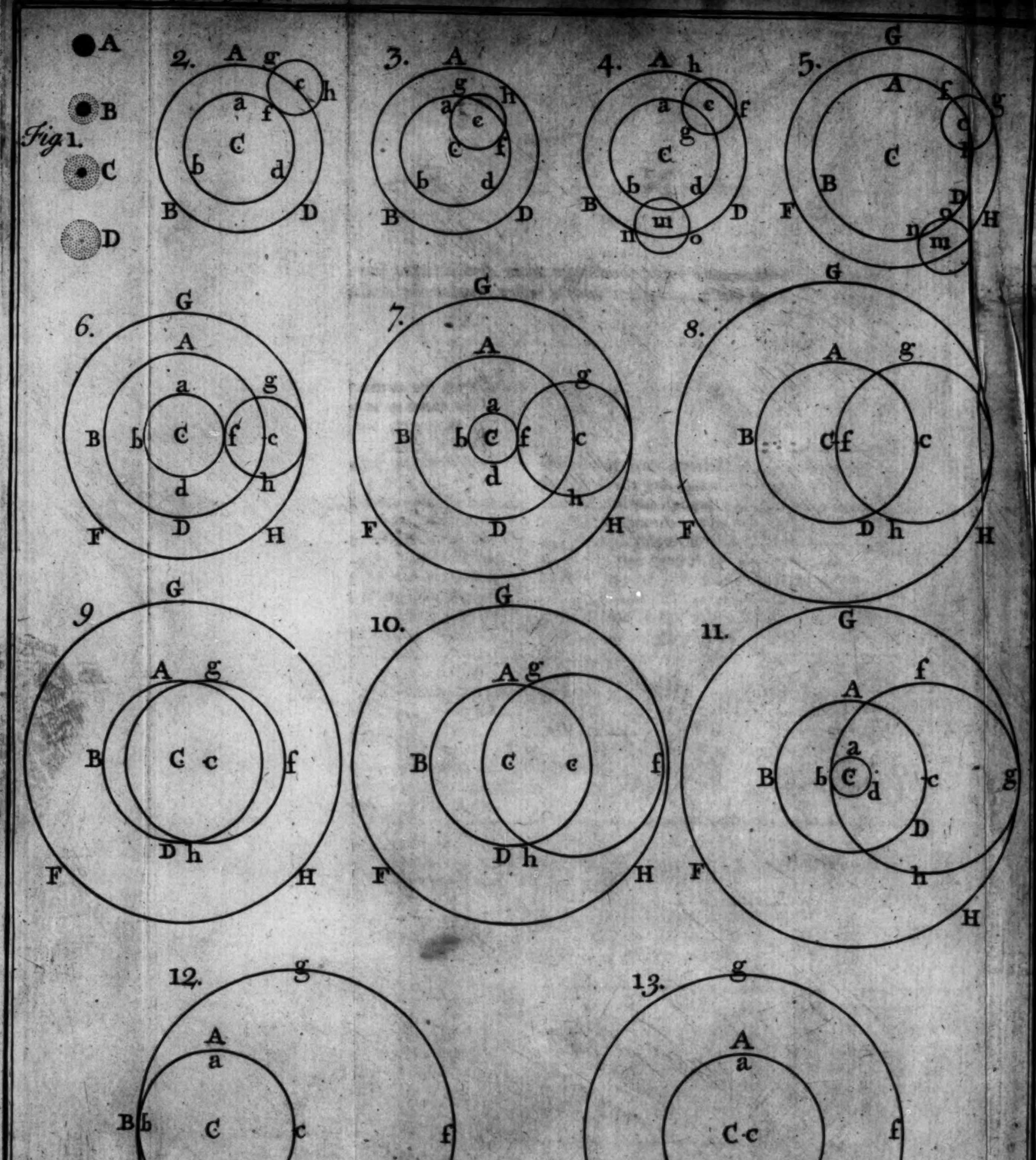
274. It will be ſo eaſy to apply what I have here ſaid of the luminous border, to the dark border mentioned in art. 258, that I ſhall ſpend no more time about it: and indeed I ſhould hardly have been thus particular about the luminous border, but that I cannot help ſuſpecting, that even the Great Sir ISAAC NEWTON himſelf was once led into a miſtake by a like appearance. At leaſt, I do not find, he has any where accounted for the extraordinary ſtrength of the ring of light, next the central black ſpot, ſpoken of in Obſ. 23. Part. I. of his ſecond Book of Opticks, nor do I ſee any way of accounting for it, but from ſome ſuch conſideration.

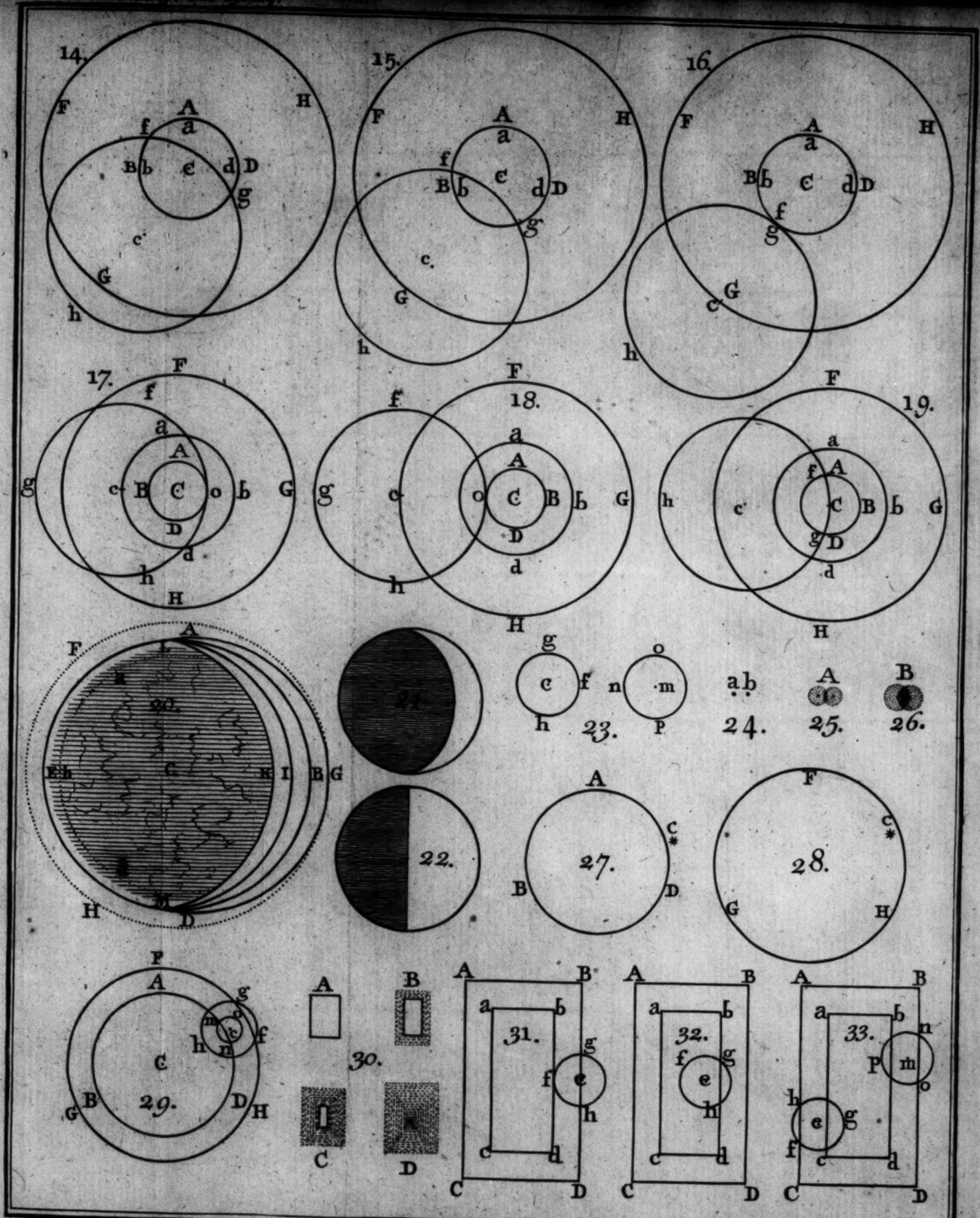
FINIS.

I N D E X.

The numbers with no letter before them refer to the Articles in the four Books, with r *before them to the Articles of the Remarks, and with* e *before them to the Articles of the Essay at the end of the Work.*

Plate 17. end of the Essay.

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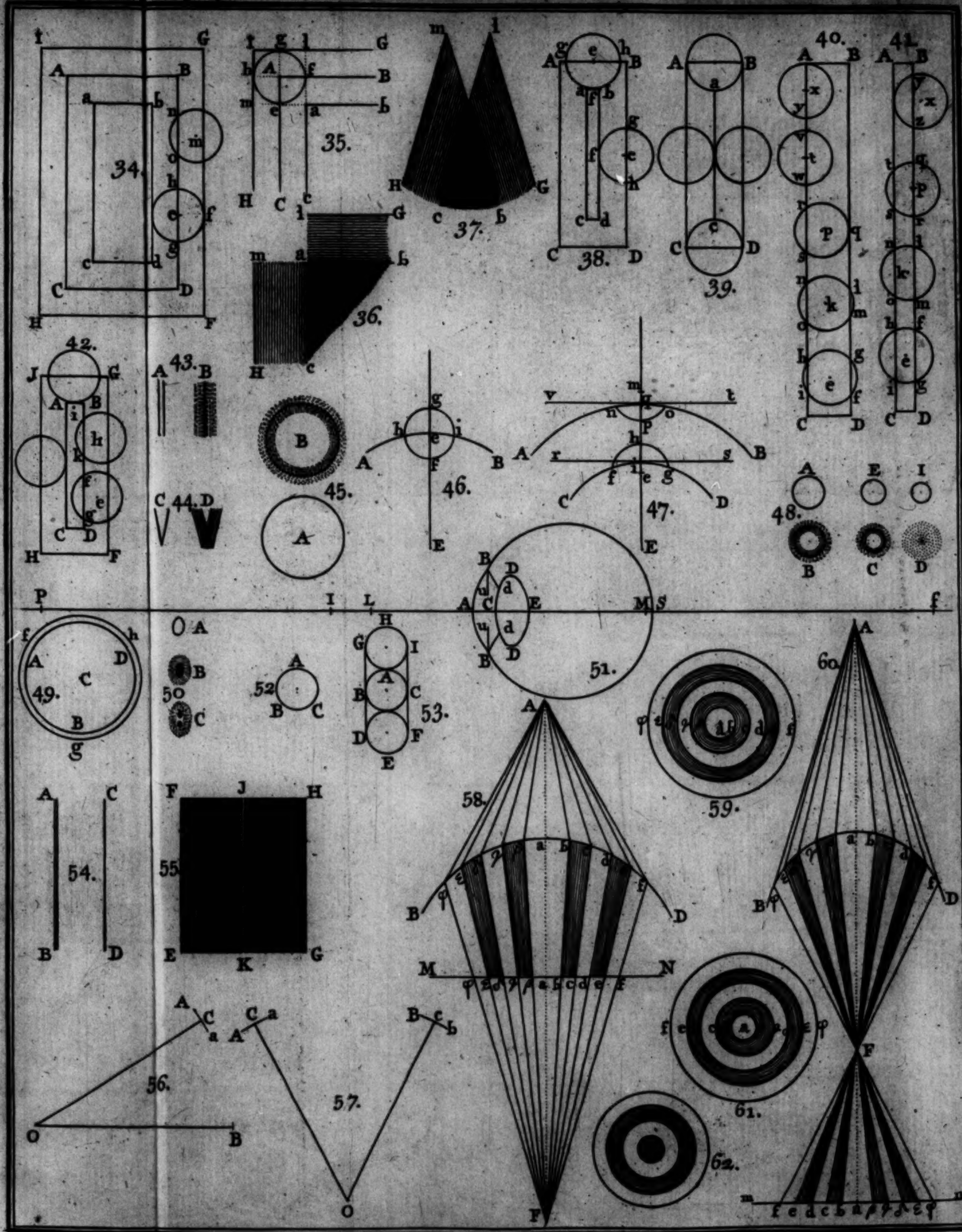
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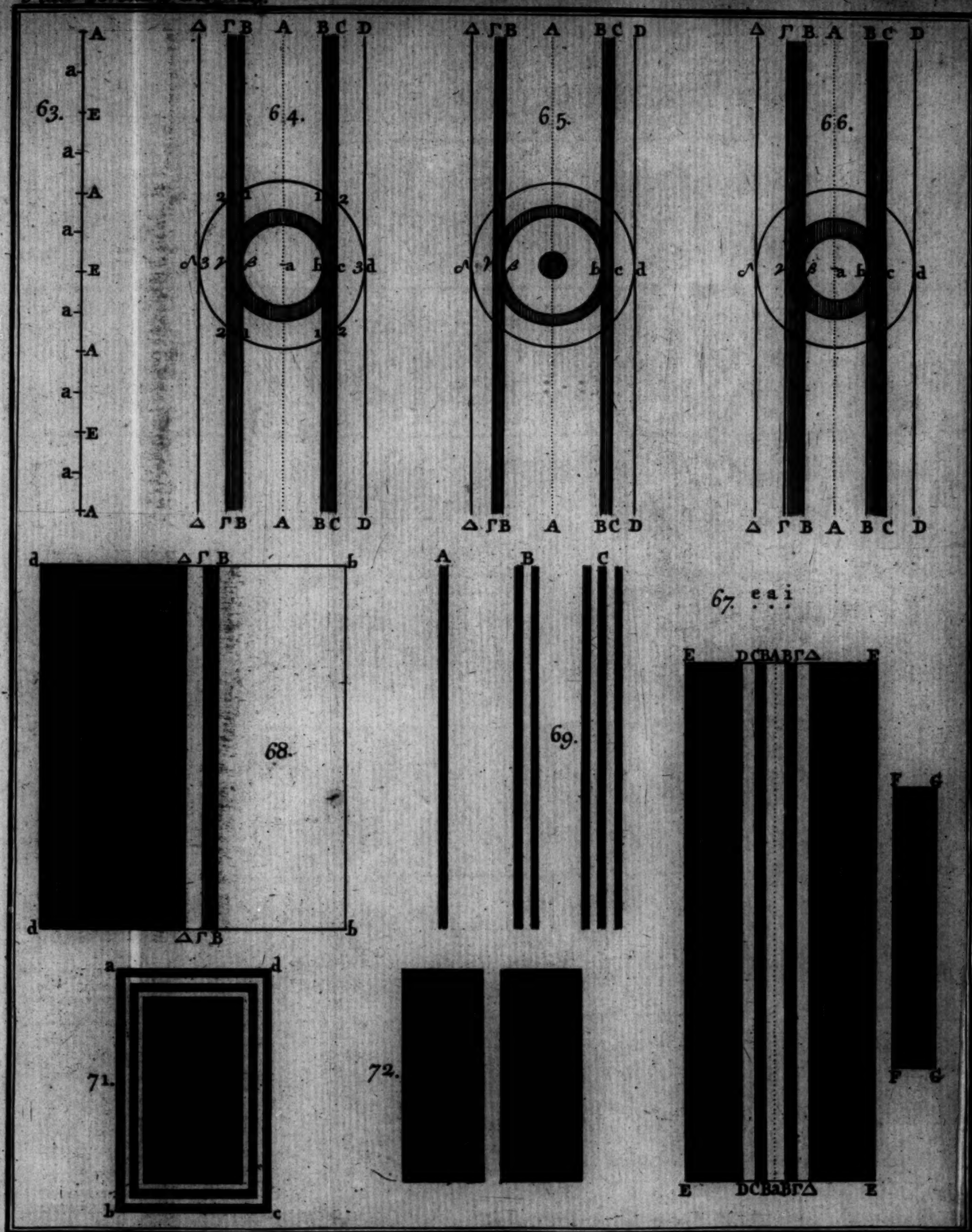
Plate 20. end of the Essay.



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ERRATA

ERRATA *in the four Books.*

Article

48. LINE 2, *read* focus F in convex glaſſes, but towards it in concaves, the *&c.* l. 9, r. conjugate or correſponding.
54. in the margin at c, r. Art. 52.
66. l. 3 from the end, r. deſcription.
81. l. 4 from the end, r. make.
95. l. 13 from the end, *dele* perhaps *and* r. ſome hundreds of miles, by Remark 384.
132. at the bottom of the page, r. *Locke*'s Eſſay B. 2. c. 9.
135. l. 3 from the end, r. ſignified.
138. l. 24, r. ſenſible of the diſtance when *&c.* l. 5 from the end, r. between the pupils.
142. l. 8 from the end, *dele* Fig. 207 in the margin.
144. l. 4, *for* Oϖ r. Ox. l. 2 from the end, r. perpendicular from A upon.
156. l. 3 from the end, *dele* in going.
160. l. 10 from the bottom of page 61, r. than we ſhould do if it were ſmaller. l. 1 of pag. 62, *dele* becauſe——compariſon to it.
182. l. 7 from the bottom of page 84, r. reſplendent.
188. l. 5, *for* reflected r. refracted.
200. in the margin, *write* Fig. 292, 293.
215. l. 7, r. the focus q.
236. l. 6, r. Ef equal to EF in the lens or ſphere, but equal to CF in the ſingle ſurface, ſay *&c.* line 11, r. ſemidiameters EF and Ef.
272. l. 3 from the end, r. remains Oπ—Oϖ.
276. l. 4 from the end, *for* $\overline{DC-d}$, $-$ d *read* $\overline{DC-d}$, $-$ c;
279. l. 16 from the end, in the denominator of the fraction, *read* $1 + \frac{AC}{a} +$ *&c.*
348. l. 1, *dele* and double microſcopes.
364. in the fourth column of the Table, *for* 199 and 398, r. 200 and 400.
380. l. 4, *read* $\frac{Pp}{Pl}$, l. 5, *read* ηXx.
289. l. 4 from the bottom of the page, *read* gZb.
399. l. 10, r. focus's I, K.
422. l. 2, *read* ABE.
432. l. 2, *read* SM, line 3, *read* SN.
465. l. 1 and 3, *read* bH, bH, bH.
485. l. 2, r. refracted ray.
496. in the margin, *add* Fig. 471.
498. in the margin, *for* Art. 179 read *Newton*'s Opticks p. 114. 8vo.
516. line 6, *read* sab Fλ, and l. 12, *read* sa,
540. l. 23, r. never ſo little.
547. l. 4 from the end, *read* GAN.
551. in the margin at [b], *dele* ſee the Remarks on chap. 10.
552. laſt line, r. EF, HK.
565. l. 5, r. paraſelene.
578. at the end, *add* ſo far *Hugenius*.
579. l. 10, *read* in g.
585. l. 7, r. cuts GQ.
589. l. 9, *read* CK, CB, BL.
618. laſt line, in the margin, r. Art. 437.
626. l. 8, in the margin at [b], *dele* 602.
645. l. 10, in the denominator of the laſt fraction, r. Q$-\frac{1}{4}$S.
656. l. 3, *read* $7pp + 4rp + 7rr$, l. 5, r. $7pp - 4rp + 7rr$.
695. l. 13 from the end, *dele* as in fact we find it.
701. l. 4, *read* IKRi.
715. l. 10, *read* $bo : bp$.
785. pag. 304 note [a] l. 4, r. Art. 798.
809. l. 7, r. any point Q.
824. l. 1, r. with a pair.
825. laſt line, *add*, ſo far Mr. *Molyneux*.
829. l. 4 from the end, r. axis cd.
840. l. 7, r. expeditious; l. 11, r. elevation.
850. in the margin, *write* Fig. 590.
852. *dele* the laſt ſix lines. How far, *&c.*
860. l. 5, *for* axis r. bar; l. 7, *for* this axis r. an axis.
877. l. 21, for ib r. ik.
892. l. 4 from the end, *dele* ſame.
911. l. 4, r. for the year 1713, and *dele* but *&c.*
915. l. 4, r. as the like error. See Remark 694.
921. laſt line, *add*, ſo far Mr. *Molyneux*.
945. laſt line, *add*, ſo far Mr. *Hadley*.
969. l. 4 from the end, r. been giving.
973. l. 15, r. the ſide next to.
976. l. 2, r. rings or holes.
977. l. 4, r. outwards, horizontal and; l. 10 and 15, r. apparent points; l. 25, r. each other, upright and.
983. l. 13, *dele* (as you ſee in the 641ſt Figure) and *dele* Fig. 641 in the margin.
1021. l. 4 from the end, *dele* he places the object——and by the eye.
1022. l. 2, *for* would generally cover, r. were nearly equal to.
1077. l. 3, *dele* from thoſe——by *Galileo*.
1090. l. 3, r. acronychal.
1101. r. diſtances of the Satellites 5. 965, 9. 494,

9.494, 15.141, 26.630 from *Newton's* Princip. pag. 391, l. 22
1110. margin, r. spheroidical.
1133. last l. but one, r. visible.
1152. l. 3, r. disappear.
1158. l. 2, r. 1932″.
1160. l. 4, r. 37″, the types sometimes fail in these strokes ″ elsewhere.
1200. l. 8 from the end, r. plate upon a board *kl*.

ERRATA *in the* REMARKS.

Remark.
3. LINE 5, for he, read the.
4. 16, r. Art. 145, 146 and Rem. 506.
8. l. 6, r. that nearly as great.
10. l. 7, r. crystalline.
12. l. 6, r. as must be made in perfect eyes in order to see.
29. l. 13, r. from that of the humour.
31. l. 5, r. into the bottom of the eye. l. 6, *dele* very.
40. l. 9, r. similar to.
45. l. 5 from the end, r. exposed naked to the.
50. l. 5, r. can hardly be caused.
55. l. 1, r. EQ.
79. l. 6, r. Anno 1363.
80. l. 3, r. 14th Century.
82. l. 7 from the end, r. diaphanous.
101. *dele* the last line.
108. l. 8 from the end, r. telescope does.
138. l. 5 from the end, r. *q l*.
141. last line, *add*, after refractions and reflections.
147. last line, *add*, see the Tables in Art. 364, Rem. 482, 684.
152. l. 2 from the end, r. TcS.
156. l. 10, *dele* in.
191. l. 2, r. confirmed.
209. in the margin, *dele* solved.
226. last line but one, *for* decreases r. increases.
229. l. 3 and 4 from the end, *dele* rather.
268. last line but one, r. appears always
291. l. 8 from the end, r. equal to *o*C.
295. last line, *add*, see Rem. 516.
297. l. 11, r. PE, RE.
298. last line, r. EP, ER.
304. last line, *add*, see Rem. 516.
306. last line, *add*, see Rem. 532.
310. last line, r. Art. 614.
323. l. 7, r. black line.
346. l. 4, *for* it r. this point.
370. at [a] in the margin, r. Art. 961.
397. l. 3 from the end, r. 53°, 35′.
417. l. 4, r. as *n* to *m*.
420. l. 22 from the end, r. all times of the day.
465. l. 5, r. age of 34.
507. l. 5, *for* convex r. concave.
610. l. 5, *for* 11, r. 1, 1.

ERRATA *in the* ESSAY.

PAGE 116. col. 2. line 37, r. print.
126. col. 1. l. 5. r. 18″
p. 135. col. 1. line 30, r. makes.

ERRATA *in the* Figures *of the four* BOOKS.

Plate
13. FIG. 176, C wanting where EO cuts the circle.
38. fig. 473, *x* wanting at the end of the line ANX.
41. fig. 503, *for* vexter, r. vertex.
50. fig. 604, β and γ wanting at the slit above the center of the plate EF.
57. fig. 642, the flames of 4 candles, called *ll* in Art. 985, are wanting in the focus of the speculum SS.
59. fig. 649, between *b* and *c* write 1, 2, 3, overagainst the prick'd arches.
60. 61, 62, 63, *read* pag. 409, 425, 433, 456, respectively.

ERRATA *in the* Figures *of the* REMARKS.

Plate
5 FIG. 54, put in *r* as in fig. 56, 57.
56, join A, O and bisect CE in T.
8, fig. 103, for K write γ.
9. fig. 105, place *r* as in fig. 106.
fig. 115, put L at the top of the line MF produced.

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THE

BOOK-BINDER

Is desired to correct the Paging of the 4 last Plates of the first Set, thus:

Plate.		Page.
60	facing	409.
61	facing	425.
62	facing	433.
63	facing	456.

The paging upon the other 20 Plates of the Remarks and Essay is right.

5